**Matrix Computation and its applications**

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**Lecture - 26**

**Coordinate of a vector with respect to ordered basis**

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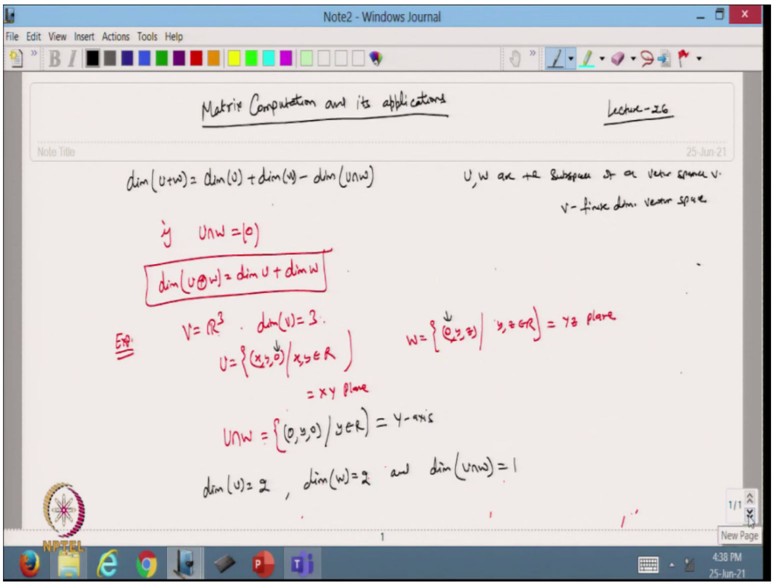


So, today I am going to discuss whatever we have done. In the previous lecture we have shown that the dim(U+W)=dim(U)+dim(W)-dim(U$\cap$ $W$). Where U and W are the subspaces of a vector space V, And, V is a finite dimensional. So, suppose I take the dimension or maybe I can say that, V is a finite dimensional, finite dimensional vector space.

Now, from here I can say that, if U $\cap$ W = 0. Just it contain the 0 element from here we can say, then I can write that the dim(U+W)=dim(U)+dim(W). So, this thing, we can write down.

For example, suppose, I take the vector space V=$R^3$. So, dim(V)= 3. So, let we take the subspace U. So, I just take the subspace and I take the subspace U={(x,y,0)|x,y∈R}. So, all the elements, I can say that this is x y plane. I can take the another subspace W={(0,y,z)|y,z∈R}.

So, I can say that the elements coming from y and z space. So, this is I just write y,z∈R and it is basically a y-z plane. I In fact, I can also write here in that x, y∈R and that is equal to the x-y plane. And, maybe I can write the y-z plane. Now from here, you can see that in this case if I take U∩W.

So, it will contain all the elements, which have elements coming from the U. So, U is if it is U coming from U, then it will contain the third coordinate 0 and if it is coming to W the first coordinate 0. So, it means if we take the U∩W that will contain only all the elements that which has $U \cap W = \{(0,y,0) | x,y \in R\}$. So, basically it is equal to a y-axis.

So, it contains all the elements lying on the y-axis. And, we also know that the dim(U)=2, dim(W) = 2 and dim(U∩W)=1. So, that is just a y-axis.

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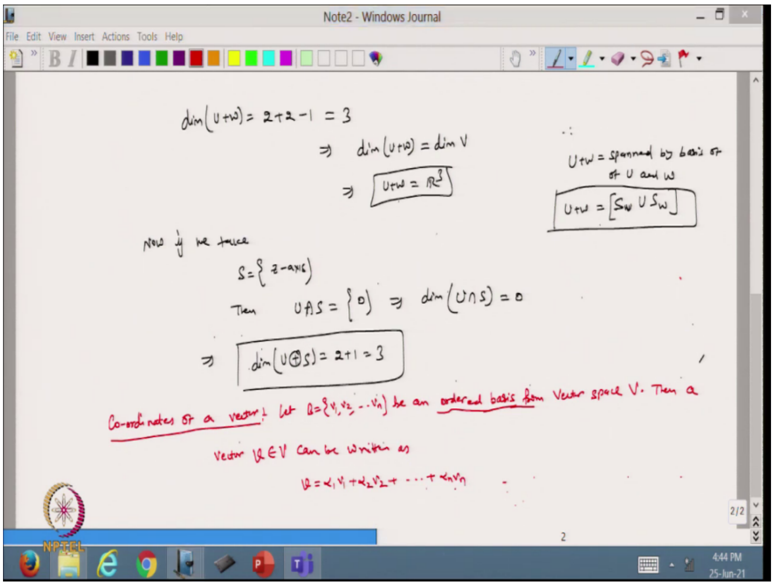


So, now from here I can write directly, that the $\dim(U+W)=\dim(U)+\dim(W)-\dim(U\cap W)=2+2-1=3$. And, from here it says that since dim(U+W)=dim(V), which implies that U+W is itself $R^3$. And, that is also true in this case, because I am taking all the elements from the X-Y plane and from the Y-Z plane. So, U+W will contain all the basis of it because it is spanned by we know that U+ W is spanned by basis. So, if I take the set as that is the spanning set for U and spanning set of W. So, I just take the span of this one. So, it becomes the U+W that we already know. So, from here I say that U+W becomes in fact, the whole vector space V. Now, if we take W or maybe if we take another subspace S, that contains only the z axis. Then, if I take $U+S$. So, then from here you can see that U is just the X-Y plane and S is a Z axis, so it will contain only 0 elements.

And, from here I can say that the dimension of $U \cap S$ will be 0, because it will contain only 1 element that is a 0 element. So, from here I can say that the

$\dim(U + S) = 2 + 1 = 3$, because the $\dim(S) = 1$. So, from here I can write that this will be a direct sum so, this one we can find out that dimension of a direct sum it invert the direction, then we can use this theorem.

So, after this one we define another terminology and that is called coordinates of a vector. So, let us define this one, let I take a set $B=\{v_1,v_2,\ldots,v_n\}$ to be an ordered set from vector space V. So, it is an ordered set that means we cannot change the order. If it is a $v_1$, then it will be $v_1$, $v_2$, whatever the order is there the element of the vector will lie in the same order. So, if it is an ordered set from the vector space V, then a vector v from V can be written as so, now, take v.

So, it is ordered set from the vector space v, then a vector v. So, it is not the ordered set I can say it is the ordered basis from vector space v. Then any vector v from can be written as some $\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n$ So, now, I have written the linear combination.

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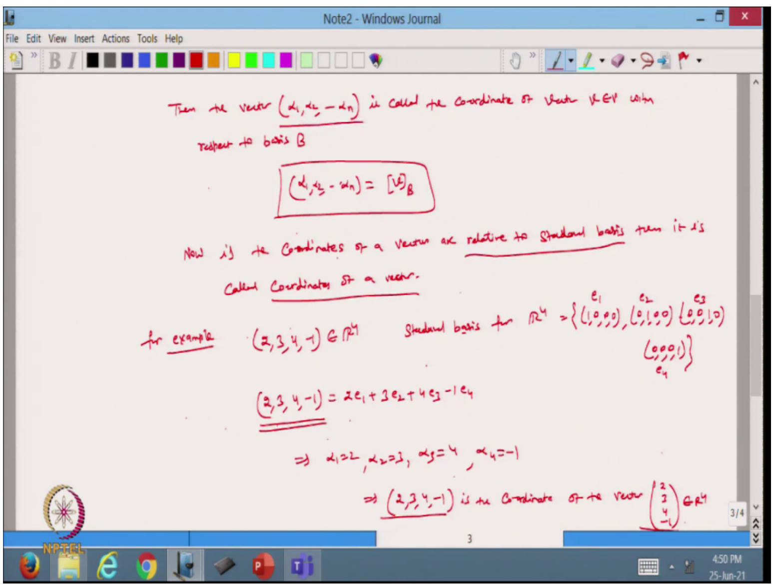

Then, then the vector that is $\{\alpha_1,\alpha_2,\ldots,\alpha_n\}$. The coordinate is called the coordinate of a vector of vector v belonging to V with respect to basis B.

So, from here I can write like this one. So, this $\{\alpha_1,\alpha_2,\ldots,\alpha_n\}$, it can be written as if the coordinates of V are related to the basis B. So, this is represented like this one. Now, this is the way we can define. Now, if the coordinates of a vector are related to standard basis, then it is called coordinates of a vector, because here it is coordinates of the vector v with respect to the basis B.

But, if we take the coordinates of the vector related to the standard basis, then they are just called the coordinates of the vector. For example, suppose I take the vector {2,3,4,-1} that belongs to $R^4$. Now, we know that the standard basis for $R^4$ so, this is just {1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0} and {0, 0, 0, 1}. So, these are called the standard basis. And, we are represented by $e_1$, $e_2$, $e_3$ and $e_4$.

So, these are called the standard basis. Now from here I can write that, 2, 3, 4 minus 1, it can be written directly as $2\ e_1+3e_2+4e_3+1e_4$. And, so, I am writing a vector v as a linear combination of the standard basis of $R^4$.

So, from here you can check that my $\alpha_1=2$, $\alpha_2=3$, $\alpha_3 = 4$, and $\alpha_4 = = 1$. And, which is the same as the coordinates of this vector {2, 3, 4,-1}.

So, from here I can say that the set {2, 3, 4,-1}. So, this is the coordinates of the vector {2, 3, 4,-1}. So, this one I can write here because this vector belongs to $R^4$. So, this is the vector and its coordinates are this one. So, from here I can say or maybe I can say that so, if the coordinate or the vector this. So, I am writing like this one. It means I am talking about the standard basis.

So, a coordinate means the whole vector we are taking. We are talking about the whole vector this one and the elements of these are the coordinates.

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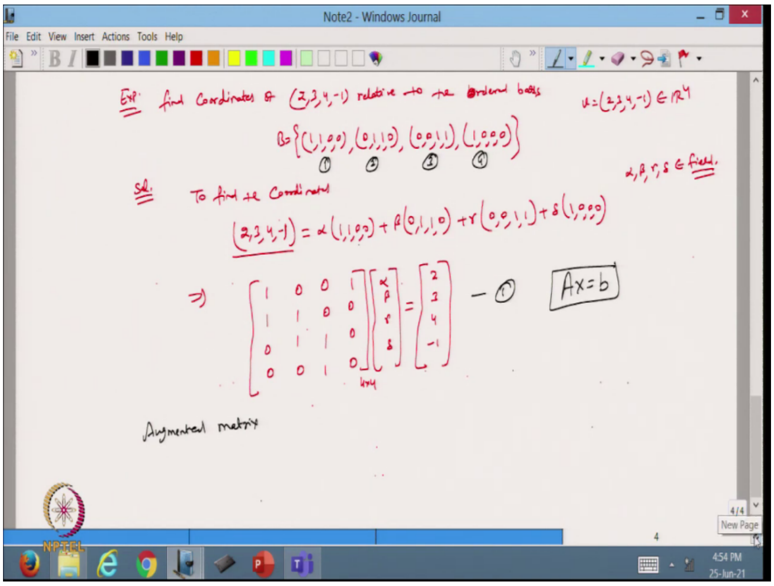


Now, let us take another example. The question is to find coordinates of {2, 3, 4,-1} relative to the ordered basis. So, here I am taking the basis. So, the basis is {1, 1, 0, 0}, then {0, 1, 1, 0}, {0, 0, 1, 1} and {1, 0, 0, 0}. So, these are the basis I am taking. And, we are talking about, so, this vector V is basically {2, 3, 4,-1} this is the same.

So, it belongs to $R^4$ that we already know. So, it is the basis of this one. So, it means that these vectors are linearly independent and span the whole $R^4$ so that is the basis. Now, I need to find out the coordinates of this one related to the ordered basis. Now, so, for this one we need to write. So, to find the coordinates we need to write the vector {2, 3, 4,-1} as a linear combination.

So, suppose I take here{2, 3, 4,-1}= α{1, 1, 0, 0}+β{0, 1, 1, 0}+γ {0, 0, 1, 1} +δ{1, 0, 0, 0}, where this α, β, γ,δThey belong to the scalar field. So, these are the coming from the field basically, these are the scalar and coming from the field. So, I am taking this as a real number here. Now, we need to find this α, β, γ, *and* δ. So, it is because these are the coordinates of this vector related to the basis B. So, from here if you see it is a just linear combination. So, if I put a matrix [1 0 0 1 1 10 0 0 1 1 0 0 0 1 0 ][α β γ δ ] = [2 3 4 − 1 ] . So, it is a 4× 4 system. Now, in fact, here we cannot change the order. So, order means this is my 1st, 2nd, 3rd, 4th, because if I change the order, then this value of α, β, γ, *and* δ will also change.

So, we cannot change the order, because we know that if we swap some columns of the given matrix, then the result will be changed. So, now, from here I will get this. So, this one we need to solve. So, it is a 4× 4 system. So, you can check that this is equal to A x = b and we need to find the solution for this one. So, the best thing for this one is to reduce this into the echelon form and then find the solution.

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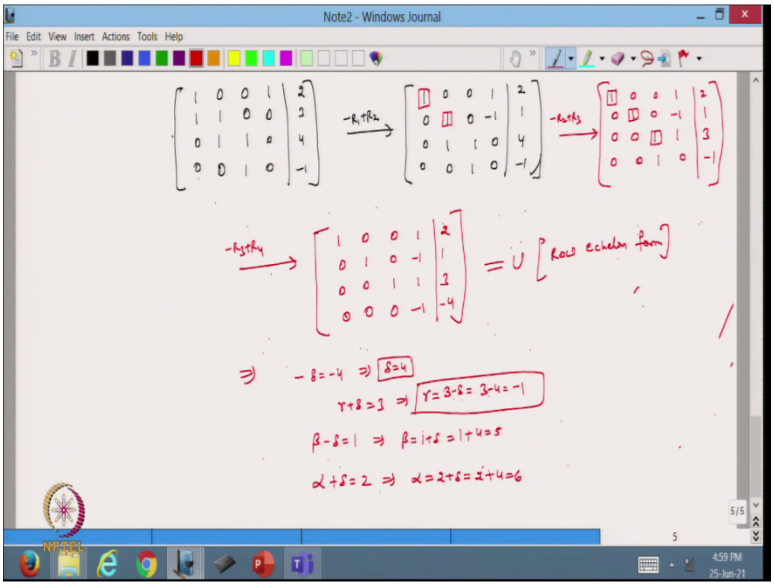


So, for this one I will write the augmented matrix. So, the augmented matrix I can write directly. So, this will be my augmented matrix. So, I can write from here. So, it will be {1, 1, 0, 0}, {0, 1, 1, 0}, {0, 0, 1, 1} and {1, 0, 0, 0}. And, this one I will write as {2, 3, 4, -1}.

Now, from here I will write - $R_1$ +$R_2$, because I want to make this element 0. So, it will be 1, 2. So, -1 + 1 = 0. So, it will be 1, 0 and -1. So, it will be - 1 and -2 +3 = 1. And, all other elements will be the same {0, 1, 0,-1}. So, from here I get this matrix. Now I need so, this is my pivot element, this is also pivot.

So, I need to make this element 0. So, from here I will write

-$R_2$ +$R_3$. So, it will be 1, 2 and then {0, 0, 1 1}. So, I get this value. Now from here you just check it is my pivot, this is a pivot, this is a pivot. So, the last one I can apply for is -$R_3$ + $R_4$. And, ultimately I will get {1, 0,0, 1}, {0, 1,0, -1}, {0 0 1 1}, {0,0,0,-1}.

Now, it is echelon form and because it is a $4 \times 4$ matrix, so, now from here so, this is my equal to U, the row echelon form. So, from here I can directly write that. So, I can write from here $-\delta = -4$. Because, we have to do the back substitution and from here I can write $\gamma + \delta = 3$. So, that gives $\gamma = 3 - \delta$.

So, 3-4 = -1. So, $\gamma$ is coming -1. Now, from the second last equation or the second equation, it will be $\beta - \delta = 1$. So, that gives you beta is equal to $1+\delta$. So, it is $1+\delta = 4$. So, that is equal to 5. And from the first equation it is $\alpha - \delta = 2$. So, that gives me $\alpha = 2 + \delta$. So, it is $2 + 4 = 6$. So, it is $\delta = 4$, $\gamma = -1$, $\beta = 5$ and $\alpha = 6$.

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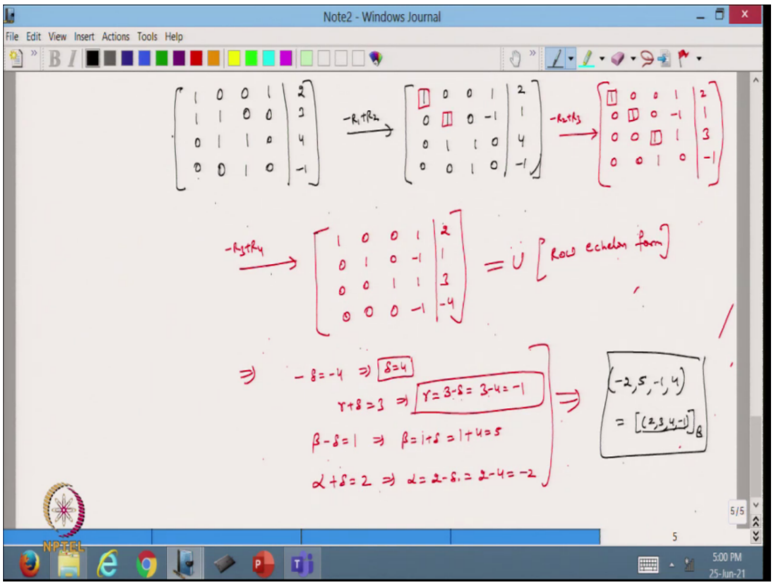


So, now from here I can write $\alpha = 2 - \delta$. So, it is 2-4, it is - 2. So, now, from here all this we can from here I can write my coordinates. So, this is the coordinates {-2,5,-1,4}. So, now we can write that this is the coordinates of the vector. So, the vector we have started with is {2, 3, 4, -1} with respect to the basis B. So, this one I can write. So, these are the coordinates of this vector with respect to the basis B.

So, that is {-2,5,-1,4}. So, this is the way we can find out the coordinates of any vector related to the given basis. So, that is one of the ways we can define the basis.

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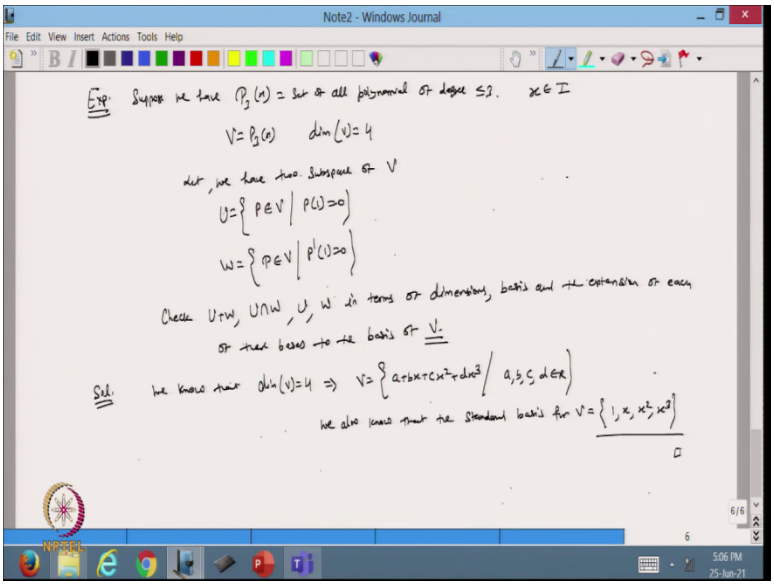


Now, so, based on this one I want to discuss 1 more important thing that how we can use the extension theorem to extend the basis. So, for this one I just want to do 1 important example. So, suppose we have a vector space $P_3(x)$. So, that is a set of all polynomials of degree less than equal to 3. And, suppose x belongs to some interval I. So, I take the vector space V as $P_3(x)$ and also I know that the dim(V)=4, because it is a polynomial of degree less than equal to 3. So, its dimension is 4. We know that.

Now, let we have two subspaces of V. So, I take the first subspace U={p$\in V | p(1) = 0\}$ It means, I am taking all the polynomials from this vector space V, such that 1 is the root of that polynomial.

And, I take the another subspace W that is P all the polynomials from V such that and W={p $\in V | p'(1) = 0\}$. Now, the question is check U +W, U ∩W, U, W in terms of dimension, basis, and the extension of each of these bases. So, each of these basis means the basis of U +W, U ∩W, U, W, basis to the basis of V the basis of V, it means so, to the basis of V.

So, means first I to need to find out what are the basis of U +W, U ∩W, U, and W. And, then we have to write it in dimension and then we should be able to extend these bases to the basis of V. So, that is the question. So, we know that the dimension of V is 4.

Because, it is V is basically all the polynomial of type V=={a+bx+c$x^2 + dx^3 \in V | a, b, c, d \in$ R}. So, it is the space of all these polynomials. And, we also know that the standard basis for V is {1, x, $x^2$, $x^3$}.

So, this is the standard basis for the set of polynomials of degree less than equal to 3. And, it is a 4 number so; we know that its dimension will be 4. So, we will continue, so let me stop here. So, in this question we need to find out how we can find out the dimension of the basis for different subspaces. And, we will continue this example in the next lecture. So, thanks for watching.

Thanks very much.