# Matrix Computation and its applications

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## Lecture – 01
## Binary operation and groups

Hello viewers, welcome to the course on Matrix Computation and its application. So, this is the 1st lecture of this course. So, the outline of this course is as follows.

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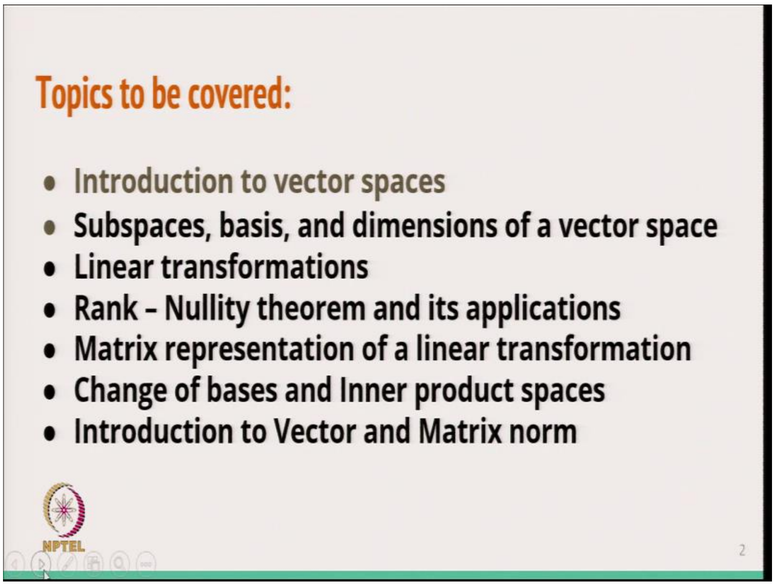


So, in this course we are going to cover, we start with the introduction to vector spaces and then we will discuss the subspaces, the basis and the dimension of the vector spaces. So, this is an introduction to basically linear algebra and then we will discuss the linear transformations that are involved in the vector spaces and we will also discuss a very important theorem that is the rank nullity theorem and its applications.

After that, because this course has the name matrix computation, we will discuss how the linear transformations lead to the matrix representation. And then after this we are going to discuss the change of basis and inner product spaces because in this course we are going to change we will get the different types of methods in which we will discuss how we can transform the different types of matrices into the simpler form of matrices.

So, that comes under the change of basis and also we will discuss the inner product spaces. And after that we will start with an introduction to vector and matrix norms, how we can find out the difference or how we can measure the magnitude of a matrix.

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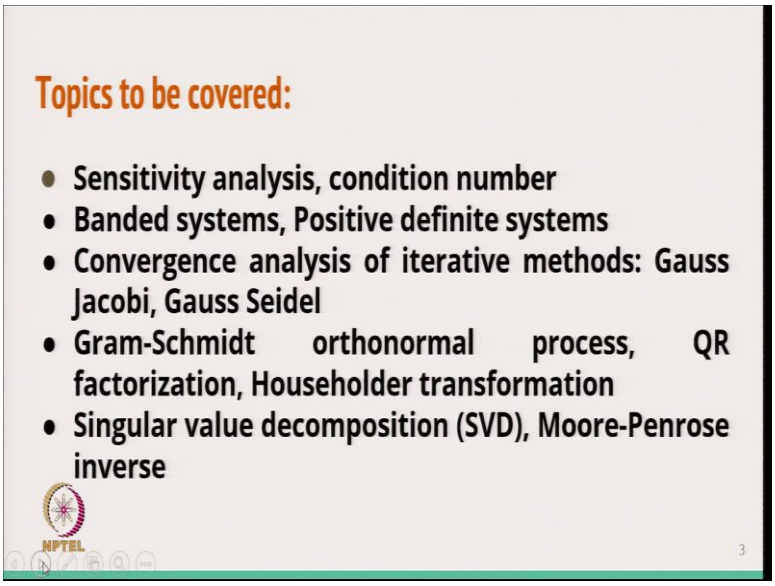


Then we will also start with sensitivity analysis and condition number of the matrix. And then we will discuss the special type of matrices that is called the banded system and positive definite system. We will also cover the convergence analysis of iterative methods.

For example, Gauss Jacobi, Gauss Seidel; because in this case we need to find out the matrix norms that will be used in this one and then we will discuss the Gram-Schmidt orthonormal process, QR factorization and Householder transformation. So, these transformations are very much important for us whenever we have data and then data is involved with the matrices.

So, these types of things are very essential dealing with this one and then we will also discuss the very important point that is called the singular value decompositions because in this case we are going to discuss the general eigenvalues.

Because when the matrix is a singular or  is not a square matrix then how we can find out the eigenvalues of that matrix, so, that will be discussed by the singular value decomposition. And then we are also going to discuss the Moore-Penrose inverse or pseudo inverse because when the matrix is not a square matrix then how we are going to take the inverse of that matrix. So, we are going to discuss the Moore-Penrose inverse.

So, in this course, let us start with the basics and the first part of this course deals with linear algebra.

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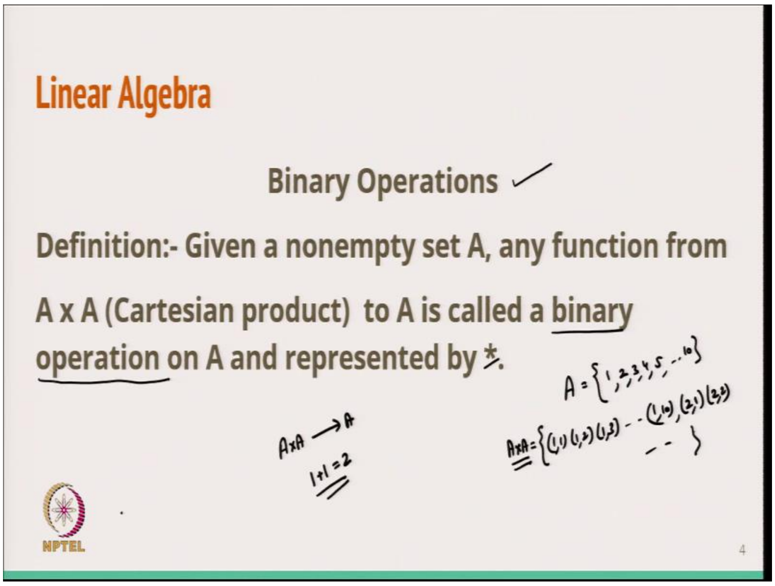


So, in linear algebra we start with the very basic relation that is called the binary operations. So, binary operation means, binary means, it is discussing the two things. So, this is the definition of the binary operation that given a non-empty set A, any function from A cross A because suppose I have some set A.

Suppose I take a set, it may be A={ 1, 2, 3, 4, 5,...,10} and suppose it goes upto 10 then any function from A x A. So, this is the Cartesian form. So, Cartesian forms gives you

AxA={(1,1),(1,2),...,(1,10),(2,1),(2,2),...}, A x A is a Cartesian product in which we are going to have one element from A and another element also from the A. So, this is the Cartesian product.

So, a binary operation given a non-empty set A any function from A x A to A is called a binary operation on A and represented by a star. So, we are going to discuss that we take A x A and that goes to A. So, in this case we are applying some operations here like I take 1 + 1. So, this is going to be 2 and 2 is also in the A. So, I can say that I have defined this binary operation as a plus sign. So, in this way we can find out whether we can apply or we can define the binary operations.

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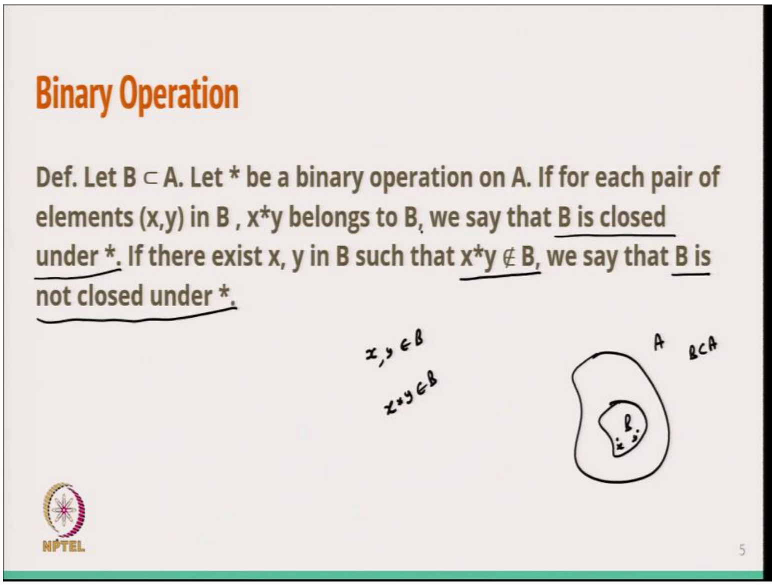


Then so, after we apply or we define a binary operation then we will say that let I take a; so, now suppose this is my some set A and I take a subset. So, some subset is there. So, that is a subset B. So, I will take that B ⊂ A. So, and then let * be a binary operation on A. So, binary operation on A, it is well defined here on A.

Now, if for each pair of elements x, y in B, so, now, I take two elements x and y that belongs to B. So, I just choose two elements one is here maybe or one is another here x and y and then if I take apply the same binary operation x * y and that also belongs to B, we say that the B is closed under that binary operation *. And if x and y in B such that the x * y does not belong to the B then we say that B is not closed under that operation star.

So, this way we can define whether that binary operation that the set is closed under that binary operation or not. So, that we can discuss.

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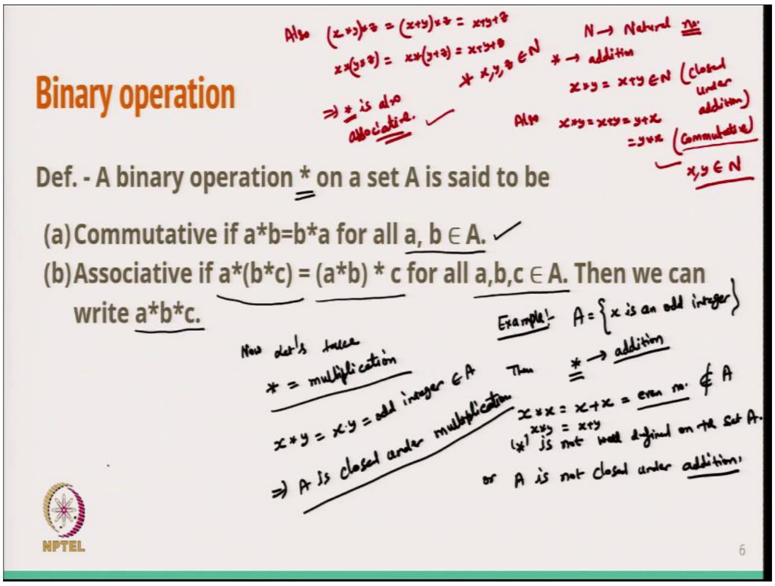


Now, we will define that as a binary operation. So, the binary operation we represent by * on a set A is said to be commutative if a * b = b * a, for all the elements a, b belongs to the set A and another property is that associativity. So, we will call it the binary operation * is set to be associative if we take a*( b * c) = (a * b) * c.

It means that a * b * c we can put the bracket anywhere among them. So, and if this is true for all values of a, b, c then we can write this one as a * b * c. So, if this is satisfied then I will say that the binary operation in this case is associative. So, in this case I will start with an example of how we can define.

So, let me take the example. So, let me take A ={ x is an odd integer}. So, I take the set A is the set of all the odd integers then I define the * binary operation as addition. So, in this case I know that if I put x * x = x + x and I know that the odd + odd = even.

So, this is an even number in this case because I know that 3 + 3 = 6, 3 + 9 = 12. So, all are even numbers. So, in this case I can say that and this does not belong to A. So, from here I can say that this binary operation * is not closed or I can say that this is not well defined on the set A or I can say that A is not closed under addition. Because only then you will say that it is closed under addition when I apply the binary operation that belongs to that set only then we will say that B is closed under that one.

So, in this case I will say that it is not closed under addition. For example, if I choose, this is the first * I have taken. Maybe I can take another star. So, let us take now let us take this binary operation as equal to multiplication. So, let us take this one. Suppose I take x * y =xy.

So, x * y = xy. Now, x is odd and y is odd. So, I know that I am just taking a simple multiplication and odd integer into odd integer that is again an odd integer; because I know that odd into odd in this case because you can say that 3 x 3= 9; 1 x 3 = 3. So, in this case I can say that this belongs to A. Here I can say that A is closed under multiplication.

So, this is closed under multiplication. So, for the same set I can define different type of binary operations and for some binary operations I can say that this is closed and for some binary operation it is not closed. So, let us take another binary operation. Suppose I take a set of natural numbers and in this case suppose I take the binary operation * as +.

So, in this case I know that if x * y = x + y and adding two natural numbers is again in the natural number. So, that belongs going to be N. So, it is closed under addition and also I can have x * y = x + y =y + x = y * x.

So, from here I can say that this is true for all elements x and y belongs to N. So, in this case I can say that this binary operation is a commutative operation. So, it is a commutative and if I take three natural numbers, x * y * z = (x+y)+z = x+(y+z).

So, from here I can say that this is true and it is immaterial where I put this bracket. So, in this case I can say that * associative.

So, it is associative in the case of natural numbers and the binary operation we are taking is addition. So, in this case it is commutative as well as associative. Now, we can also change the star with the subtraction. So, you will see that in this case it is not even a closed under subtraction, so, the set of natural numbers. So, this way we can define the binary operations and then we can check whether it is closed or not and commutative or associative or not.

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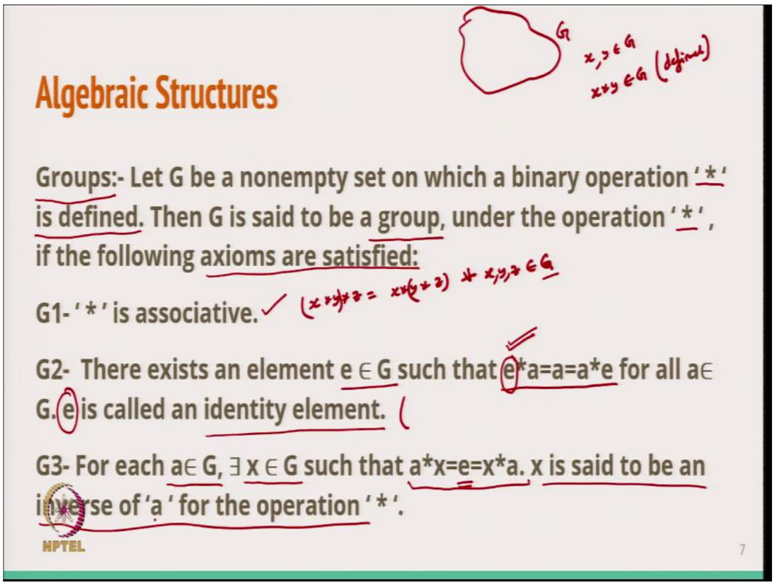


So, after this one the binary operation, we can discuss some algebraic structures that are very important to define the vector spaces. So, the first one is I am going to define the groups. So, let G be a non-empty set. So, suppose I take some non-empty set G. So, it is a non empty set on which a binary operation * is defined. So, * is defined means when I take the two elements x and y belong to G, x * y also belong to G.

Then G is said to be a group under the operation star if the following axioms are satisfied. So, these axioms are satisfied.

G1) * is associative e.i (x * y) * z =x * (y * z)

G2) Then there exists an element e ∈ G such that e * a = a * e =a ∀ a∈ G and e is called an identity element.

G3) For any a∈ G there exists an element b such that a * b=b * a=e, then b is called an inverse of a.

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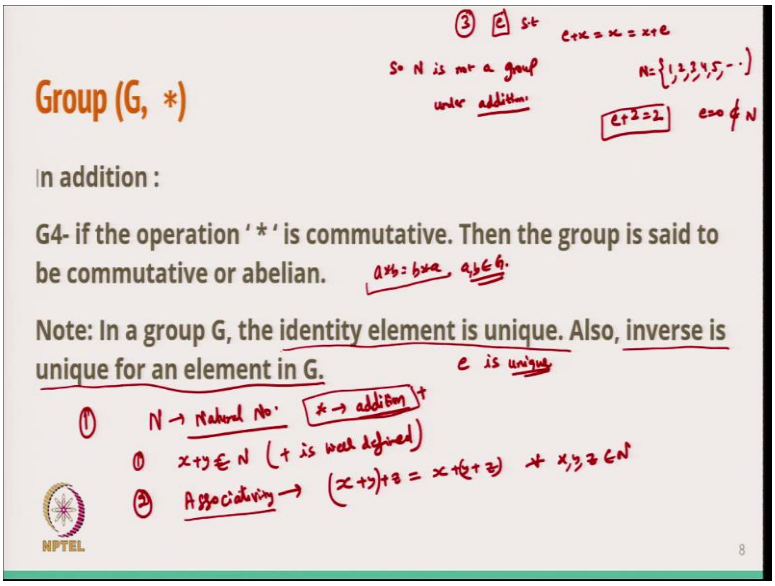


Now, we define

G4) if a * b = b * a for all a,b ∈ G then the group G is said to be a commutative or an abelian group.

So, you note that in the group G the identity element that we have defined is always unique and also the inverse is unique for an element in G. So, it is also you have to keep in mind that element e is unique. So, everything depends upon which type of elements or which type of binary operation we are going to define.

Let us take some examples about the group. So, in the group I just start with the first example. I take the set of natural numbers and in this case I will define that star as an addition. So, in this case let us see whether it is going to be a group or not. So, I am defining the star as an addition. So, addition means this one. Now, I know that the first part is that if I choose any x + y then the natural number plus the natural number is also a natural number.

So, it belongs to the N. So, addition is well defined. So, I can say that this binary operation is well defined. Second one is that I am going to discuss associativity. So, associative means, I have x * y * z = x + y + z. So, this is true for all natural numbers and then this is true for all x, y, z, so, belongs to the N.

So, it is associative. Now, the main thing is that I will start with the third one. Now, we have to choose the element e such that if I take e + x = x = x+e.

Now, I know that the natural numbers N ={1, 2, 3, 4, 5,...}. So, in this case suppose I take 2 then what should be the e so that I get 2? So obviously, in this case my e should be 0, but that does not belong to N. So, this property is not satisfied. So, from here I can say that N is not a group under addition. So, this is not a group under addition.

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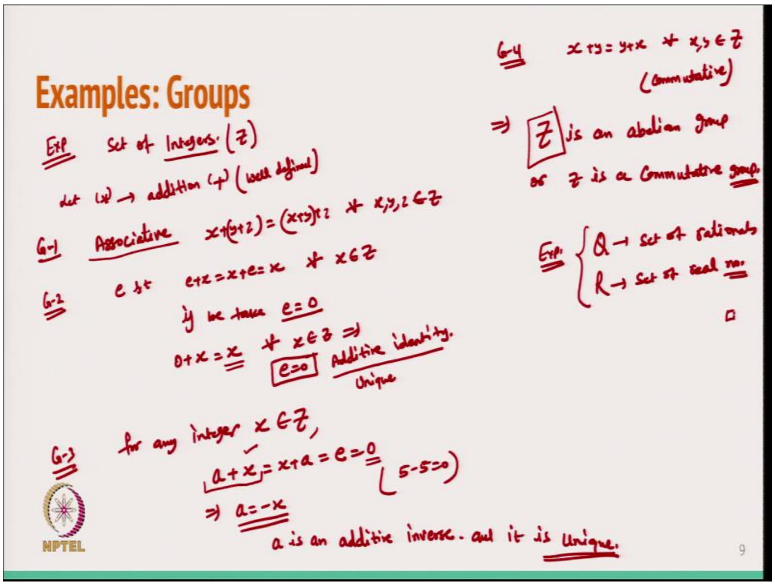


Then I take another example. So, instead of the natural numbers, suppose I start with a set of integers Z. So, in this case I will take that let the star binary operation we choose addition by +. Now, we know that the sum of two integers is an integer. So, this is well defined. So, binary operation is well defined. Now, I will start satisfying the properties.

So, G1 ( associativity) is satisfied.

So, now, we know that the sum of any three integers is going to satisfy the associativity because x * y * z =(x * y)*z=x*(y*z)

Now, G2)

So, I need an element e such that e + x=x.

So, now, in this case I know that if we take e = 0. So, from here I can say that my e is 0 and that is we call it additive identity. So, this is an additive identity and we know that this is unique in this case because there are no other elements we can find from the set of integers such that we add to x and we get this value of x other than 0. So, in this case it is unique.

Then G3. So, in this case for any integer, suppose I take x that belongs to Z, I need an element 'a' such that if I put a + x =e=0. So, from here I can say that a = - x because I know that if it is 5 then - 5, it is going to be 0.

So, in this case we can choose -x, whatever the element is there I choose just choose the minus of that one and I will get the value 0. So, it is also unique.

So, from here I can say that this is a group. Now, I will satisfy the 4th property and from here I know that x + y = y + x, this is true for all x, y belong to the set of integers. So, in this case I can say that this group is this set and this binary operation is commutative. And from here I can say that the set of integers is an abelian group or we can say that Z is a commutative group. So, this way we can say that the set of integers is an abelian group.

Similarly, we can discuss other examples. For example, I can choose a set of rationals or R the set of real numbers. So, in this case I can define the set of rational or the real numbers under different types of binary operation and then we can check whether it is going to be a group or not. So, that we will discuss in the next lecture. So, we stop here.

So, today we have started with the course and in that we just defined the basics about what is the binary operation and how the binary operation can be commutative or associative and then we discuss the other definition that is groups. So, in the next lecture we will continue with this one.

So, thanks for watching. Thanks very much.