Matrix Computation and its applications
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Lecture - 14
Examples of basis and standard basis of a vector space

Hello viewers. Welcome back to the course on Matrix Computation and its application. So, in the previous lecture, we have discussed about the basis and the dimension of a vector space and in this lecture, we will continue with that one. So, let us start with this. So, this is lecture number 14.

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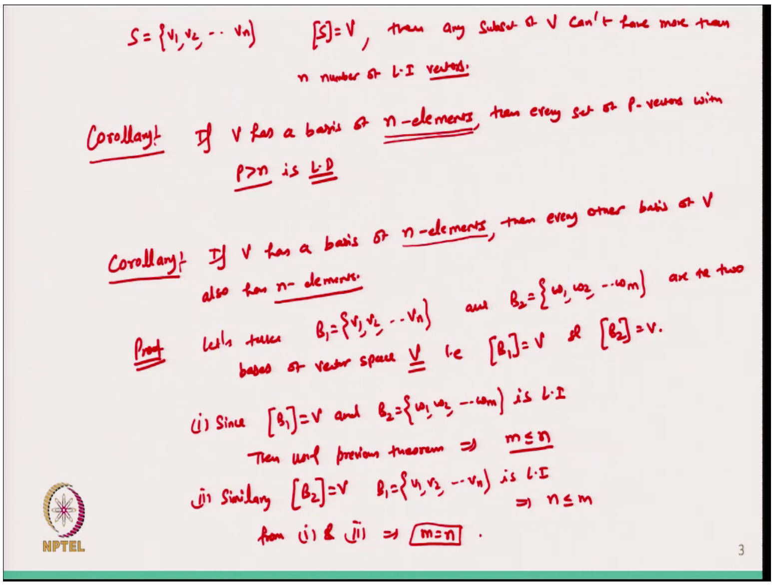

Now; so, in the previous one, we have discussed that if I have a set S which contains n number of elements and that S spans the whole vector space V. Then then any subset of V can, then any subset of V cannot have more than n number of n; more n numbers of linearly independent vectors. So, that we have seen.

Now, there is a corollary. So, in this corollary; it says that if V has a basis; if the V has a basis of n-elements, then every set of P-vectors with P greater than n is linearly dependent. So, that we already we also seen using the one example, because the V has a basis which contains n-elements; it means, that V is n-dimensional and then every set of P-vectors with P greater than n is L-D. So, that we have already seen also.

Now, after this one we can have some another corollary. If the vector space V has a basis of n-elements, then every other basis of V also has n-elements; it means, if one basis has n-elements then, if I choose any other basis that also contain n-element.

So, the proof is. So, let us take basis B 1. Suppose, that contains v 1, v 2 up to v n. And I take B 1 and B 2 that contains w 1 w 2 up to w m. So, let take B 1 and B 2 are the two bases of vector space v; that is, B 1 spans V and b 2 also spans v.

Now, we have to show that n is equal to m. Now, since B 1 spans v and B 2 is w 1 w 2 up to w m is L I then, using previous theorem. We have that m will be less than equal to n, in the previous theorem we have discussed. So, this is the 1st one, 2nd one. Similarly, B 2 spans v, because it is a basis. So, if it is basis definitely span.

And B 1 that is containing v 1 v 2 up to v n elements is L-I. So, it spans V and this set is L-I which implies that V which implies that n definitely will be less than m. So, from these two condition. So, from 1 and 2 we get m is equal to n. So, if I choose any two basis then, they will contain that the set will contain the same number of elements in that basis. So, this is the proof we have done now.

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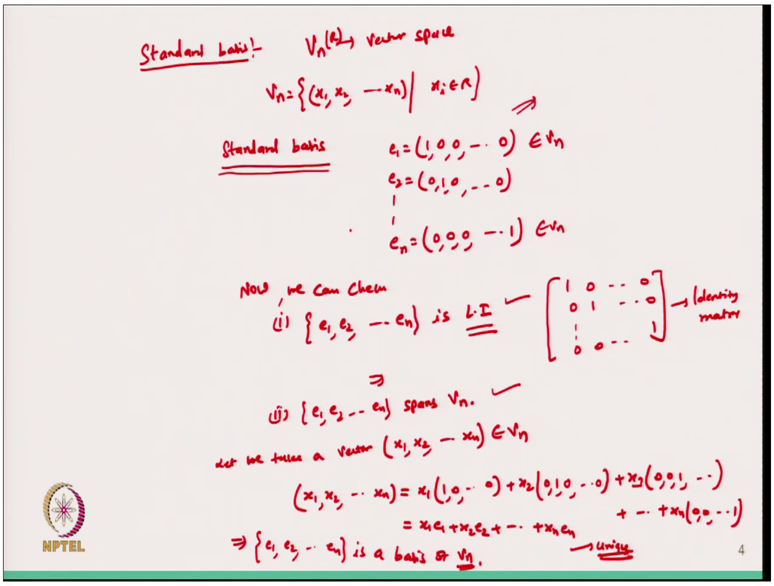

So, after this one we can discuss about the standard basis. So, standard basis we can discuss like we have a V n; V n is a we know it is a vector space and V n contains all the elements like this 1 x 2 x 1 x 2 up to x n such that each x i belongs to real line.

So, this is vector space over the real line that we already know this one. Now, and in this case, we can define the standard basis. The first basis I can define as e 1 that is I can write 1 here and 0 0 0, another I can define the vector 0 1 0 0 so on and in the last I can define e n that contain 0 0 0 1. So, this all belongs to V n.

Now, how they are the standard basis. Now, we can check the set e 1 e 2 up to e n. So, this is the set is L-I. So, why it is L-I? Because the corresponding matrix will get 0 1 0 0 0 up to 1. So, you will get only unit matrix or identity matrix. So, we call it and this is always invertible. So, in this case this matrix identity matrix, so which implies that the set of vectors are linearly independent. So, the 1st claim is over. The 2nd one is that the set e 1, e 2 up to e n spans V n.

So, that is very easily we can show that let we take a vector say x 1 x 2 up to x n belongs to V n. So, this case I can write x 1 x 2 to x n; this can be written very easily like x 1, 1 0 0 plus; x 2, 0 1 0 up to 0; x 3, 0 0 1 and x n 0 0 up to. And I maybe, I can write x 1 e 1 plus x 2 e 2 x n e n. So, this vector can be written like this one and this is unique. So, we can write this uniquely.

So, from here, I can say that it is linearly independent and it spans V n. So, from here I can say that the set this e 1 e 2 e n is a basis of V and we call it standard basis, because it contains only one non-zero element and all other elements 0. So, that we can always use if somebody ask about some basis of any vector space, then we can the simplest basis we can have with the standard basis.

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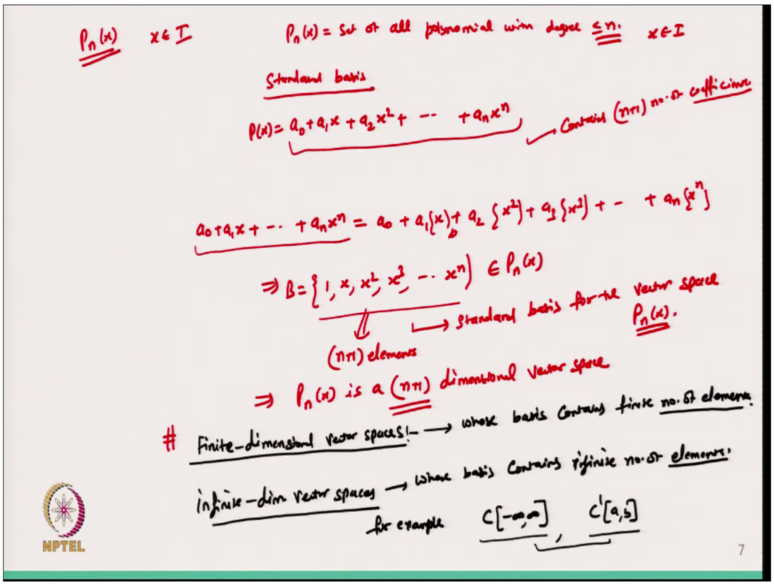

Now. So, after discussing this one, what about the vector space P n x where x belongs to some interval I. So, we know that P n x is set of all polynomials with degree less than equal to n and x is coming from the interval.

So, we know that it is a vector space. So, in this vector space, the standard basis. Now, if I take the element any element from here. So, let me choose, let me write a 0 plus a 1 x plus a 2 x square up to a n x n. So, this is the polynomial I have from here. So, that it is I call it P x. So, in this P x you know that it contains n plus 1 number of co-efficient.

So, this I can write as a 0 a n x n. So, this I can write as a 0 plus a 1 and then, x plus a 2 x square plus a 3. So, this is just a vector I am taking, a 3 x 3 a n x power n. So, from here, you can say that if I choose any vector from here, then I just take a set containing the element 1 x x square x cube x n.

So, this is a set I call it maybe, I call it B. So, this is the set that belongs to P n x, because 1 a also belongs to P n x x is also belongs to P n x. And if I take any element from this P n x, it can be written as a linear combination of this vectors and this is the simplest one.

So, from here, I can say that this is the standard basis for the vector space P n x and from here. And this vector contains n plus 1 number of elements. So, from here, I can say that P n x is a n plus 1 dimension space. So, it is n plus 1 dimensional vector space.

Now, from here, we can define now, from I can define the terminology finite dimensional spaces; final dimensional vector spaces where. So, these are the those whose basis contains finite number of elements like we have seen V n point P n x like this one.

And another is infinite dimensional, where basis contains infinite number of elements. For example, if I take the vector space of all the continuous function defined from minus infinity to infinity or I take the space c 1 defined from a to b. So, set of all the continuously differentiable function whose derivative is also differentiable or whose derivative is also continuous and defined from a to b.

So, this is c 1 a b, this is c minus infinity to infinity. So, all these functions make the vector space and this is infinity dimension, because they cannot be represented by finite number of elements from that set. So, these are called the infinity dimensional spaces.

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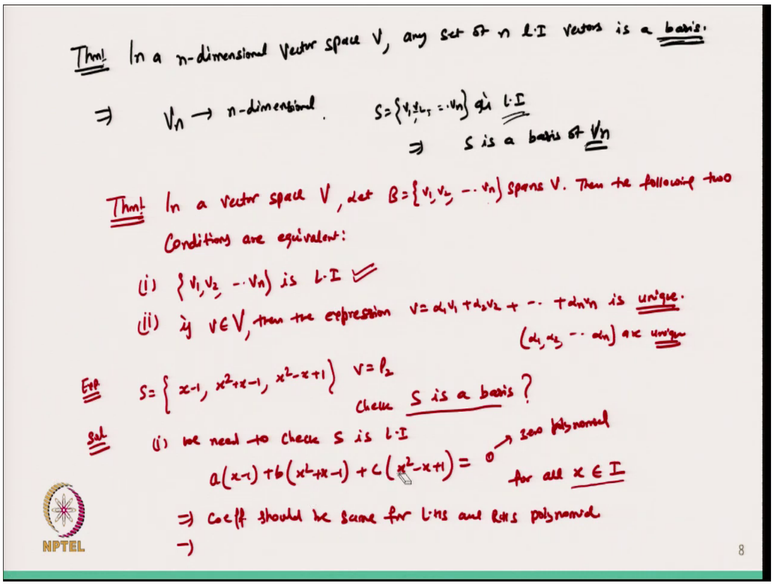

Now, from here there is a one more result that in a n-dimensional vector space V, any set of n linearly independent vector; linearly independent vectors is a basis, because if it is n-dimensional vector space, then you take any set which contains n linearly independent vector and that vector, that set will be a basis for that vector space V. So, this is we can seen very easily; it means that which implies that suppose, I take V n element V n set. So, this is n-dimensional.

So, if I take a set as which contains n number of elements v 1, v 2 to v n and this set is linearly independent, then I can say that S is a basis of V n. So, if it is linearly independent, then it will definitely it will span the whole vector space V n. So, that is why we call it that it is also a basis of this one.

So, once they are linear independent that is enough that this spans the whole V n, because it is a n dimensional space. So, this is just the application of the previous theorems based on this vector spaces the dimension and the basis. So, it is understood we have also given lot of example based on this one.

Now, after this one, one more terminology we want to use is that theorem that in a vector space V. Let I take a basis B spans V then the. So, I just write a set B which spans V. I do not know what is whether it is a basis or not. Then, the following two conditions are equivalent. The 1st one is the set v 1 v 2 up to v n is linearly independent.

So, it spans V that is ok. Then, I can say that v 1 v 2 v n is L-I or I can say that if I take any element v from V then, the expression v is equal to alpha 1 v 1 alpha 2 v 2 alpha n v n is unique; it means, that if I write this expression then this alpha 1 alpha 2 alpha n all are unique. Only one representation is there that we can write V as a linear combination of v 1 v 2 up to v n.

So, this is unique or maybe, I can say that alpha 1 alpha 2 alpha n are unique. So, either it is L-I or it is unique both things are same. If it is L-I, then it is unique and if it is unique, then definitely it is L-I. So, just it can be seen very clearly, because it spans this one. So, if it is span, then any element from V can be written a linear combination of this one.

And if this is linearly independent also, then we have seen that the corresponding matrix we have seen that this is non-singular and we can have the unique solution.

So, in that case the representation will be unique and if the representation is unique, then this linearly independent. So, we can just understand from this one and it is up to the viewers to understand and to do the proof for this one. It is very simple. Now, after this one, let us do few examples based on this. So, let us write some question.

Suppose, I said that I have a set S. So, this is example let I take the set S which contains x minus 1 and x square plus x minus 1 x square minus x plus 1 and suppose, my vector space V is P 2 ok. So, I want to check S is a basis or not. So, this one we have to check.

And now, I am taking the V as P 2. So, P 2 means set of all the polynomials of degree less than equal to 2. So, this one I want to check. Now, 1st we need to check S is linearly independent.

So, for this one, what I am going to do is. I will take a x minus 1 plus b x square plus x minus 1 plus c x square minus x plus 1 and then it equal to putting equal to 0; 0 means 0 polynomial, but here we. So, ok let us solve this one. Now, this is true for all x belongs to the interval. This expression is true for all x belongs to the interval.

Otherwise, if you put like this one, then maybe you can find the roots, but the condition here is that this is true for all x belongs to a.

So, if it is true for all x; implies that the coefficient should be same for left-hand side and right-hand side polynomial. So, from here which implies that now, I can choose the terms corresponding to x square. So, it is b plus c.

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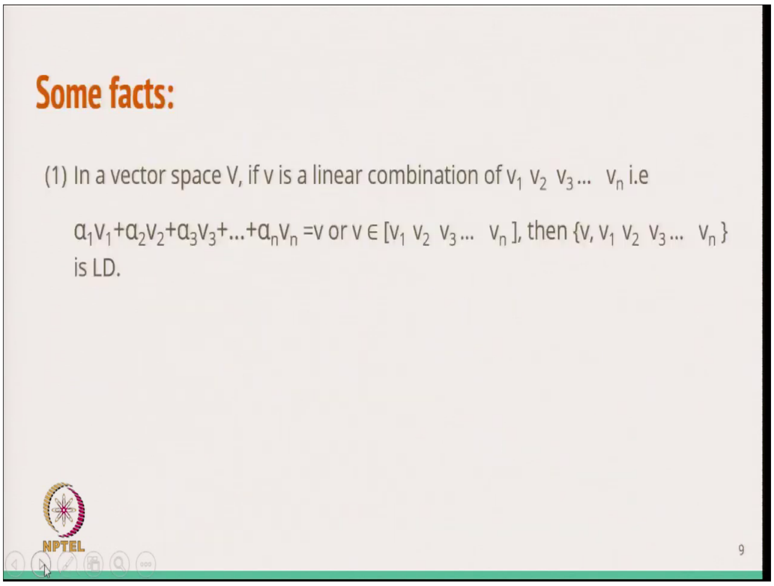

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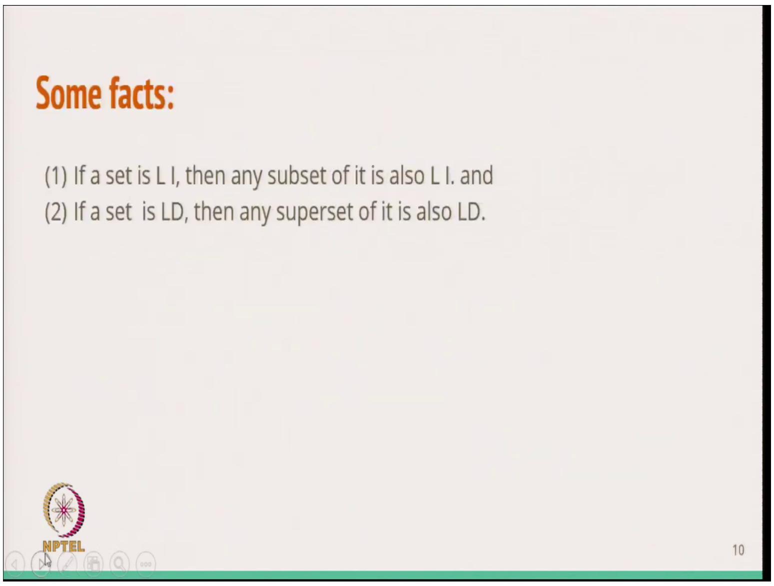

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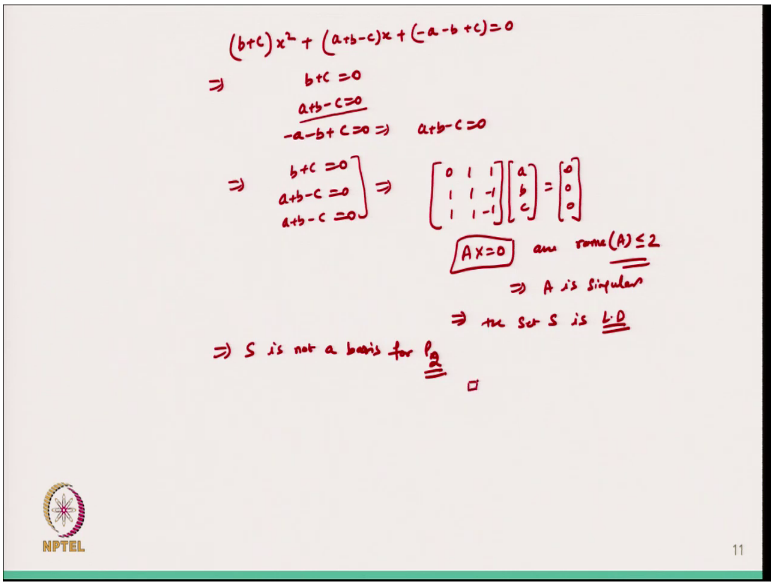

Now, from here I can write b plus c x square; that is, I can find then with respect to x I can write a plus b minus c x plus the constant term. So, I can write here, minus a minus b plus c equal to 0. So, this one I can write. From here, I can write b plus c is equal to 0, then a plus b minus c equal to 0 and from here I can have minus a minus b plus c equal to 0.

Now, from here, this one I can write expression as a plus b minus c equal to 0 and that is already there. It means this system is, because corresponding to x is a plus b minus c and this is minus a minus b plus c and this is both are same. So, it means I have a system verification that is b plus c equal to 0, a plus b minus c equal to 0 and a plus b minus c equal to 0.

So, this one equation is a redundant equation. So, from here, I can say that this is the matrix with rank 2, because if I write the matrix correspond matrix. So, it is 1 0 1 1, 1 1 minus 1 and 1 1 minus 1 a b c equal to 0 0 0 and from here you can check that this matrix cannot be.

So, A X is equal to 0 and a rank of A in this case is always less than equal to 2 it cannot be 3. Because just I can reduce this last row as a 0 here, no problem and from here, I can say that A is singular and which implies that the set S is linearly dependent.

So, if it is a linearly dependent then, from here, I can say that the set S is not a basis for P 2, because if it is not a L-I, then from here, we can say that this set is not going to be the basis for this one, because for the basis, it should be linearly independent and should span this one.

So, first condition is not satisfied from here only we can conclude that this S is not the basis for the set of polynomial of degree less than equal to 2 ok. So, from here. So, I think we can stop here. So, today we have discussed about the meaning of finite dimensional spaces and infinite dimensional spaces and discuss few examples. So, in the next lecture, we will continue with this one. So, thanks for watching.

Thanks very much.