Matrix Computation and its applications
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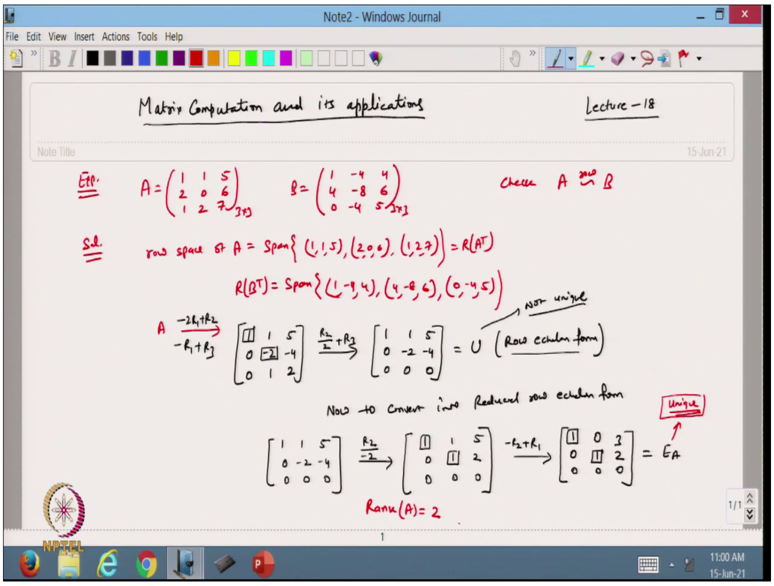

Hello, viewers. So, welcome back to the course on Matrix Computation and its application. So, in the previous lecture we have started with some relation in the about the range space of A and B 2 matrices. So, we will continue with that one and let us discuss few more example based on that.

So, let us start with the example. So, now, suppose I have a two matrices A and this matrix is 1 1 5, 2 0 6, 1 2 7 and I take another matrix B 1 minus 4 4, 4 minus 8 6, 0 minus 4 5. So, this is we are taking 3 cross 3 matrix and we want to check whether A is row equivalent to B or not. So, this one we want to check.

So, now we know that the row space of A, so, that will be span by this row vectors 1 1 5, 2 0 6 and 1 2 7 that would be span by this one and we know that this is equal to the range of A transpose. Similarly, in the case of B also the range space of B that will be span by 1 minus 4

4, 4 minus 8 6 and 0 minus 4 5. The only thing is that we do not know whether these 3 vectors are linearly independent or not the same case is here.

So, in this case we cannot talk about the dimension of the range space A transpose or range space of B transpose. Now, what we do is that so, this is we can construct with the rows. Now, I will take a and try to convert this one into the row echelon form. So, in this case I want to make this element and this element 0. So, just I will try to make these two elements 0. So, I have taken here 2, 1.

Now, so, I will take minus 2 R 1 plus R 2 and minus R 1 plus R 3. I am just taking the linear combination; scalar multiple linear combination addition. So, that we are going to take. So, it will be 1 1 5, then this will be 0 because I am taking minus 2 plus 2 0 minus 2. So, it will be minus 2 and then it is minus 2.

So, minus 10 plus 6. So, it will be minus 4 and then from here I am just taking minus of this one adding here. So, it will be minus 1 minus 1 0 then minus 1 2 is 1 and minus 5 plus 7 is 2. So, in this case you can check from here that it is coming minus 2 and minus 4 and it is 1 and 2.

Now, this is my pivot element. Now, this is my pivot element. So, I will want to make this element 0 to make in the echelon form; row echelon form. So, what I am going to do is now I want to make this element 0. So, I will take just R 2 divided by 2 and then add to R 3.

So, it would be 1 1 5, 0 minus 2 minus 4 and then from here it will be 0 then it will be 0 because I am just dividing by 2. So, it will be minus 1 plus 1 0 minus 2 plus 2 0. So, this is my U, that is row echelon form. And you can see from here that this is not unique, because I am having minus 2 here maybe I can multiply by minus 3 and then I will get minus 6 minus 12. So, it does not matter, but it will be in the still in the row echelon form.

But, now suppose I want to convert it into our reduced row echelon form. So, in that case, now convert into reduced row echelon form. So, what I do is that in the reduced row echelon form I have my matrix here 1 1 5, 0 minus 2 minus 4, 0 0 0. So, now I will take R 2 and divide by it minus 2 just to make, to bring the element 1 here and I will get 1 1 5, 0 and I will get 1 here and I will get 2 here 0 0.

Now, the pivot element should be 1 there and now all the elements, other elements in the pivot column should be 0. So, that can be done from here. So, I will take minus R 2 multiplied by minus 1 and I add to R 1. So, this will be 1 1 it will be 0. So, let us in this case first I will write here 0 1 2 and it will be 0 0 0. Now, I multiply by minus 1 and add here. So, nothing will change here. So, this will become 0 and minus 2 plus 5 so, it will be 3.

Now, so, this is my pivot element, this is my pivot element. So, that is my reduced row echelon form and the good thing with this reduced row echelon form is that this is unique because here it should be the one 1 1 and the all other element should be 0. So, if we find out by any of the operation ultimately in the end we will get this reduced row echelon form and that is the uniquely determined. So, that is the benefit with this one.

Now, in this case so, the number of so, now, I can say from here that the rank of A is coming 2. So, this is our rank is 2. Now, now I take the matrix B. So, B is coming 1 minus 4 4, 4 minus 8 6, 0 minus 4 5. So, this is also 3 cross 3. Now, I will convert this into the echelon form.

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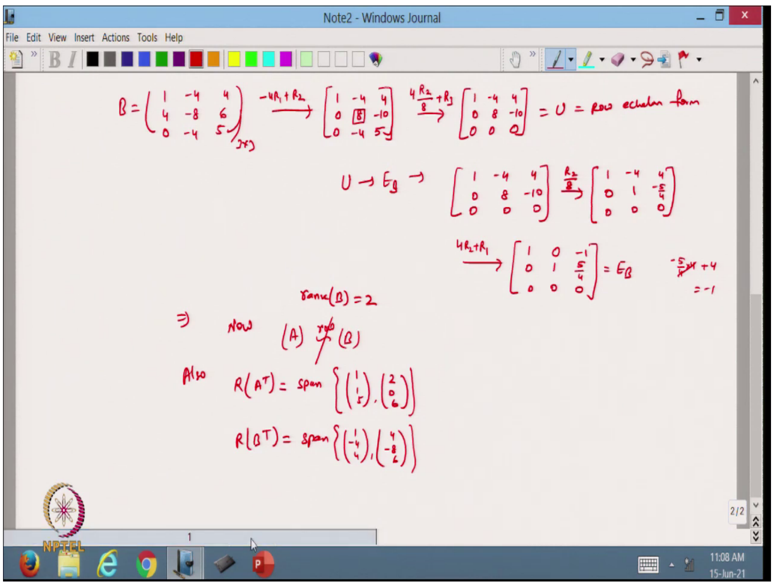

So, I will from here I will get 1 minus 4 4. So, I will make this element 0 by multiplying. So, I will take minus 4 R 1 and adding to R 2. So, from here I will get this value 0 and minus 4 and minus 4 it will be 16 minus 8. So, it will be 8 minus 4 into minus 4, 16 minus 16 plus 6 minus 10 and that is already 0. So, it will be minus 4 and this is 5.

Now, from here this is ok. So, this is my next pivot is this 8. Now from here I have to make the element 0 here. So, 1 minus 4 4. Now, in this case it is 0 8 minus 10 I want to make this element 0. So, what I do is that I will take R 2 divide by 8 and multiply by 4 and add to R 3 ok because I will divide by 8. So, it will be 1 and then multiply by 4 and then add.

So, this will make it 0 this will also make it 0 because basically I am dividing by 2 here. So, if I divide by 2 here. So, it will be minus 5 plus 5. So, that will be 0 ok. So, this is a equal to U, the echelon form; the row echelon form. Now, from here if I convert this into the E A so, let us check that one. So, I want to convert this into E B.

So, let us see what is going to happen. Now, 1 minus 4 4, 0 8 minus 10, 0 0 0 I will divide by 8 to make it 1. So, I will take R 2 divide by 8. So, it is 1 minus 4 4, 0 1 and it is minus 10 by 8. So, minus 5 by 4 and this is 0 0 0. So, now it is 1 1. So, now, I have to make this element 0. So, what I am going to do is I am multiplying by 4 R 2 and adding to R 1. So, 0 1 5 by 4 0 0 0.

Now this is 1, now this become 0 because minus 4 4 minus 4 0 and this is minus 5 4 into 4 plus 4. So, that will be minus 1. So, it will be minus 1 and that is my E B. Now, in this case you can check from here that the rank of B is also 2, it is the rank of A is also 2; 1 0 3, 0 1 2 and from the B also it is coming this one. So, now, from here now A matrix A is not because they definitely the both have the same rank, but they are not agree means the rows vectors are different. Here it is 1 0 minus 1, here it is 1 0 3 and 0 1 2.

So, it is 0 1 5 by 4 also the range space of A transpose is span by. So, in the corresponding A I have 1 and 2. So, first or second row I can take. So, I just take span of 1 1 5 and 2 0 6 and range space of B transpose span of 1 minus 4 4 and it is 4 minus 8 6 and both have the same. So, range space is. So, dimension of is in both the cases it is coming to. So, that is the way we can do.

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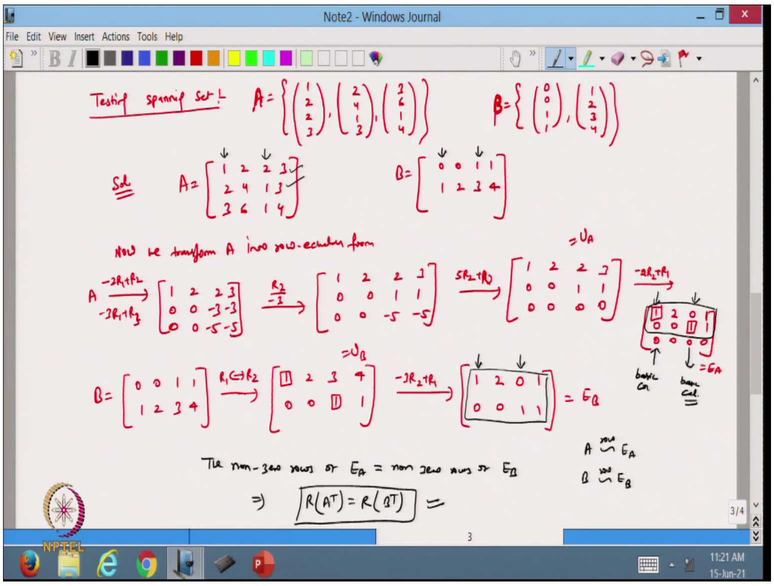

Now. So, now, we will take one more example testing spanning set. So, I have a two set A that is 1 2 2 3, 2 4 1 3 and 3 6 1 4. So, this is your set one A and another I am taking the set B is 0 0 1 1 and 1 2 3 4. So, I am taking this two sets and let us check whether they are spanning the same set or not subspaces.

So, now we take the matrix A. So, I just call it alpha and beta. So, I constitute the matrix A made up of these rows 1 2 2 3, 2 4 1 3, 3 6 1 4 and I constitute the other matrix B that is 0 0 1 1 and 1 2 3 4. So, we want to check whether they are going to span the same subspace or the space of A transpose or B transpose. So, that we are going to check.

Now, this A so, I will convert this one to the echelon form. So, what I going to do is. So, let us say now we convert or we transform A into row echelon form. So, A will transform. So, what I am going to take is just I am taking minus 2 R 1 adding to R 2 and minus 3 R 1 adding to R 3.

So, I am going to have 1 2 2 3 and then we have going to have 0 0 because minus 2 plus 2 minus 4 minus 4 plus 1 minus 3 and minus 6 plus 3 minus 3 and then 0 0 because I am multiplying by minus 3 and adding here so, minus 6 plus 1 minus 5 and minus 9 plus 4 minus 5. So, I get this value.

Now, from here I want to make this element 0. So, 1 2 2 3 and then so, I can take just minus 3 common no problem. So, I can write R 2 divide by minus 3. So, it become 1 1 and 0 0 minus 5 minus 5 ok. So, I will convert this one into the 1 1 1 and now I am applying 5 R 2 plus R 3. So, it would be 0 0 0 0. So, that will be coming. So, this is the echelon form and now, I will reduce into the row echelon form. So, this is equal to U and I will convert this into the reduced row echelon form.

So, what I am going to do is that I am taking minus 2 R 2 plus R 1 adding here and from here I will get 0 0 1 1, 0 0 0 0. 1 2 and that become 0. So, minus 2 plus 3, it is 1. So, from here I will get this and this and so, it is a E A. Now, I go to the value of E A and that is why I am converted this into E A because it is a of unique form.

Now, I take the matrix B. So, 0 0 1 1, 1 2 3 4. So, I will first I will take this and the second row and the first row. So, I will swap. So, I will or R 1 will swap with the row R 2. So, it is 1 2 3 4 and 0 0 1 1. So, this is already in the echelon form from here. Now, this and this. So, I will this is the echelon form U of maybe I can write B and this is U A and then I will convert the reduced echelon form. So, I will make this element 0. So, I will apply minus 3 R 2 and adding to R 1. So, it would be 0 0 1 1 and that will be 1 2.

So, I am taking minus 3 plus 3 that would be 0 and minus 3. So, it will be 1. So, this is my E B. Now, from here you can check that this is the rows nonzero rows and this is the corresponding nonzero rows. So, 1 2 0 1, 0 0 1 1. So, in this case you can check that the nonzero rows of E A is same as nonzero rows of E B, because I know that the; so from here I know that A is row equivalent to E A and B is row equivalent to E B.

Now nonzero rows of E A is equal to the nonzero rows of E B. So, from here we can say that which implies that the span or from here I can say that the range space of A transpose would be same as the range space of B transpose ok because these two vectors if we made the matrix they are row equivalent row.

So, this nonzero rows of the matrix A and the nonzero rows of the matrix B they will span the same space. So, the range space of A transpose will be same as the range space of B transpose. So, this is we can say.

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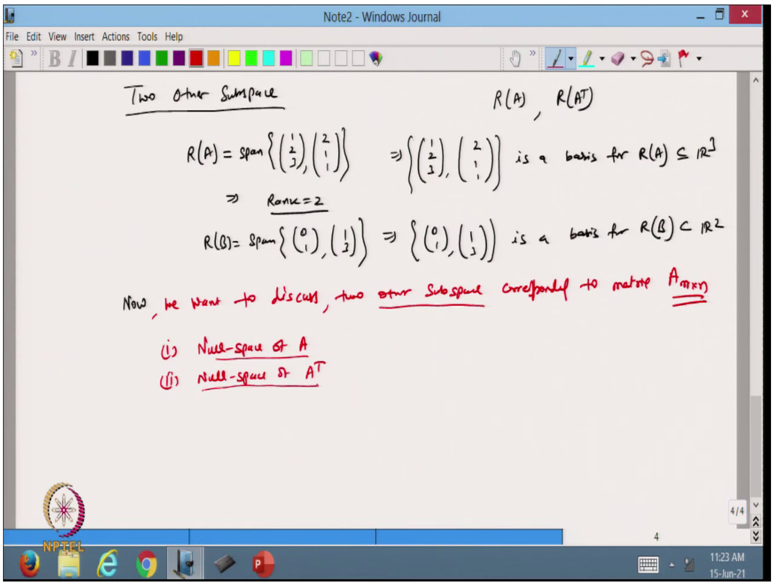

Now, after doing this one we will introduce; so we have introduced the two subspaces; two other subspaces. Till now we have spaces the range space of A and the range space of A transpose because in the range space of A will be the corresponding column ok. So, suppose I want to find out the before that one I want to find out the range space of A.

So, the range space of A will be span by. So, I will take the E A. So, E A this one. So, this is my basic column and this is also basic column ok. So, this one and this one. So, I will take the corresponding columns, corresponding to basic column. So, it is this first 1 and the third one. So, 1 2 3 span by 1 2 3 and then 2 1 1 and they also are linearly independent. So, that is the range space of A.

So, that is span by two vectors and that is equal to the dimension is definitely we know that rank is 2 and the rank of A is equal to A transpose that we already know so no problem. In the same thing in the B; so, in the B if you see this is my first and the third column. So, they are the basic columns basically. So, that will be again first and the third.

So, I can say from here that the range space of B is span of 0 1 and 1 3 ok and from here and also these are the linear independent. So, from here I can say that these are the basis. The vectors 1 2 3 and 2 1 1 so, these are the basis; is a basis for range space of A and this is 0 1, 1

3 is a basis for range space of B ok. So, it is basically a subset of R 3 and this is a subset of R 2 ok. I can say the range space is a subspace of R 2 here and it is a subspace of R 3 here.

So, based on this one I can make the column spaces and the row spaces. So, row spaces gives you the range space of A transpose and column space is gives you this one, but these are not seen because we know that if A is column equivalent to B only then the row space will be same. Since now it is coming from R 3 and it is coming from R 2.

Now, so after discussing this one, now we want to discuss two other subspaces corresponding to matrix A that is m cross n. So, two other subspaces are first one I am going to find out null space of A and the second one we are going to discuss null space of A transpose because whenever we have a A, its transpose is always there. So, we want to find out another two subspaces. So, one is null space of A and another is null space of A transpose. So, that we are going to discuss in the next lecture.

So, in the today lecture we have discussed example based on that one and we showed that if the two matrix are row equivalent it means we have showed that if their row; reduced row echelon form is same, then they are going to span the same spaces subspaces. So, that we have discussed and in the next lecture we are going to introduce the null space of A and the null space of A transpose. So, I hope you have enjoyed this lecture. Thanks for watching.

Thanks very much.