Matrix Computation and its applications

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Lecture - 37
Continued

Hello viewers. So, welcome back to the course on Matrix Computation and its application. So, now, we will continue with the previous problem that we started in the previous lecture: how we can write a matrix corresponding to the linear transformation related to the new basis. So, we will continue with that one.

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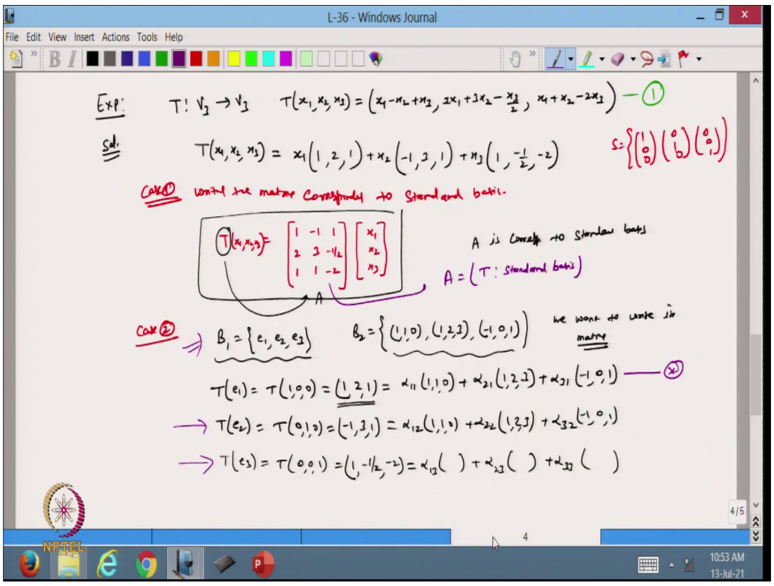

So, in the previous lecture we showed how we can write the matrix corresponding to the linear transformation that is given to me in terms of standard basis and in terms of the new basis.

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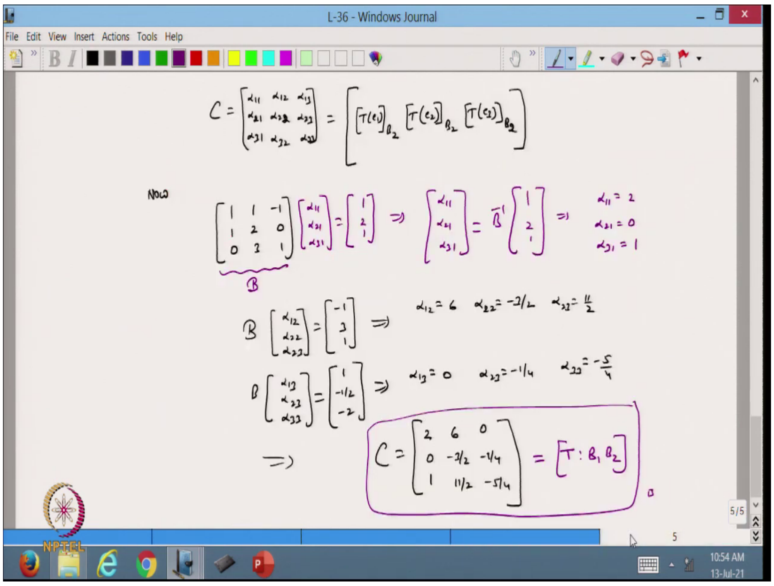

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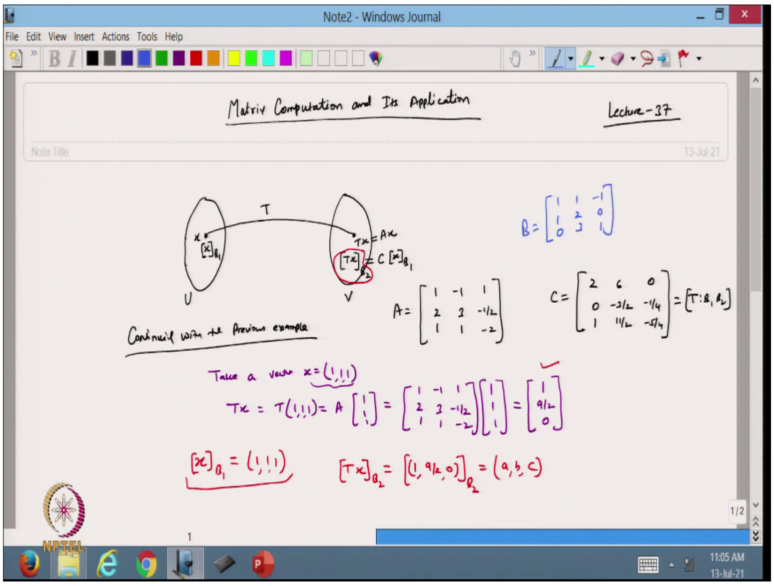

Now so, basically what we are doing here is that, suppose I have a this is my vector space U and this is my vector space V. Now here for a given x I am taking the linear transformation T and this vector in the V that is my Tx. Now from here I am able to write the matrix for T corresponding to the standard basis and this is equal to my Ax.

So, this way we can write directly from here and this is what we have done here with this A. Now what do we have done the same vector I am taking, now I am writing this vector as a new coordinate of this vector with respect to the basis B1. So, the same element, but I am writing a new vector with new coordinates of this vector x with respect to the basis B1.

So, basically here it is equal to x not equal to x the new x it is coming. So, whatever the coordinates will come from here I am going to get Tx that is in the V. So, I am writing the coordinates of this with respect to basis B2 and then we know that we are able to write the matrix. So, this matrix is my C matrix. So, this will be equal to C and x of B1. It means that you take any transformation of any matrix, this transformation linear transformation T each it is mapping the same element into the same element.

The image of x will be Tx. It does not matter which basis I am taking. Because every time I represent the same vector x with a new coordinate vector and here the same thing is happening. So, this is the relation between how the matrix will change with respect to the new basis. So, let us verify this one. Now this is my matrix C and this is the corresponding matrix. So, let us take this one.

Now, continuing with the previous example. So, from here I get my matrix A. So,

$$A=\begin{bmatrix}1 & -1 & 1\\ 2 & 3 & -1/2\\ 1 & 1 & -2\end{bmatrix} \quad C=\begin{bmatrix}2 & 6 & 0\\ 0 & -3/2 & -1/4\\ 1 & 11/2 & -5/4\end{bmatrix} = [T: B1B2]$$

Now, I want to check that. So, let me take a vector. So, I take a vector x =(1, 1, 1). So, just take a simple vector (1 1 1). Now I want to find it the same as. So,

$$Tx = T(1,1,1) = A\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}1 & -1 & 1\\ 2 & 3 & -1/2\\ 1 & 1 & -2\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}1\\9/2\\0\end{bmatrix}$$

Now, I want to check whether I am going to get this value with respect to the new basis also or not. So, this one thing we need to do is that, I have to verify this relation. So, what I am going to do is the same x I am taking now and I am writing on the basis B1. Now if you see from here the basis B1 is again we have taken just for the simplicity we have taken as a standard basis.

So, I think it is not going to change anything here. So, the coordinates will be the same (1, 1, 1) no problem. So, this is what I have taken now. Now from here so, this is the value the

I have taken. Now I want to find the value corresponding to these. So, I want to find the left hand side of this one.

$[Tx]_{B1} = [(1,9/2,0)]_{B2} = (a,b,c)$

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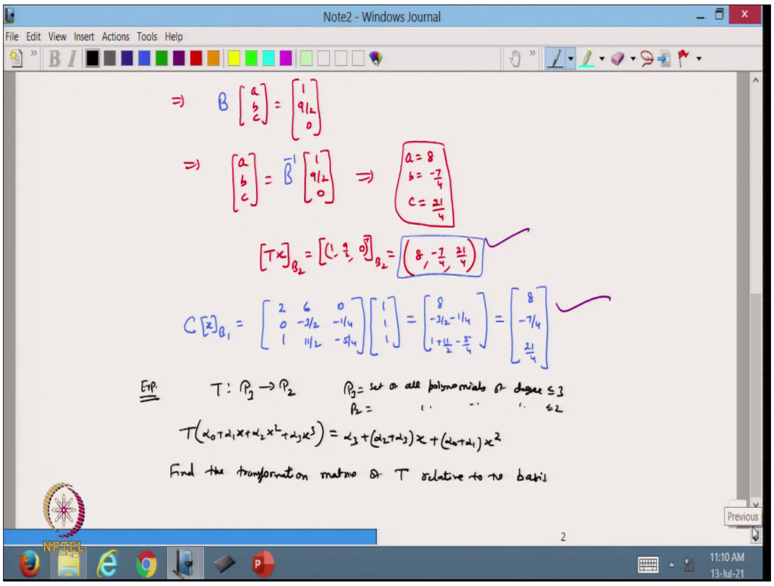

So, this one I can write from here that my matrix C is this one. So, it should be equal to matrix C and then the vector (a b c) = (1 ,9/2, 0). So, I can take the linear combination of these vectors and I know that C is a 3 by 3 matrix which is invertible.

So, from here I can find the value of a, b, c and this will be equal to C inverse and (1, 9/2, 0). So, this one we can write and if we solve this one. So, that we can solve easily we will get that the a =8, b =-7/4 and c =21/4. So, this value is coming.

So, now from here I can say the coordinates of Tx. So,

$[Tx]_{B2} = [(1,9/2,0)^T]_{B2} = (8,-7/4,21/4)$

Now, I want to verify whether the same value is coming for $C[x]_{B1}$. Now I want to find out what will happen to $C[x]_{B1}$ because $[x]_{B1}$ is the coordinator of the same x with respect to B1.

So, $[x]_{B1}$ is (1 1 1) no problem. So, now, from here I can write that this is C in just a minute. I just want to check pneumatics made up of no this is B basically.

So, I have to just write here changing them. So, C is this one and B is this one. So,

$$B=\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

So, this is my basis matrix. So, basically we are not writing here C, it is B basis matrix it is B that we have taken. So, B is the basis that is basically made up of B2. So, this is a made up of B2 thus we have already seen that I have taken the basis B2 and from the B2 I made this matrix this B.

So, this is the B I am taking and based on this B we are able to write this form and from here it is my B inverse and that gives the value. Now I want to find out. So, C is

$$=\begin{bmatrix} 2 & 6 & 0 \\ 0 & -3/2 & -1/4 \\ 1 & 11/2 & -5/4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} 8 \\ -7/4 \\ 21/4 \end{bmatrix}$$

So, you can see from here that this vector and this vector. So, this and this are the same. So, from here this is verified that the same x is going to the image Tx, now in the new basis we are writing the same vector x with a new basis and the image also we are writing with the new basis and that is also equal to this one. So, that makes the verification for this one.

So, this is the way we are writing the same linear transformation with respect to the new basis. So, let us take one other example of how we can define this example. So, let us take one I have a linear transformation T from the set of polynomials of degree less than equal to 3 to the set of polynomials of degree less than or equal to 2. So, where P_3 is a set of all polynomials of degree less than equal to 3 and P_2 also.

Now, from here I define the linear transformation T

$$T(\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3) = \alpha_3 + (\alpha_2 + \alpha_3)x + (\alpha_0 + \alpha_1)x^2$$

So, this is the image we are defining T from P_3 to P_2. Now, based on this one I want to write. So, the question is to find the transformation matrix of T relative to the basis. So, this is the basis we are going to take.

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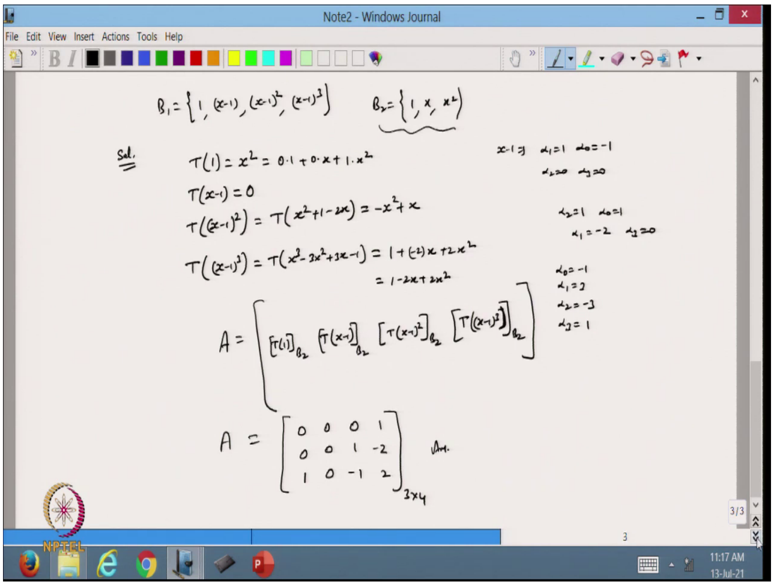

So, we are defining the basis B1 $=\{1, (x-1), (x-1)^2, (x-1)^3\}$. So, this is the basis B1 and B2$=\{1, x, x^2\}$. So, this is my linear transformation. Now, if somebody asked me what will be the matrix corresponding to the standard basis. So, we know that.

So, now, we need to go directly from here. So, we know that the linear transformation is completely determined by the image on the basis of P_3. So, we want to find out what will be the T(1)

$T(1) = x^2 = 0.1+0.x+1.\ x^2$

$T(x-1) = 0$

$T(x-1)^2 = T(x^2+1-2x) = -\ x^2+x$

$T(x-1)^3 = T(x^3-3x^2+3x-1) = 1-2x+2x^2$

So, this is what we want to find out. So, from here I have my $\alpha_2 = 1$, $\alpha_0 = 1$, $\alpha_1 = -2$ and $\alpha_3 = 0$. So, if I substitute here $\alpha_3 = 0$, $\alpha_2 = 1$.

Now from here you can say that $\alpha_0 = -1$, $\alpha_1 = 3$, $\alpha_2 = -3$ and $\alpha_3 = 1$ and if you put the sign here.So, that is the image. Now from here we can write. Now this is my standard basis basically.

So, from here if you see my matrix I can write my matrix A that will be. So, it will be

$A=[[T(1)]_{B2}\ \ [T(x-1)]_{B2}\ \ [T(x-1)^2]_{B2}\ \ [T(x-1)^3]_{B2}\ ]$.

$$A=\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 1 & 0 & -1 & 2 \end{bmatrix}$$

So, that is the matrix we are going to get for this. So, that is my matrix for the linear transformation corresponding to the basis B1 and B2. So, that is my answer because we are taking the linear transformation from P_3 to P_2. So, it is dimension 4 and dimension 3. So, we are getting the matrix 3 by 4.

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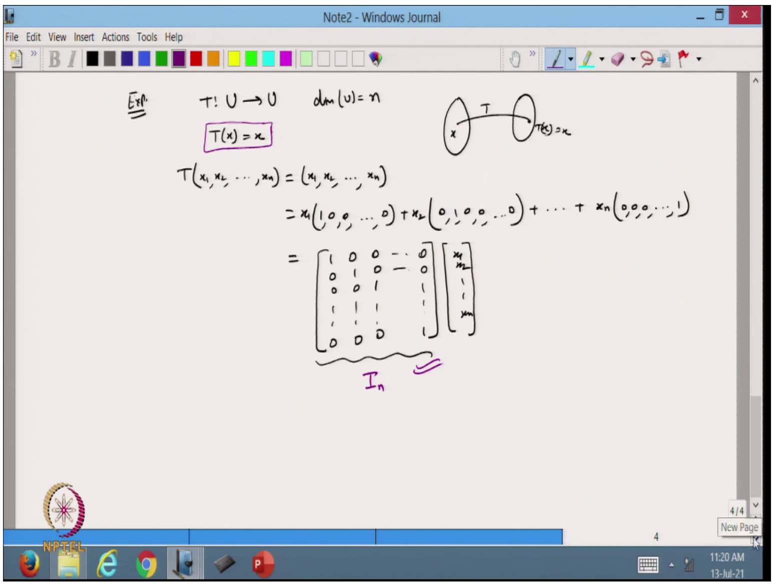

Now, the same way we can define another example. Suppose I have a transformation T from the vector space U to the vector space U and let dim(U)= n, then suppose I take this transformation T(x) =x. So, now, if I want to write the corresponding linear matrix corresponding to the transformation, then from here you can see that it is like T(x)= x. So, I am having the transformation from this that is my x taking the transformation T.

So, what I am going to do is that, I am going to write the matrix x ∈U. So, definitely my

$T(x_1, x_2, \ldots x_n) = (x_1, x_2, \ldots x_n)$

$$= x_1 (1,0,0\ldots 0) + x_2(0,1,\ldots 0) + x_n(0,0,\ldots 1)$$

$$= \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

So, this matrix if you see then this is identity matrix of order n * n and also we know that this transformation identity transformation. So, we are writing the matrix corresponding to the standard basis then you can see that this identity matrix gives us the identity matrix. This identity linear transformation gives the identity matrix for this one. So, this way we can write the matrix corresponding to the given transformation.

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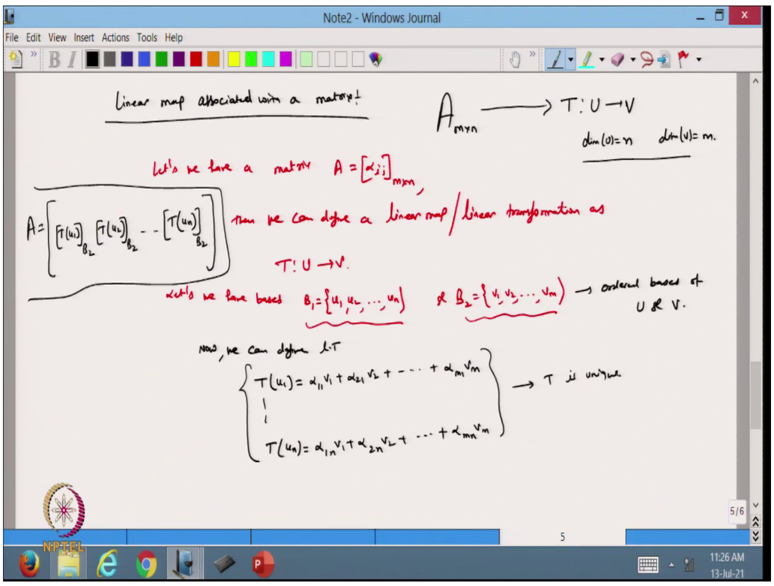

Now, after doing this one the next question is that. So, we are able to write the matrix corresponding to the given linear transformation can we do the other way. So, we want to do the converse of that one. So, we write down a linear map associated with a matrix.

So, now in this case we have a matrix A that is supposed to be of order m cross n and from there I want to find the linear transformation T :U -> V. So, dim(U)=n and dim(V) = m. So, this way we want to write about how we can write these things. So, let us see how we can write the linear task transformation from the given matrix. So, let us have a matrix.

So, I have a matrix A =[α_{ij}] of order m cross n now then we can define a linear map or linear transformation as. So, how can I define it? So, I can define by T from U to V right. So, we can define the linear transformation T from U to V. So, now, everything depends upon which basis we are going to take. So, if there is no basis then we can define that this is a standard basis so, no problem.

But let we have; so, let us have basis B1. So, that is the basis we are taking that is

B1={u_1, u_2 ,….,u_n} and B2 ={v_1, v_2 ,….,v_n} this is the basis of V and this is the basis of U. Now I know how we can define the linear transformation. So, these are the ordered bases of U and V.

Now, I can define. So, now, we can define the linear transformation like this T of I will take u_1. So, that will be equal to now this is the corresponding matrix we have. So, this is the and I know that this matrix is made up of the coordinates. So, I know that A=[[T(u_1)]$_{B2}$ [T(u_2)]$_{B2}$ …. [T(u_n)]$_{B2}$].

Now we can define Linear transformation

$T(u_1) =\alpha_{11} v_1 + \alpha_{21} v_2 + \alpha_{31} v_3 +\ldots+ \alpha_{m1} v_m$

$T(u_j) =\alpha_{1j} v_1 + \alpha_{2j} v_2 +\ldots+ \alpha_{mj} v_m$

:

$T(u_n) =\alpha_{1n} v_1 + \alpha_{2n} v_2 +\ldots+ \alpha_{mn} v_m$

So, let us take one example and I know that this is the basis. So, this T is unique. It is uniquely determined from corresponding to this basis.

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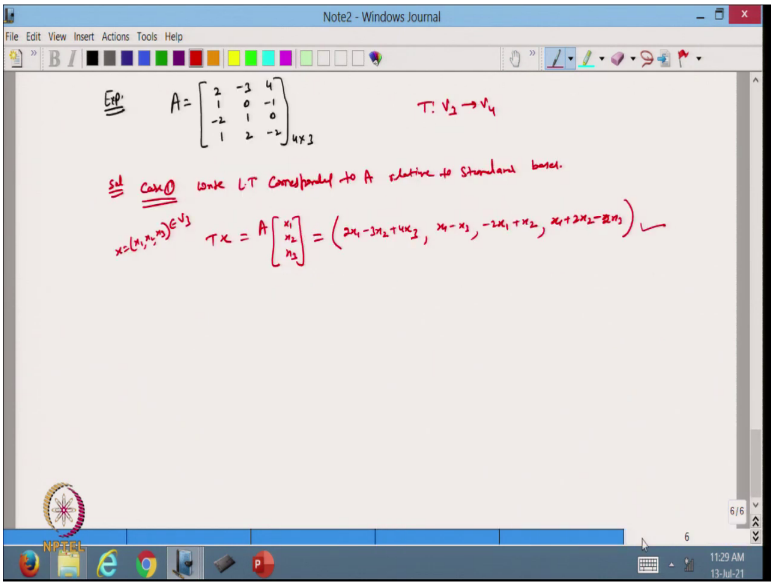

So, for example, let us take one example: I have a transformation. So, suppose I have a matrix A = $\begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & -1 \\ -2 & 1 & 0 \\ 1 & 2 & -2 \end{bmatrix}$

So, write linear transformations corresponding to A relative to standard bases. So, I have a standard basis then in that case I will just. So, from here I know that my transformation T :U -> V where my or maybe I can just write from here that T will be from V_3 to V_4.

So, from here I can write that my matrix A taking the element from V_3 that is

$$Tx = A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

I am taking $(x_1, x_2, x_3) \in V_3$ and if you write from here then I can write my A. So, image will come.

$= (2x_1-3x_2 +4 x_3 , x_1- x_3, -2x_1+ x_2 , x_1+2x -2x_3)$

So, that is my linear transformation corresponding to the standard basis. Now the same linear transformation I want to write is related to this basis B1 and B2. So, this is ok means we are able to write very easily the corresponding linear transformation for the with respect to the standard basis.

Now, we want to write the linear transformation corresponding to the basis B1 and B2. So, let me stop here today. So, in today's lecture we have discussed various examples based on how we can find out the A matrix corresponding to the linear transformation with respect to the basis and then we define the converse of that one that we have a matrix and we want to find out the linear transformation corresponding to that one.

So, in the next lecture we will show how we can write the linear transformation corresponding to the new basis B1 and B2 as we are going through this example. So, thanks for watching.

Thanks very much.