Matrix Computation and its applications
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Lecture - 10
Continued…

Hello viewers. So, welcome back to the course on Matrix computation and it is applications. So, we will continue with the some other examples of linearly dependence and lineraly independence. And, we are also going to discuss some important facts about that one. So, let us start with that.

So, this is a lecture number 10. Now, in the previous lecture, we have started with some the definition of linearly dependence and independence.

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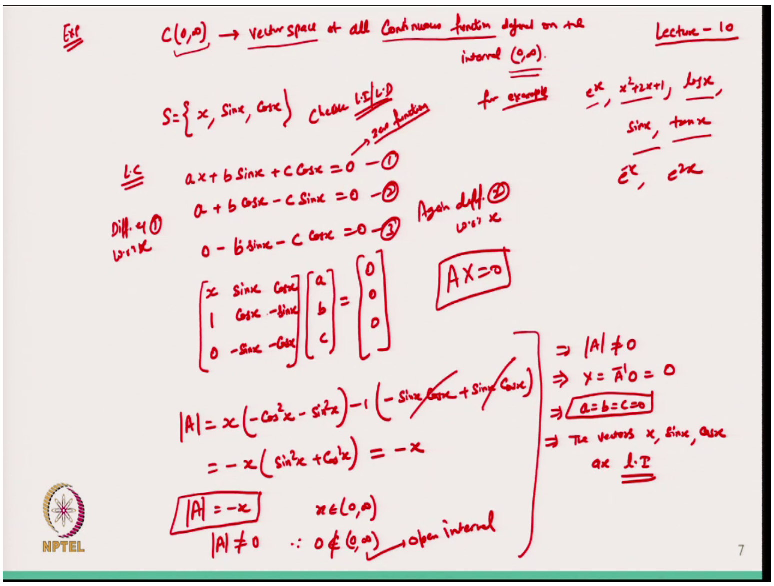

So, in this case I am taking one example that belongs to the C 0 infinity. So, it is a space vector space of all continuous function defined on the interval 0 infinity. So, this is vector space, another type of vector space. In which all the continuous functions falls, which are defined on the interval 0 to infinity.

So, for example, if I want to check the elements of this vector space, then you can say that exponential belongs to this or maybe polynomial x square plus 2x plus 1 belongs to this or maybe I can take log function, because x is not 0 so, no problem. So, this belongs to this one or sin x tan x. So, all these type of functions they belongs to this category ok.

So, these type of function and it contain infinite number of functions, it is just I am giving few examples. So, it can be a e minus x, it can be a e 2x. So, if you see from here then it contains infinite number of elements belongs to this space. So, it is a vector space.

Now, in this case suppose I take a set of the elements and element I am choosing just taking x, and suppose I am taking sin x and suppose I am taking cos x so, just three elements from this one. And, I want to check linearly independence or dependence of these vectors. So, I am talking the about this vector. These are the three vectors belong to this one.

Now, in this case also. So, let us take the linear combination. So, linear combination is I am taking ax plus b sin x plus c cos x equal to 0 ok. So, in this case my x is coming from this interval. And, a b c are the scalars so, that we are going to find out just to check whether they are linearly independent or dependent. So, I just taken the linear combination and putting 0. So, this 0 is a 0 function.

So, it is basically if you want to check the C 0 infinity is vector space or not then we have to take the 0 function as a additive identity. So, this is the additive identity 0 functions. So, now, from here I get this. Now, the question comes that how we can find out the value of a, b, c, because this is just the 1 equation.

So, in this case we just take the help of the some other concept of the function. Now, we know that this function belongs to the C 0 infinity, but from here also, I know that x I can is a polynomial and even I can take the derivative this also, sin x also I can take the derivative cos x also, I can take the derivative. So, this function are differentiable. So, I can just I just write 1.

So, I can just differentiate equation 1 with respect to x. So, I just differentiate. So, from here I can get it is a plus b cos x minus c sin x equal to 0. So, I get the second equation, but I need to take the help of one more equation, so, that I will get 3 by 3 system. So, again so, after this one again differentiate 2 with respect to x.

So, this will be I am taking just a x and sin x the derivative x is 1. So, here now it become just constant. So, it is 0 plus, then minus b sin x, because the derivative cos x is minus sin x and this will be minus c cos x it could be 0. So, this is my third equation. And, we the function which are differentiable that is also member of this one, because a function is a differentiable it definitely it will be a continuous function.

So, from here I get this value now from here I will just write this into the form of a matrix. So, it will be a, b and c and then it is I can write from here x, it is sin x, it is cos x, 1 cos x minus sin x 0, minus sin x minus cos x and that is equal to 0 0 0. So, from here I get a matrix a homogeneous system A x equal to 0.

Now, I have to find out the determinant of this one. So, let us take the determinant. So, I just want to find the determinant of A. So, let us find out. So, it is starting from the first column so, x and then minus cos square x and then minus sin square x minus 1. So, this is minus 1. So, it will be minus sin x cos x and then plus sin x cos x. So, that is my in determinant and this is cancel.

So, from here I can get this value minus x and it would be sin square x plus cos square x and that is 1. So, from here I will get the value minus x. Now, the determinant of this is minus x. Now, from here if you see from that x is coming from interval, from 0 to infinity and 0 is not the member of this interval, because it is a open interval.

So, from here we can say that the determinant of a is not 0 in this case, because 0 does not belongs to this one it is a open interval. So, from here you can check. So, from here since my determinant is not equal to 0. So, I am going to have x is equal to A inverse 0 and that will be 0.

So, from here I can have my a, b, c both all the three are 0 and from here you can say that the vectors so, the vectors x sin x and cos x are linearly independent ok. So, from here if this type of things we have to do, when we deal with the space of continuous function or differentiable function like that one.

So, this is the way we can do. Now, so, after doing this example so, let us take one more example based on this because sometime I just take one more example.

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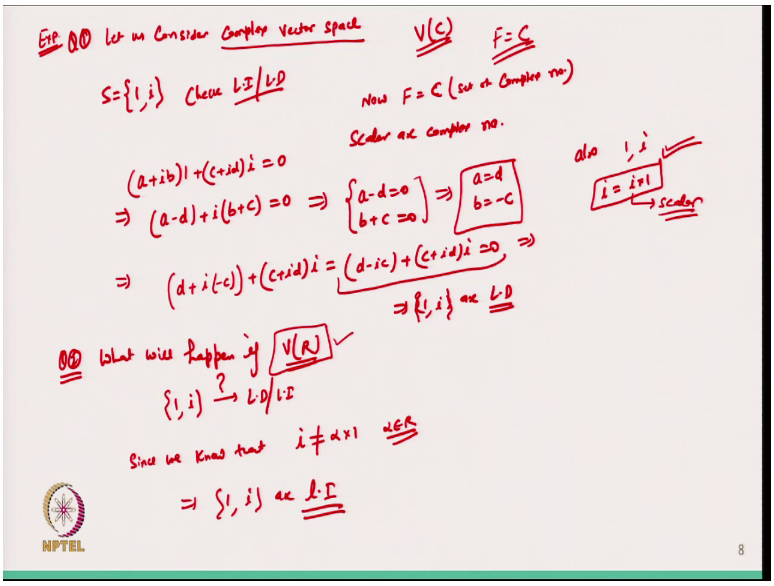

So, let us consider complex vector space. So, complex vector space means the vector space V define on the complex number C. So, institute to the real the field of real numbers we are taking the field as set of complex numbers. So, in this case we are taking this one.

Now, so, let us take so, let us I choose the set S that is I just take 1 and i. Now, I want to check whether they are linearly independent or dependent, linearly independent or linearly dependent. So, I will take the linear combination. So, in this case now the field F is set of complex number. So, that is set of complex numbers. So, in this case my scalars are complex numbers ok.

So, let us take scalars. So, I just take the scalar a plus i b, this is the scalar plus 1, c plus i d i equal to 0. So, I just take the linear combination. So, this is my linear combination and from here I can just collect the real and imaginary part.

So, from here I can say that a is real and if I take i here so, it will be minus d. So, it will be a minus d plus i b plus c, that is going to be 0 and this is the 0 complex number. So, comparing the real and imaginary part. So, from here we can write that we get a minus d equal to 0, and b plus c is equal to 0.

So, this is the we got a minus d equal to 0 and b plus c equal to 0. Then from here a is equal to d and b is equal to minus c. So, I got from here now from here. I can write this as a is d. So, I can write d plus i b is minus c plus c plus i d i. So, this one I can write as so, d minus i c and c plus i d i equal to 0. So, this one I get from here this part.

Now, so, from here I can have this combination and one more also if I take the elements 1 and i. So, I know that, I can be written as i into 1. So, this one I can write, that 1 vector can be written as the scalar multiple of another vector. So, another vector is 1 and this is my scalar ok.

So, from here you can also check that 1 and i are linearly dependent, because even they no need to do all these one, we can also verify from here, that 1 vector can be written as a scalar multiple of the other. So, in this case I will say that these two vectors are L D, 1 and i they are linearly dependent.

Now, so, the same question. So, it is a question number 1, I can write question number 2, what will happen if we take the vector space V over the a real line. It means, I have the set 1 and i and I want to check whether it is L D or L I. So, this one I want to check, but now I am taking this vector space. So, vector space of real numbers.

So, scalar is coming from here. Now, we know that, since we know that that I cannot be written as sum alpha into 1. Where alpha belongs to real number it not possible unless until alpha is i, but we cannot choose alpha i is a i because we are taking the field as a real number only.

So, in this case I can say that the set 1 and i are linearly independent. So, the same set of vectors, but we are taking the different vector space, here it is a scalar field is equal to real number, then they become L I and if you take the vector space over the complex number then they are L D. So, this is one of the examples of having the same set of vectors, but coming from the different vector spaces.

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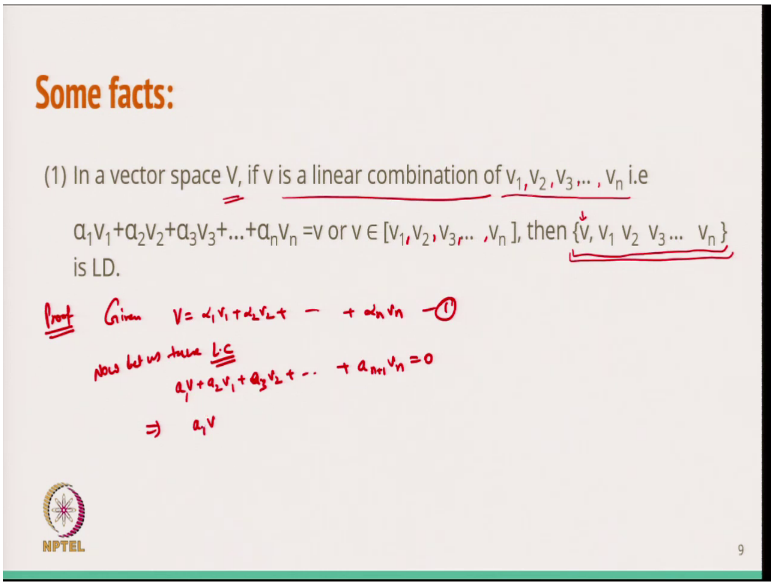

So, this one now there are some few facts we want to discuss, that in a vector space V. If, I take that v is a linear combination of v 1, v 2 and v n, that is I can write v as a alpha 1 v 1 plus alpha 2 v 2, because, this vector v is a linear combination of the vectors. It means that v belong to the span of this. So, it is belongs to the span of this vectors.

Then, if I take as a set of vectors adding that vector v then they are always L D, linearly dependent. So, this is just the fact here and the proof is very simple, it can be a theorem also, but just we are discussing in the same of facts. Now, given that v belong is a combination of alpha 1 v 2, alpha 2, alpha n v n.

It means, v can be written linear combination of these vectors. Now, I want to check what this set so it is given. Now, I just take the linear combination of v a 1 v 1 plus a 2 v 1 a 3 v 2 a n plus 1 v n equal to 0.

So, let us take, now let us take linear combination. So, this is the set of vectors and I take just linear combination. Now, from here that is given to me and I want to find out what is going to happen for the scalers, that it is whether it is going to be a LI or ID. Now, from here I can write this as a 1 v I just take on the other side.

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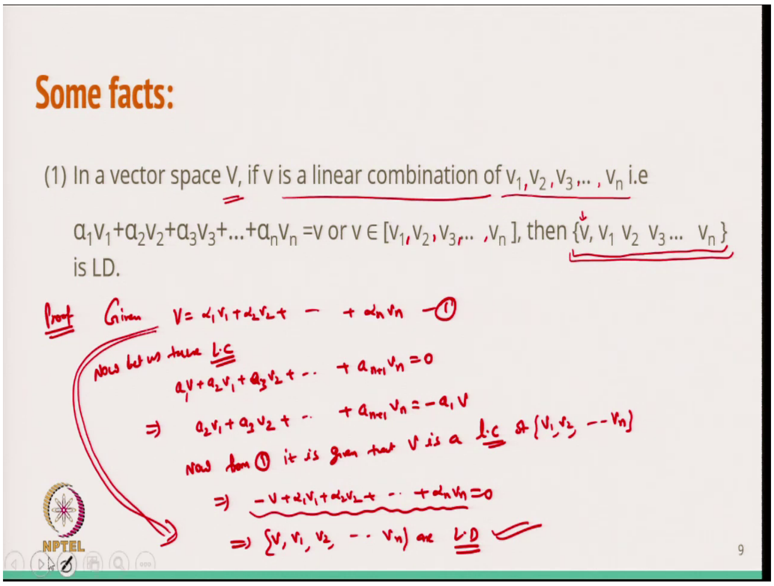

So, maybe I can just write this as I can write a 2 v 1, plus a 3 v 2, a n plus v n, I can write as a minus a 1 v this one I can write ok. Now, from 1 it is given that v is a linear combination of v 1, v 2 up to v n ok.

So, from here I just so, from here I can write this minus v 1, sorry minus I can write minus v plus alpha 1 v 1, alpha 2 v 2, alpha n v n equal to 0 from 1, because just I am taking the minus 1. And, now it became the linear combination of this one of v, v 1, v 2, up to v n and the coefficient is coming minus 1 always ok. So, from here I just can say that the set of vector v, v 1, v 2, v n are linearly dependent.

So, this is just became the linear dependent. So, from here it just choose my a 1, I just can divide by a 1 or I can choose my a 1 is equal to minus a 1 is equal to 1 and then it becomes the same thing. So, from here we get that this is linearly dependent.

So, maybe I can take this rules concept directly from here also, just I will apply this then take the minus v and then become the linear combination and then we get the linear dependent ok. So, from here we can say that they are linearly dependent.

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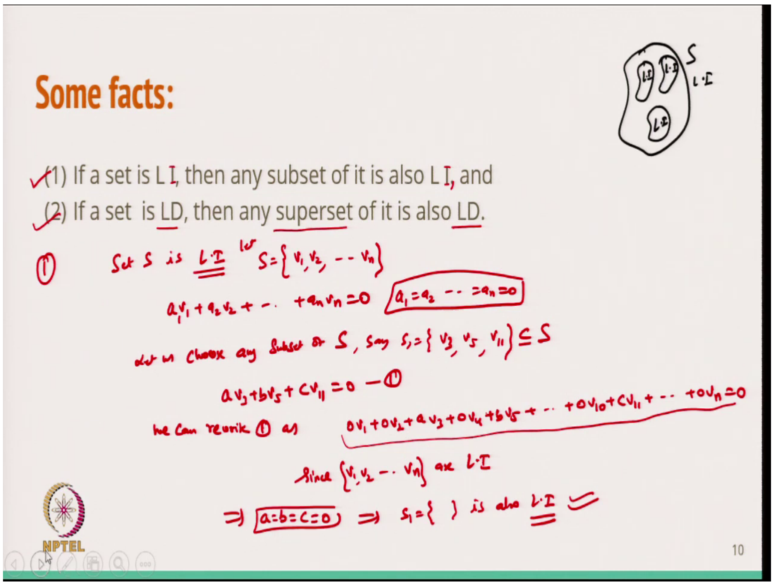

Now, the another fact is that let the set so, the first one, if the set is L I. So, L I means linearly independent, then any subset of it is also linearly independent. And, if the set is L D linear dependent then n is superset of it is a L D. So, let us do this one the first one. So, suppose I have a set S is linearly independent.

So, suppose I just take the S as v 1, v 2 up to v n ok. So, let and suppose this is LI. Let us say this one and S is LI. So, from here if I take a v 1, a 1 v 1, a 2 v 2, a n v n that is going to be 0 ok. So, that is there and all my a 1, a 2, a n all are 0. So, that is the meaning of linearly independent.

Now, I take any subset. So, let us let us choose any subset of the set S. So, let us take so, say I choose S 1 as just v 3, v 5 and maybe v 11, just 3 members I choosing ok. Then, I just want to check what is going to happen in this case. So, I can write from here a v 3 plus b v 5 plus c v 11 and I put this equal to linear combination like this one.

Now, this is coming from here. So, this is the subset of S. So, these elements are coming from here and S is itself a linear independent. So, this one I can write, we can rewrite, it can be rewritten, we can rewrite 1 as so, I can write this 1 as 0 v 1 plus 0 v 2 plus a, v 3 plus 0 v 4 plus b v 5. Similarly, 0, v 10 plus c, v 11 0 v n that is equal to 0, I am just adding 0, 0, 0 from here.

Now, this became the linear combination of the vectors belonging to the S and which is a linearly independent. So, from here since v 1, v 2 up to v n set of these vectors are linearly independent. So, from here I can say that b is equal to c is equal to a all are 0.

So, from here I can say that S 1 set of vector is also linearly independent. So, these vectors are also linearly independent. So, if I take any subset, then it is coming linearly independent. So, this is now we can take the next one that if the set is LD, then the superset is also LD.

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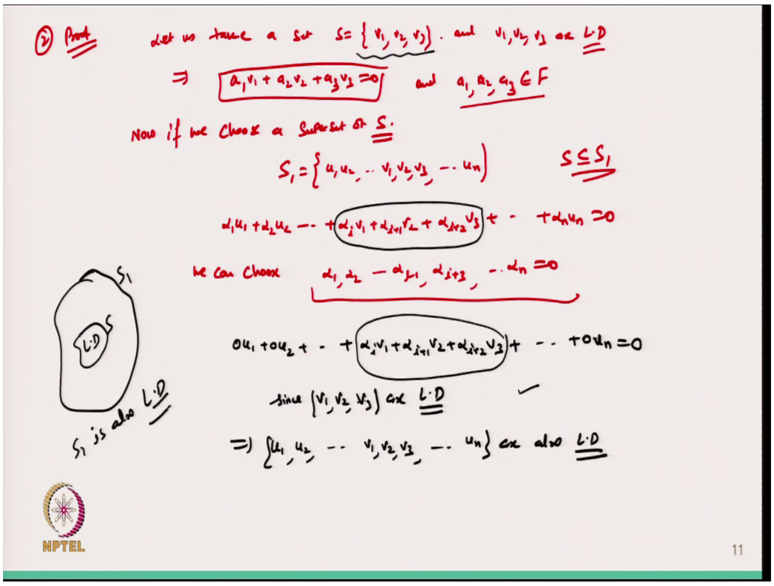

So, the proof, so, this is the second one. Now, let us take a set S suppose I take v 1, v 2, v 3 and let we take the set S and v 1, v 2, v 3 are linearly dependent. So, from here it means that suppose a 1, v 1 plus a 2, v 2 plus a 3, v 3 are 0. So, this is equal to 0 and a 1, a 2, a 3 belongs to the field F and they are need not be 0. So, they are LD ok, it may be 0 or not does not matter.

Now if we choose a, so, if we choose a superset of S. So, superset means that is set which contain this S is a superset of this S. So, let us take it S 1. So, S 1 will contain the elements some elements I can say as a u 1, u 2 some elements is there then v 1, v 2, v 3 are there and u n something like this one. So, that is so, from here I can say that S is a subset of S 1. So, S 1 is a superset of this.

Then, I can find out from here that if I take the linear combination, like alpha 1, u 1, plus alpha 2 u 2, plus some alpha i, v 1, alpha i plus 1 v 2, alpha i plus 2 v 3 and some alpha n u n that is going to be 0.

Now, in this case I want to check what is going to happen for this set of vectors. Now, from here you can check from here, that we can choose all alpha 1, alpha 2, alpha i minus 1 and then alpha i plus 3, alpha n equal to 0, we can choose no problem ok. Because linear combination this so, I can choose any alphas.

So, suppose I choose this one. Then, I will only left with these 3 points. So, from here I can have 0 u 1 plus 0 u 2, alpha i v 1, plus alpha i plus 1 v 2, plus alpha i 2 v 3 0 u n equal to 0.

Now, and this is already known that these are L D. So, we know that this set v 1, v 2, v 3 are LD, since v 1, v 2, and v 3 are linearly dependent L D. So, it means that this it can be 0 or not be the 0 so, but other elements are 0. So, from here I can say that even the set u 1, u 2, v 1, v 2, v 3 and u n they are also are also linearly dependent.

Because, in this case we have taken a sum set S belongs to this which contain only three element, that are linearly dependent and then we chosen we have taken one superset of this one and then we showed that this is also linear dependent. So, it means that if I choose any set S, that is linearly dependent and this is contained in any set S superset, then the whole superset I just take it S 1.

So, S 1 is also linearly dependent, but in this case I have sum set S is L I linearly independent. Because, I have taken this set L I and then I can choose any subset or like this one from this one. So, this is also L I, this is L I, this is also L I. So, they are linearly dependent. So, one is taking the superset and then finding in subset. And, another is we taking the small one and finding the superset of that. So, this is the way we can define these two facts.

So, we will stop here now. So, today we have discussed few examples new type of examples about to check the linearly dependence or independence. And, then we have discuss two facts about the superset or the subset of the linearly dependent or independent sets. So, in the next lecture we will continue with this one I so, thanks for watching.

Thanks very much.