Matrix Computation and its applications
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Lecture - 12
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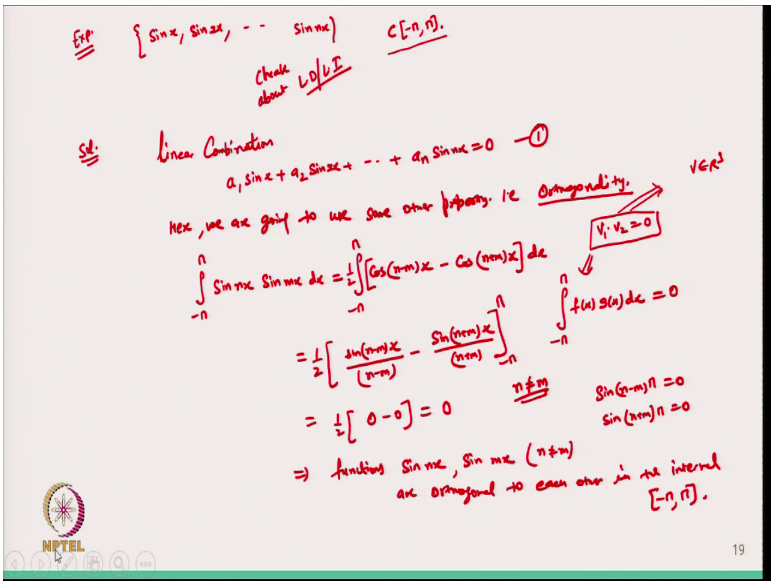

So, in the previous we have discussed one example that suppose we have set of vectors sin x, sin 2x up to sin nx and that belongs to the. So, in this case that belongs to the space of continuous function defined from minus pi to pi and we want to check whether this check about linearly dependent or linearly independent.

So, in this case what we are need to do is, because for checking the linearly dependent or independent I will take the linear combination. So, linear combination I am going to take. So, let us take it a 1 sin x plus a 2 sin 2x up to a n sin nx equal to 0, where 0 is a 0 function defined in this interval. So, now, so, in the previous lecture we have discussed that if I take the derivative and we n times and then we will get n by m matrix.

So, still we are unable to check show that whether these factors are linearly dependent or independent. So, here we are going to use some other property and that is orthogonality. So, here we are going to use the property of orthogonality. So, in this case we know that two vectors are orthogonal when their inner product or the linear.

So, suppose I have a vector V 1 and V 2 and till now we know that V 1 dot V 2 if it is 0 then we say that these two vectors are orthogonal to each other, but here we do not have the vector of this form. What we have used to do like we have a V belongs to R 3 or R 4, the Euclidean space.

So, here we extend this orthogonality to the integration from minus pi to pi and then taking function f x and g x dx. So, this or the concept of dot product becomes the integration from minus pi to pi f x dx because here we have a vector that is of discrete type, but here we have a vector that is a function. So, if this is equal to 0 then we say that the function f x and g x are orthogonal to each other in the interval from minus pi to pi. So, here we are using the same property.

So, now what we are going to do is we know that from minus pi to pi if I take sin nx and if I take sin mx dx then let us check what is going to happen here. So, here we are going to discuss the formula for sin nx and sin mx. So, this formula can written as half cos n minus mx minus cos n plus mx dx.

So, here I am using the formula for the sin inequality and now from here it can be written as half then I taking the integration. So, it become sin n minus mx divided by n minus m minus sin n plus mx divided by n plus m and then substituting the limit. Now, it is a n and m are the integers here.

So, in this case I know that the sin n minus m will be some integer pi. So, this will be 0 and similarly sin n plus m is also integer value pi that is also going to be 0 and similarly for minus pi. So, if you see from here if I substitute the limit then I am going to get half and then 0 minus 0. So, that is 0. So, the dot product we are defined like this one. So, it is inner product basically we are defined. So, in this case it is going to be 0.

So, from here we can say that the functions sin nx sin mx. So, here we are considering one condition that when n is not equal to m because it may happen that when n is equal to m. So, if n is equal to m then it is going to be the different thing because when I take n is equal to m then it will be sin square nx and then we can very easily can integrate this one.

So, the function sin nx and mx when n is not equal to m are orthogonal to each other in the interval from minus pi to pi. So, this one we can define now from here. So, let us use this one this property.

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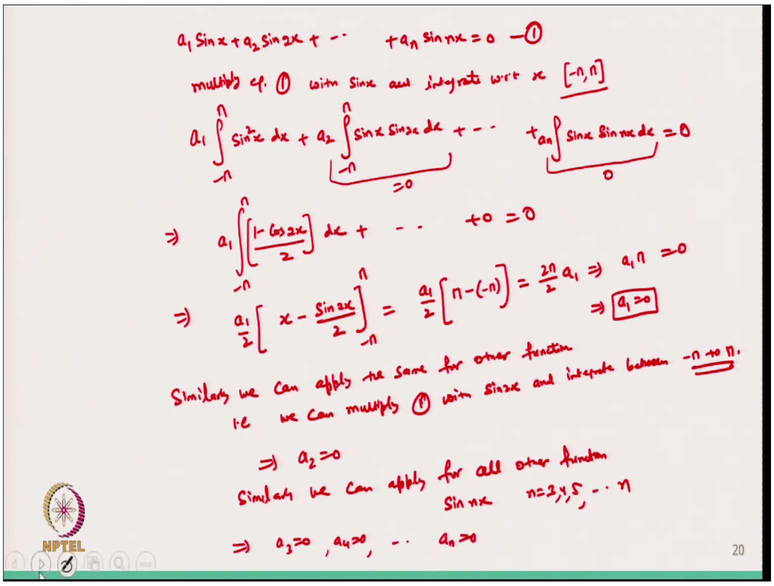

So, now I have a 1 sin x plus a 2 sin 2x up to a n sin nx equal to 0. So, this is the equation we already have 1. Now, what I do is that multiply equation 1. So, here I am having the sin x. So, multiply equation 1 with sin x and integrate with respect to x. So, what I will get?

I will get a 1 integration from minus pi to pi. So, I am taking the integration from minus pi to pi and x belongs to this one. Then it become sin x square dx plus a 2 minus pi to pi sin x sin 2x dx and so on and this is a n. So, I can take my a n outside. So, this is my a n and I can write sin x sin nx dx is equal to 0.

Now, using this property we know that sin x and sin 2x that is going to be equal to 0, this is also going to be 0. So, all the terms will be 0 except the only this term and this term from here you can see that I can write a 1 and this integration you can it can be written as 1 minus cos 2x divided by 2 and then doing the integration from minus pi to pi dx. So, definitely its value is not going to be 0. So, plus everything is 0 and that is equal to 0.

Now, from here you can check that this is a 1 by 2 and then I do I can do the integration of this one. So, it is x minus sin 2x by 2 and substituting this integral. So, it can be written as a 1 by 2. So, sin 2 pi and sin minus 2 pi they are 0. So, I will get only here pi minus minus pi and from here I can write down that it is 2 pi by 2 a 1 and that is I am going to have a 1 pi and this is going to be 0.

So, from here you can check that a 1 is going to be 0. So, the first coefficient a 1 is becoming 0. So, similarly we can apply the same for other functions that is I can multiply. So, that is we can multiply 1 with sin 2 x and integrate between minus pi to pi. So, in that case also we will get a 2 and all other terms will be 0 because here also we are using the property of orthogonality.

So, from here I will get my a 2 equal to 0. So, similarly we can apply for all other functions that is sin nx, n can be 3, 4, 5 up to n and then from here we will get a 3 0, a 4 0 and a n 0.

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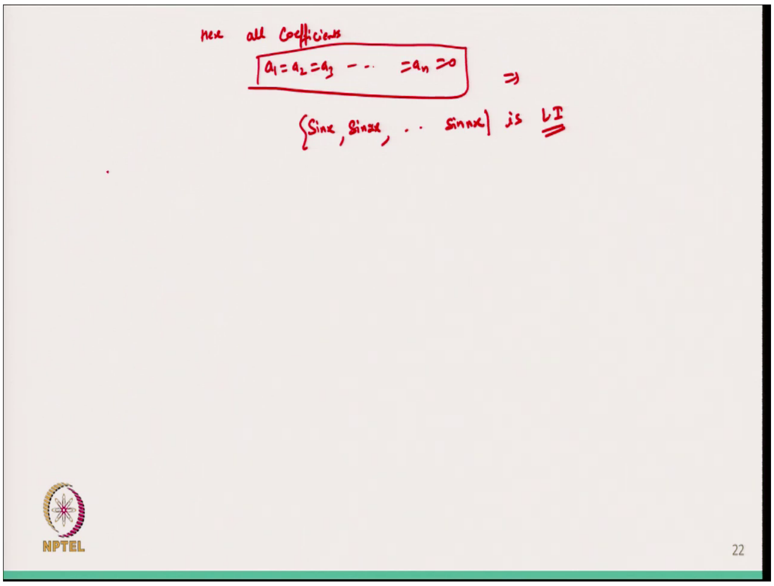

So, from here we can check that in this case here all coefficients that is a 1, a 2, a 3 a n all are coming 0 and from here we can see that the function sin x, sin 2 x, sin nx. So, this is a set is linearly independent. So, this is linearly independent set ok. So, from here we can check that this is the linearly independent. So, this is we are done.

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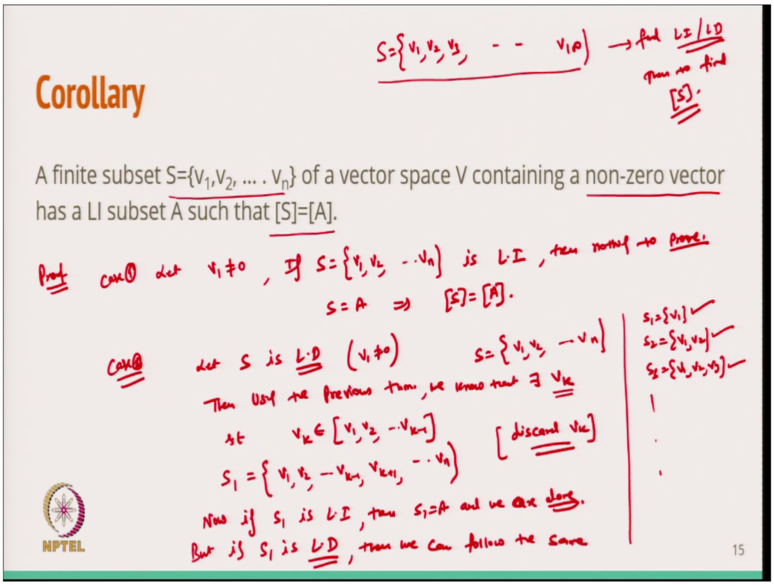

Now, in the previous class we have also started this property, this is very useful property. So, I want to discuss this property again here. So, in this case what is we are having? That we have a any finite subset of the vector space V containing a non-zero vector has a LI sub set A such that this one. It means suppose somebody gives me a set S and that contains suppose V 1, V 2, V 3 up to maybe V 100.

So, it contains 100 number of elements and somebody asked me to find whether it is LI or LD ok and then to find span. So, in this case it is very difficult sometimes to check whether the functions whether this set is a linearly independent or dependent. So, then what we can do? We can remove from this set those vectors which are making this set S as a linearly dependent.

Because we know that if I remove a element which is also a linearly also a linear combination of the previous one that we have already done in the theorem then at least we can remove those vectors and then we know that the span of S will be equivalent to that span of that vector span of that set.

So, these things we are going to do. So, this is very useful when we have a large number of vectors there. So, we have done this one. So, in this case my S is given to me. So, let we assume that V 1 is not equal to 0 ok. So, if the set S is linearly independent then nothing to prove because in that case the S is itself a A and from there the span of S is equal to span of A ok. So, this is ok.

Now, the so, it is a case 1. Case 2, let S is linearly dependent LD and V 1 I have we have taken that not equal to 0. Then using the previous theorem, so, we have a set S V 1, V 2, V n. So, what we are going to do? We are going to check because V 1 is not equal to 0. So, I will I can make a set like this one; S 1 that will contain V 1 only; S 2 V 1, V 2; S 3 V 1, V 2, V 3 like this one.

So, we can because this is a linear dependent only one vector is there then we can check whether it is linearly independent or dependent then we can check this is a linearly independent or dependent. So, then from there using the previous theorem we know that there exists a vector V K such that V K belongs to the span of the vectors residing this one. So, that we have already done in the theorem because it is linear dependent.

So, from here we can find a vector say V K which is which can be written as a linear combination of the all the vector residing this one. So, from here we can say that. So, then we can write a subset. So, we can write a subset may be S 1 that we can write V 1, V 2, V K minus 1, V K plus 1 up to V n. So, we have discarded V K and then we left with this one.

So, now, if so, now, if S 1 is linearly independent then S 1 is equal to A and we are done. But, if S 1 is linearly dependent then we can follow the same procedure.

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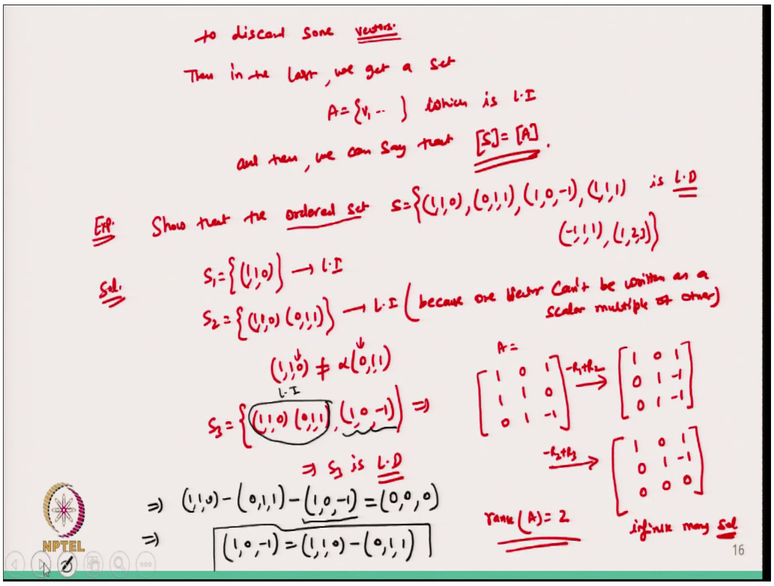

Whatever the procedure we have just defined we can follow the same procedure. to discard some vectors. Like we have discarded V K then again we can discard the vector which is a linear combination of the previous one. And so, this way we can discard the vectors whatever the vectors are there. And then in the last we get a set we call it A that is V 1 up to this one is which is linearly independent and then we can say that the span of S will be equal to the span of A.

So, this is the way we can use this theorem. So, now, let us do one example of this one. Based on this one we want to do one example. So, let us do one example. So, the question is show that the ordered set, so, this is the set we are taking 1, 1, 0; 0, 1, 1; 1, 0, minus 1; 1, 1, 1. So, suppose this is my set S. So, I want to show that this set is LD or maybe I can have a two more elements. Maybe I can put two more elements. Maybe I can have minus 1, 1, 1 and maybe 1, 2, 3 like this one.

Suppose I take this vector. It just takes 6 vectors and I want to check whether they are LD or not. So, the one procedure is that taking the linear combination solving and then we want to show that whether it is LD or not. Other thing is that it is ordered set. So, we define a set S 1 that contains the first element and this is of course, it is LI because it is only one element and that is a non-zero element then we define S 2. So, S 2 I am taking 1, 1, 0 and 0, 1, 1.

So, I am adding one more element there and then we want to check. So, from here now we can say that this vector and this vector both are linearly independent because one vector cannot be written as a scalar multiple of other. Like I cannot write 1, 1, 0 cannot be written as a sum alpha times 0, 1, 1 because here the 0 element is in the third come position and here 0 is coming the first position. So, that is not possible. So, these are linearly independent.

Then I take S 3. So, S 3 I will take 1, 1, 0; 0, 1, 1 and then 1, 0, minus 1. Now, from here if you see I just calculate this value. So, this become a; so, I will get a matrix that is 1 1 0 0 1 1 and 1 0 minus 1 and we can reduce this matrix into the echelon form. So, it is 1 0 1 I can make it 0 by multiplying. So, I can multiply minus R 1 plus R 2. So, it will be 0 that is 1 and it is minus 1 0 1 minus 1.

Then I will again want to make this element 0. So, I will apply minus R 2 plus R 3. So, this is 1 0 1 0 1 minus 1 then 0 0 0 because these are same. And so, from here I can say that rank of this matrix whatever the matrix is there I just call it A matrix 2. So, infinite many solution, ok. So, using this one we can say that my S 3 is linearly dependent. So, when we have; so, when we have not added this element it was LI this was LI, but then we put this element then the whole set becomes the LD.

Now, from here I just check that 1, 1, 0 minus 0, 1, 1 minus 1, 0, minus 1. So, 1, minus 1 0; 1 minus 1 0 and minus 1 and plus 1 that is also 0. So, it becomes 0. So, from here you can say that the third element this one, 1, 0 minus 1 can be written as 1, 1, 0 minus 0, 1, 1 ok so, 1, minus 1, 0 and this yeah. So, from here I can say that the third vector can be written as a linear combination of the previous two one ok.

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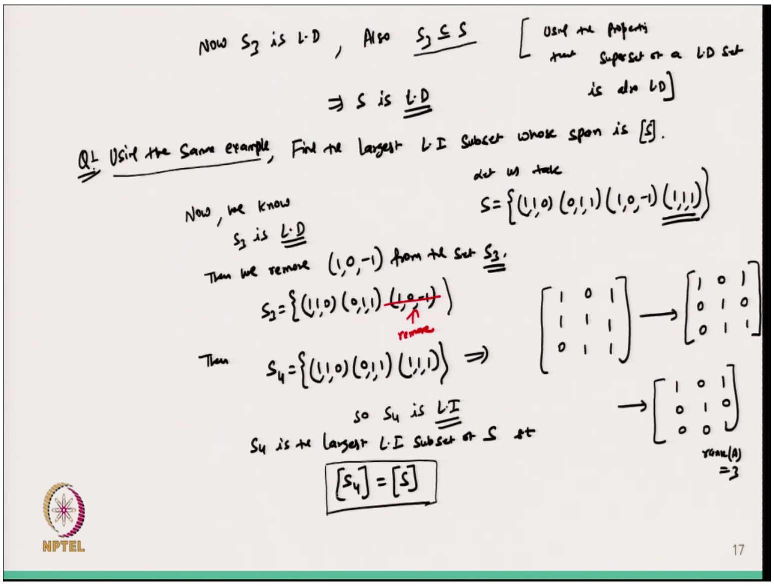

So, from here after doing this one, now S 3 is LD Linear Dependent. Now, also S 3 is a subset of S 6 because S 6 contain the 6 element. S 6 or maybe I can call it S, not x 6 let us call it S and we know that if the S 3 is linearly dependent and then the superset of this is also linearly dependent.

So, using the property that superset of a linearly dependent set is also LD. So, from here I can say that S is linearly dependent. So, this way we can find out that it is linearly dependent. Now, next question comes from here the same example. Using the same example find the largest linearly independent subset whose span is whose span is equal to S ok.

So, in this case I just for the calculation I just take, let us take S as. So, I just take the first four elements; 1, 1, 0; 0, 1, 1; 1, 0, minus 1 and 1, 1, 1. So, I just take this element four elements. So, I want to find the largest linearly independent subset. Now, we know that S 3 is LD then we remove then we remove 1, 0, minus 1 from the set S 3 ok. So, S 3 was now becoming 1, 1, 0; 0, 1, 1 and it was 1, 0, minus 1. So, I what I am going to do? I just remove this one remove.

Then I take the next set S 4 1, 1, 0; 0, 1, 1 and then the next element is this one 1, 1, 1 and from here I just want to check whether this is linearly independent or not. So, now, I can take 1 1 0; 0 1 1. So, from here I just apply the same 0 1 because minus 1 I am multiplying. So, it is 0 and then 0 1 1 and then again I am applying here. So, it is 1 0 1 0 1 0 0 minus 1 I am multiplying adding here. So, it is 0 and 1. So, rank is of this matrix is 3. So, S 4 is linearly independent and now this is.

So, we can say that S 4 is the largest linearly independent subset of S such that S 4 span is equal to S span. Because the element which was making the set linearly dependent we have removed that vector because that vector is not going to contribute in the span. And then we know that the remaining element will also span the same space as spanned by the set S. In the S we do not care about whether there is a LI or LD, it is spanning some space.

But after this one we have removed all these vectors which are making this set linearly dependent and then after getting we get the largest linearly independent subset and then the span of S 4 will definitely equal to the span of S. So, this is the proof of this one. So, ok let us stop here.

So, in the today lecture we have discussed two examples based on the set of that how we can reduce the number of vectors in a linearly dependent sets and then we can show that the span of the reduced set is equal to the span of the original one. So, in the next lecture we will continue with this. So, thanks for watching.

Thanks very much.