Matrix Computation and its applications
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Lecture - 38
Linear map associated with a matrix

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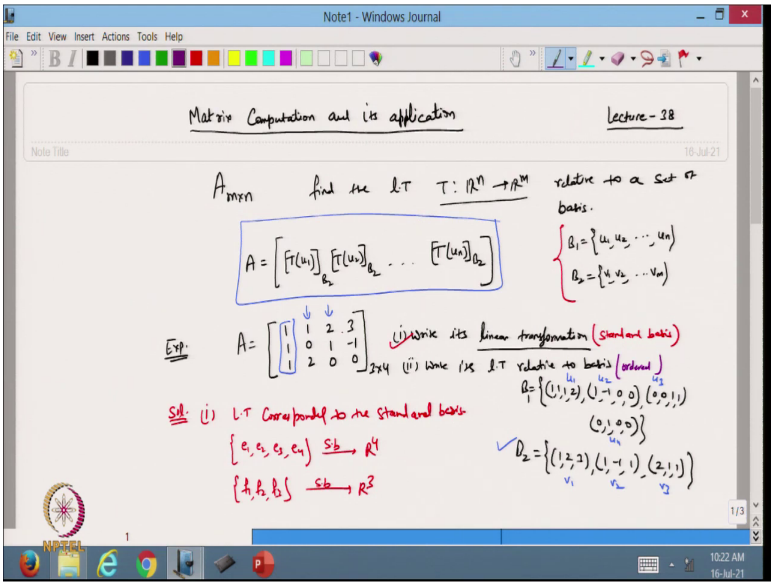

Hello, viewers. Welcome back to the course on Matrix Computation and its Application. So, in the previous lecture we have stated that suppose we have the matrix given matrix and then we want to write the linear transformation corresponding to the matrix and the basis given to us, so, then how we can write that one. So, we have started with that one. So, we will continue those things in this lecture. So, let us start with that.

So, in the previous lecture we know suppose we have a matrix A that is of order m*n and it is being asked to us that define or find the linear transformation that is T: $R^n \rightarrow R^m$ this is what we are writing here because this is the matrix of order m*n. So, we want to find the linear transformation related to the basis. So, this is what we want to do.

So, we started with the example and we also know that from the previous knowledge we know that, that A is always made up of T whatever the transformation is there T of the basis we define and then the coordinate of this one. So, suppose I have a basis let I take the basis $B1=\{u_1, u_2, \ldots, u_n\}$ and $B2 = \{v_1, v_2, \ldots, v_n\}$ then we know that this matrix $A = [[T(u_1)]_{B2}\ [T(u_2)]_{B2}\ \ldots\ [T(u_n)]_{B2}]$.

And, if this basis is not given to us, then it is understood that this corresponds to the standard basis. So, let us do one example. So, I take just one example and then we will come to know that how we can write suppose I have a matrix a that is given to me like

$$A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 1 & -1 \\ 1 & 2 & 0 & 0 \end{bmatrix}$$

So, it is 3*4 matrix. Now, in this matrix if it is asked to write its linear transformation.

So, suppose it is written like this one then it is understood that we are being asked to write the linear transformation corresponding to the standard basis. So, here I can write. So, maybe I can write this: 1st write its linear transformation, 2nd is write its linear transformation are related to the basis. So, this basis is given to me. So, suppose I have a B1={(1, 1, 1, 2), (1, -1, 0, 0),(0, 0, 1, 1) ,(0, 1, 0, 0). So, this is the basis of R^4.

And, the other basis we are talking about is B2 = {(1, 2, 3) ,(1, -1, 1),(2, 1, 1)}. So, this is given to me. Now, first I want to write about its linear transformation. So, it is understood that it is a standard basis. So, if I want to ask the first question, linear transformation corresponds to the standard basis. So, standard basis means that I have my standard basis. So, I can have my $\{e_1, e_2, e_3, e_4\}$ this is the standard basis for R^4 and I write $\{f_1, f_2, f_3\}$ this is the standard basis for R^3. So, standard basis.

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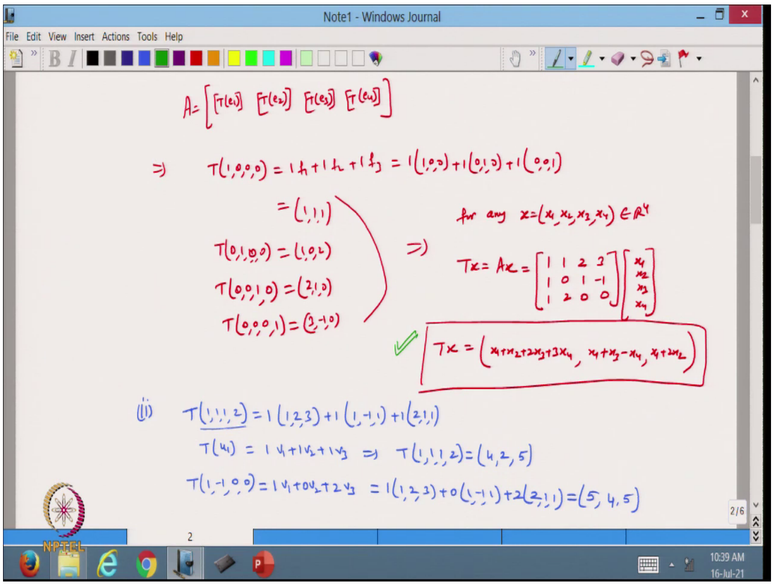

Now, from this for a given matrix, corresponding to the standard basis I know that my matrix A =[[T(e_1)] [T(e_2)] [T(e_3)] [T(e_4)]]. So, this is the matrix. So, from here it means that the first column is 1, 1, 1. So, now, our matrix is there.

So, from here I can write because I know that

T(1, 0, 0, 0)= $1.f_1+1.f_2+1.f_3 = (1,1,1)$

T(0, 1, 0, 0)=(1,0,2)

T(0, 0, 1, 0)=(2,1,0)

T(0, 0, 0, 1)=(3,-1,0)

and from here if you see I can write directly this one as T(x) where so, I can write

⇨ for any x ∈(x_1 , x_2, x_3 , x_4) ∈ R^4

⇨ $$T(x) = Ax = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 1 & -1 \\ 1 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 - x_1 \\ x_3 - x_1 + (\frac{x_1 - x_2}{2}) \\ x_4 - 2x_1 + x_1 - x_2 \end{bmatrix}$$

- $\Rightarrow$ $(x_1 + x_2 + 2x_3 + 3x_4 , x_1 + x_3 - x_4 , x_1 + 2x_2)$

So, corresponding to this matrix I am able to write this transformation. So, this is Tx. So, that is the answer of the first part that this is a linear transformation corresponding to the given matrix related to the standard basis.

Now, things change when we want to do it for this one. Now, the 2nd part I want to do. So, it is given to me that the basis is given to me. So, the corresponding matrix is this one. Now, in this case the same matrix a whatever it is given I want to write on the standard basis. So, this will be equal to T of the first basis, that is (1 1 1 2) ,(1 1 1 2) that can be written as the coordinates of the first image of the first vector in the basis.

So, it is equal to 1 and then corresponding to the basis B2 because this is what I have written and B2 is here. So, it is

$T(1,1,1,2) = 1(1,2,3)+1.(1,-1,1)+1.(2,1,1)$

$T(u_1) = 1v_1+1v_2+1v_3$

- $\Rightarrow$ $T(1,1,1,2) = (4,2,5)$

$T(1,-1,0,0) = 1v_1+0v_2+2v_3$

$\qquad = 1.(1,2,3)+0.(1,-1,1)+2(2,1,1)$

- $\Rightarrow$ $T(1,-1,0,0) = (5,4,5)$

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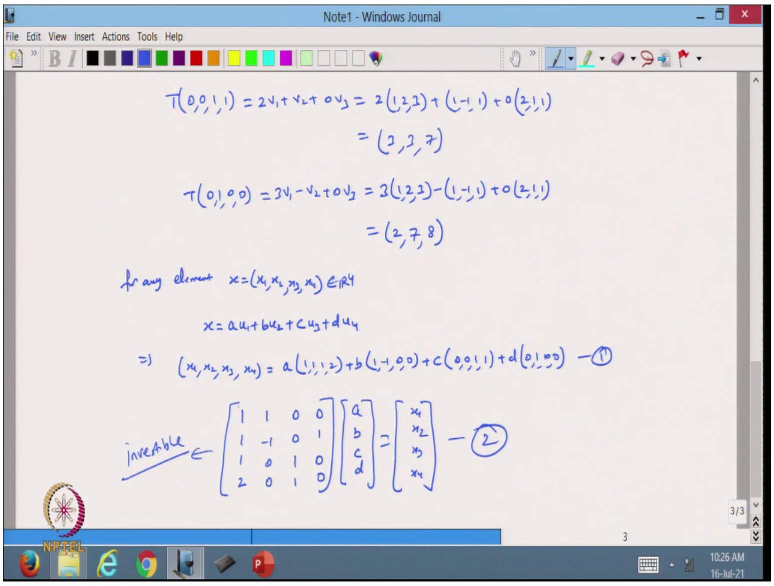

$T(0,0,1,1) = 2v_1+v_2+0.v_3$

$= 2(1,2,3)+(1,-1,1)+0.(2,1,1)$

⇨ $T(0,0,1,1) = (3,3,7)$

$T(0,1,0,0) = 3v_1-v_2+0v_3$

$= 3(1,2,3)-1(1,-1,1)+0(2,1,1)$

⇨ $T(0,1,0,0) = (2,7,8)$

So, once we are able to find this one, then we know that the linear transformation is completely determined by the image of the basis, so that we are able to find it. Now, we take that four any element x. So, $x \in (x_1, x_2, x_3, x_4) \in R^4$ I can write this x as a linear combination of the basis. So, I can write this as

$X = a u_1 + bu_2 + cu_3 + du_4$

I am writing this as a linear combination of this basis.So, from here we can write that

$(x_1, x_2, x_3, x_4) = a(1,1,1,2) + b(1,-1,0,0) + c(0,0,1,1) + d(0,1,0,0)$

So, by this way we can write any element. Now I need to find the values of (a b c d) in terms of (x_1, x_2). So, from here you can see that you will get a system of equations. So, it

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

So, this is a system of equation and it is 4 *4. So, we need to find out the solution because we know that these are the basis, so this matrix is invertible. So, if it is an invertible matrix we will get the unique solution for this system. Now, the question is how we can solve this system.

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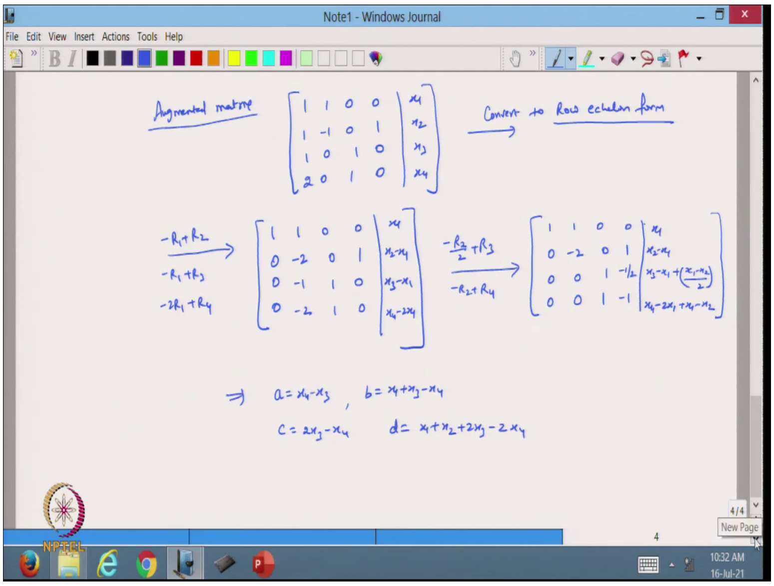

So, this system can be solved by writing the augmented matrix we know. So, augmented matrix and then so, from here I can write again the same matrix only on the last column I can put this one. So, it is

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

So, this is my matrix corresponding to the augmented matrix. Now we have to convert this one. So, convert to a row echelon form. So, this one we want to do and that we know how to do that one.

So, now, what do I want to do? I want to convert this one into the row echelon form. So, I will take this elementary operation. So, I will put minus R1+ R2, -R1 +R 3 and -2R1+R4. So, these are the things we are going to do. So, after doing this one, I will get

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & -2 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 - x_1 \\ x_3 - x_1 \\ x_4 - 2x_1 \end{bmatrix}$$

So, after doing this one, this is there. I am putting R2 /(-2). So, if I divide by minus 2 it will be 1 here and it will add to R3.

And, the same thing I can do now from here I can write that -R2 +R4.

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 - x_1 \\ x_3 - x_1 + (\frac{x_1 - x_2}{2}) \\ x_4 - 2x_1 + x_1 - x_2 \end{bmatrix}$$

And, now so, in the end so, after doing this calculation one more step we want to do then from here we can find that that after doing this I am able to get my

a= $-x_3 + x_4$, b= $x_1 + x_3$, c= $2x_3 - x_4$, d= $x_1 + x_2 + 2x_3 - 2x_4$

So, this is the coefficient I am able to get.

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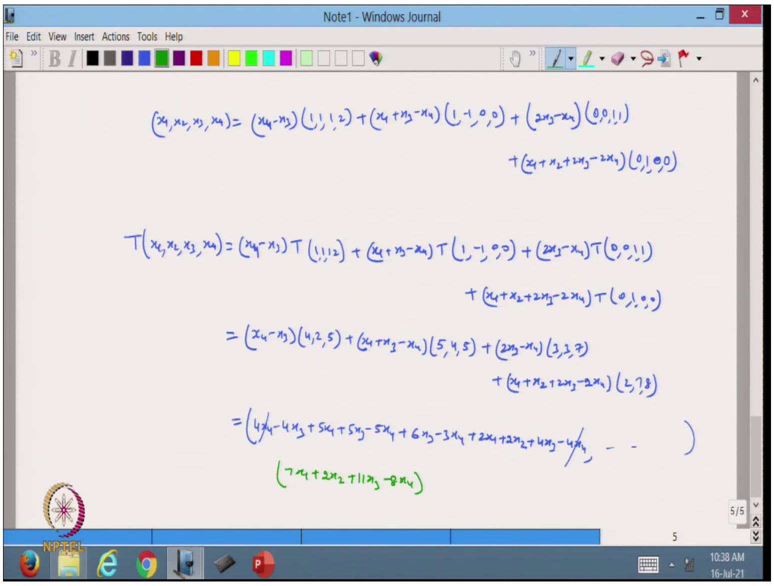

So, now, from here so, from here I get

$(x_1 , x_2, x_3 , x_4) = (-x_3 + x_4)(1,1,1,2) + (x_1 +x_3 -x_4)(1,-1,0,0) +(2x_3 - x_4)(0,0,1,1)+$

$(x_1+x_2 +2x_3 -2x_4)(0,1,0,0)$

So, now, after doing this we take the transformation on both side. So, this is again I can write like this one.

$T(x_1 , x_2, x_3 , x_4) = (-x_3 + x_4)T(1,1,1,2) + (x_1 +x_3 -x_4)T(1,-1,0,0) +(2x_3 - x_4)T(0,0,1,1)+$

$(x_1+x_2 +2x_3 -2x_4)T(0,1,0,0)$

$T(x_1 , x_2, x_3 , x_4) = (-x_3 + x_4)(4,2,5) + (x_1 +x_3 -x_4)(5,4,5) +(2x_3 - x_4)(3,3,7)+$

$(x_1+x_2 +2x_3 -2x_4)(2,7,8)$

$= (-4x_3 + 4x_4+ 5x_1 +5x_3 -5x_4+6x_3 - 3x_4+2x_1+2x_2 +4x_3 -4x_4+\ldots)$

$(7x_1+2x_2 +11x_3 -8x_4)$

$T(x_1 , x_2, x_3 , x_4) = (7x_1+2x_2 +11x_3 -8x_4, 11x_1+7x_2 +22x_3 -19x_4, 13x_1+8x_2 +30x_3 -23x_4$

So, ultimately if you calculate these then I get this transformation.

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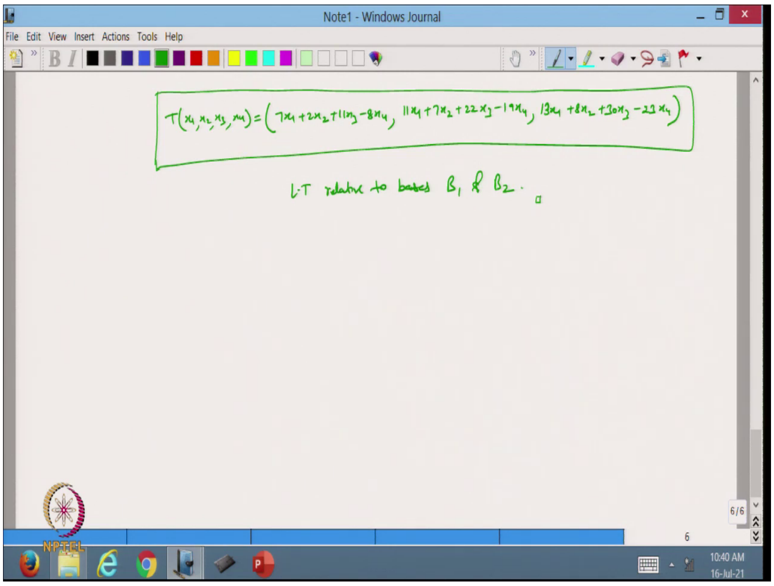

$T(x_1 , x_2, x_3 , x_4) = (7x_1+2x_2 +11x_3 -8x_4, 11x_1+7x_2 +22x_3 -19x_4, 13x_1+8x_2 +30x_3 -23x_4$

So, this is the transformation we got corresponding to the basis B1 and B2 and if you see from here this transformation is with respect to the standard basis and this transformation is entirely different from this transformation because the basis has been changed.

So, this is the linear transformation. So, this is my LT related to basis B1 and B2. So, this is the way we can write the linear transformation for a given matrix corresponding to the basis whatever the basis has been given to us. So, we follow this procedure and then we are able to write this transformation. So, I hope that this is the criteria or the method we use to find out the linear transformation.

So, we will stop here. So, in today's lecture we have discussed one example of how for a given matrix how we can write the linear transformation corresponding to the given basis and in the next lecture, we will continue with that one. So, thanks for watching.

Thanks very much.