Scientific Computing Using Matlab
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Lecture 51
Numerical Differentiation

Hello viewers, welcome back to the course on scientific computing using MATLAB. So, in the previous lecture we have tried to make some MATLAB codes for the given methods whatever we have discussed in this unit.

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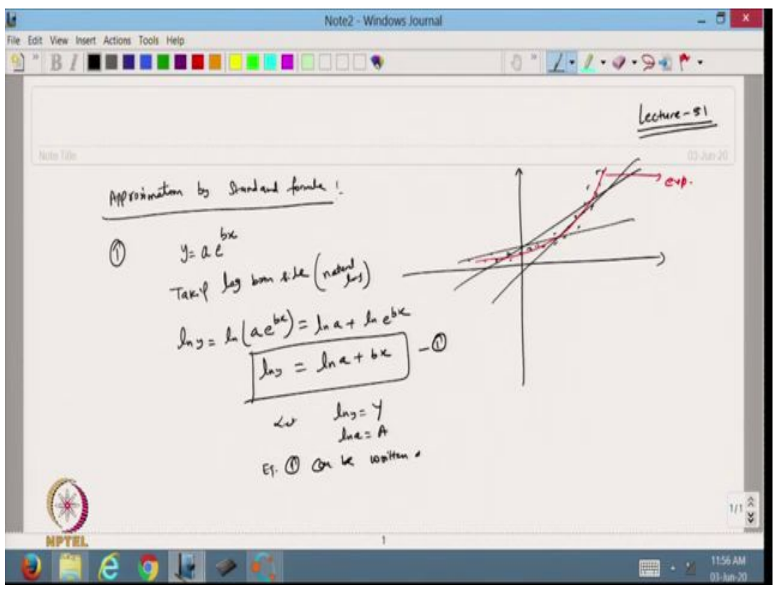

So, today I will give the last topic in this unit and then we will start with the numerical differentiation. So, today I will give you how we can extend the least square line to the data of different types. So, suppose we have data points, so that data points are distributed in the given X or Y coordinates. So suppose these other data points we are going to have. Now, if I approximate this one with the least square line, so our least square line will be like suppose this one or it may have like this one or it may be like this one.

But from here if I find that the data pattern looking at the data pattern, we find that this pattern can be approximated with a function which is not, definitely which is not a line. It may be a function of this type so like this one or I can say that this is an exponential function. So, given, looking at the pattern of what we do, in general what we do, first we try to plot the given data in the X Y plane. So, looking at the pattern of this one, we approximate the given using the least square approximation.

So, how can I extend the least square line to the function of different types? So, that is how we can do this one. So, I will try to find what is the approximation by standard formula. So, standard formula like this one in this case, so, I take case number 1. Suppose, I want to approximate this with some exponential function, so, $y = ae^{bx}$. So, this is my pattern and looking at this pattern I see that okay exponential function is fitting for this given data.

So, how can we approximate an exponential function for the given data? So let us suppose this is my exponential function and I want to fit this one in the given data. So what I do is that I will take the, so taking log both sides, natural log. So it is $\ln y = \ln(ae^{bx}) = \ln a + \ln e^{bx}$. So this is my $\ln y = \ln a + bx \cdots (1)$, so now from here you can see that I have transferred or this data that was exponential into the linear form. So, I can introduce that let my $\ln y = Y, \ln a = A$.

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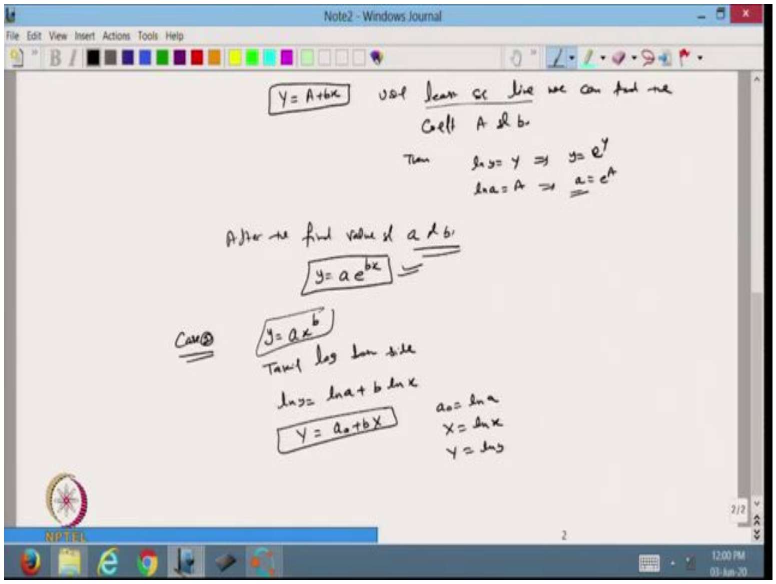

So, from here equation 1 can be written as $Y = A + bx$ and that is the equation of a line. And this equation of a line we know how to find this equation of line using the least square approximation. So, using the least square line we can find the coefficients capital A and b. So, once I know the value of capital A and b, I will put the value of A here and I will find the value of a by taking the exponential and taking the Y I am taking this y.

So, once I know that is this value then, $\ln y = Y$. So, from here I will get $y = e^{Y}$. Similarly, I can find in a that is, $a = e^{A}$ So, once I get this value I am able to find this and that is my linear approximation. So, based on this one for the given exponential function I have converted that one into the linear form. So that is called the linearization of the exponential function.

And using the method of least squares line, I am able to find this one and then I will be able to solve equation number 1. So once I have the value of a and b, then after the finding values of a and b, because a is there, and b is there, I am able to find my exponential function $y = ae^{bx}$. So this is the way we can find it. Now based on this one, I am able to find my approximation of the exponential function. So this is the way we can find out.

Now, I define case 2, let my pattern of the given data is satisfying the function of this type

$y = ax^b$. So, in this case, the same again taking log both sides, I will get $\ln y = \ln a + b \ln x$. Now, from here, I will write this $Y = a_0 + bX$, where $a_0 = \ln a, X = \ln x, Y = \ln y$. So once I get this value, this is the corresponding line I am going to have, then using the same as the least square line, I will try to find the value of a_0 and b. And based on this a_0 and b, I will try to find the value of a and x y. And then we can find out this power function using the least square approximation.

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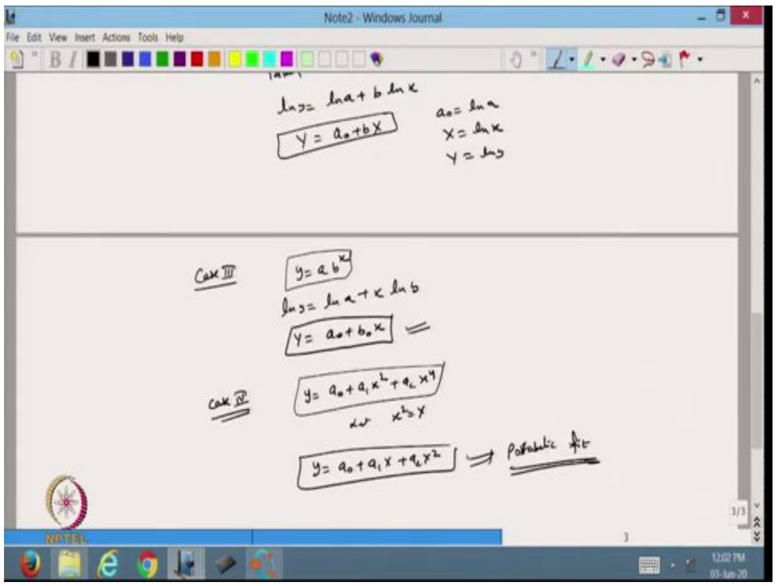

In the same way I can define another type of pattern. So case 3, y is given to of this type, so $y = ab^x$ like exponential function. So in this case also, I will take the log both sides, so it would be $\ln y = \ln a + x \ln b$. From here I can write $Y = a_0 + b_0 x$ So, again, based on this linear approximation, I can find the value of this coefficient a_0 and b_0 and then from here I can find the value of a and b. So that a and b is now known to us. So, based on this one, I can find out this approximation.

So, in this way, we can find out different, different approximations, maybe I can take case 4. Let I the data pattern of this type suppose $y = a_0 + a_1 x^2 + a_2 x^4$. So this is the fourth order polynomial. So in this case, what I do, let I put $x^2 = X$. So from here, I will get $y = a_0 + a_1 x + a_2 x^2$ and that we know how to find this one.

So, this is a parabolic fit. And we have also discussed how we can approximate this parabolic function and based on this parabolic function I can find the value of a_0, a_1 and a_2 and then I can approximate this function. So, just taking the small substitution, we can take the help of least square approximation for polynomial spline or linear polynomial parabolic then we can find the exponential feet or power feet for different, different types of data functions. So, that we can do with the help of least squares.

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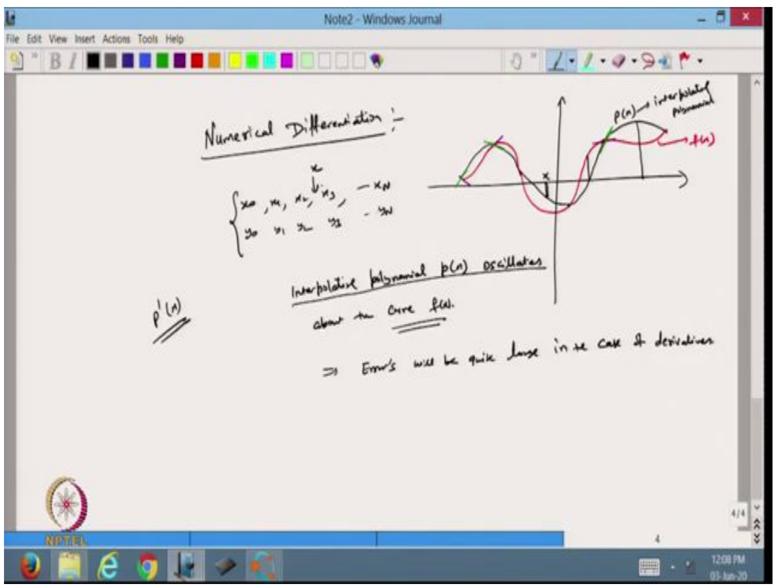

So, this is all about the unit related to the interpolation. Now, we are going to start the next unit and the next unit is numerical differentiation. So, let us try to find out how we can take the help of numerical differentiation. So numerical differentiation is that, now suppose I have the data and based on these data, suppose this is my data I have. So let us say I take the interpolating polynomial from here for maybe a cubic spline, so suppose this is my interpolating polynomial.

And suppose this data is satisfying some function, so suppose this is a function of this type, so this is my function, suppose I take this function like this one. Now from here I want to find the value of the derivative at this point, suppose I take this point. So this is the point x I am taking, and I want to approximate the value of the derivative of the function at this x, or maybe I can choose this x or I can choose this x.

So, all these points, I do not want the value of the function, but I want the derivative of the value of the derivative of the function, so in that case, we have to take the numerical differentiation. So in this case also we have the value $x_0, x_1, \cdots, x_n$ and the corresponding $y_0, y_1, \cdots, y_n$. So, based on this data, I will make an interpolating polynomial or curve fitting and based on that one we will try to find out that value of the derivative at any point x in between these data points.

So, that we are going to do with the help of numerical differentiation. Now, from here if you see that at the nodal values, this is my function, so, that is the function f(x) and this is my interpolating polynomial. So, I call it maybe P(x). Now, from here you will see that at the nodal points the actual derivative is this one, but approximate it is this one. Actual derivative at this point is this one, but if I approximate it with an interpolating polynomial, then this is the derivative here.

So the same thing is happening here, actually this one and approximating is this one. So you can see that there is a very big difference between the derivatives at these nodal values, because here you can see that actual is the positive one but the approximation is the negative one. Here the actual is the positive, but approximating is something like a parallel to x axis or negative value.

So from here you can see that, that we can say that the interpolating polynomial P(x) oscillates about that curve f(x). That we know because the interpolating polynomial has all this nodal value as the root. So that is why it will interpolate like this one. So, it will oscillate about the curves. So, in this case, I can say that the errors will be quite large in the case of different derivatives, in the case of derivatives.

So, now based on this one. So, how to try to find out the value, so suppose I have somewhere x here and I want to find what is my $P'(x)$. So this is my interpolating polynomial and I want to find the value of the derivative at any x to this one.

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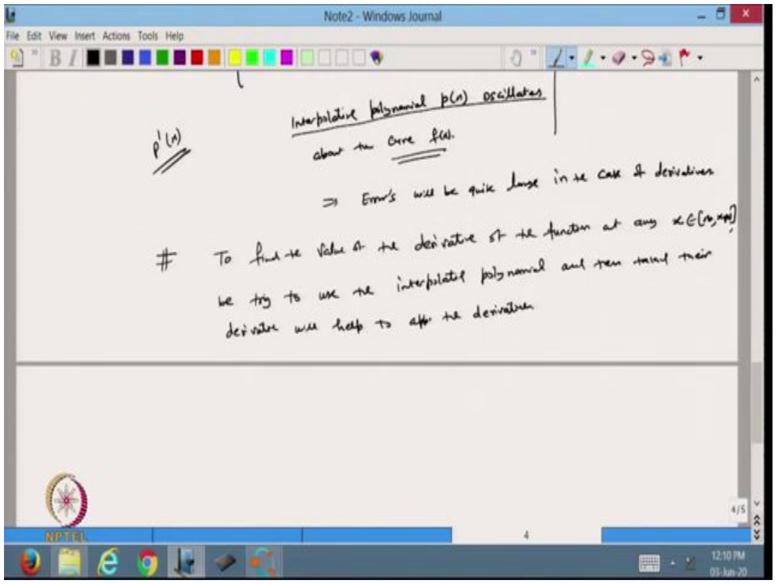

So now in this case we will take the help of the previous unit. So let us try to find out this one. Now, to find the value of the derivative of the function at any $x \in [x_0, x_n]$. So this is the point value is given to me, we try to use the interpolating polynomials and then taking their derivatives will help to approximate the derivative.

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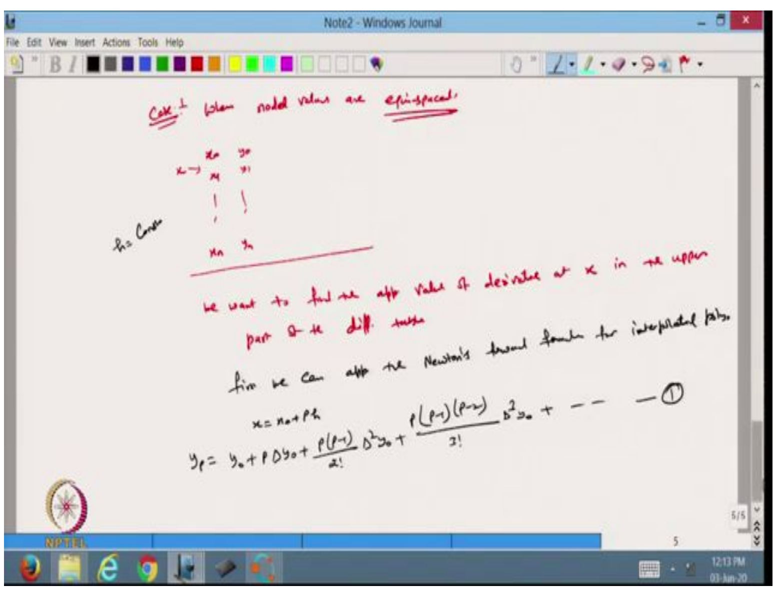

So let us try to take this one. So let us take case 1, when the nodal values are equi-spaced. So let us try to find out this one. So, in this case, I have my $x_0, x_1, \cdots, x_n$, and they are equi-spaced, I have the value $y_0, y_1, \cdots, y_n$. Now somebody asked me that give me the value of x the value of the derivative of x for the x lying here. So in this case, we want to find the approximate value of the derivative at x in the upper part of the difference table. So, this is a way we can find the difference table and suppose this is the value I want to find in the upper half.

So I know that so we can approximate. So first we can approximate Newton's forward formula for interpolating polynomials. So now we know that I can find the interpolating polynomial. So, let us assume $x = x_0 + ph$, where h is constant. So, in this case it is given to me, now I can find my y_p by interpolating polynomials that are given by as

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots \cdots (1)$$

So, this is my interpolating polynomial.

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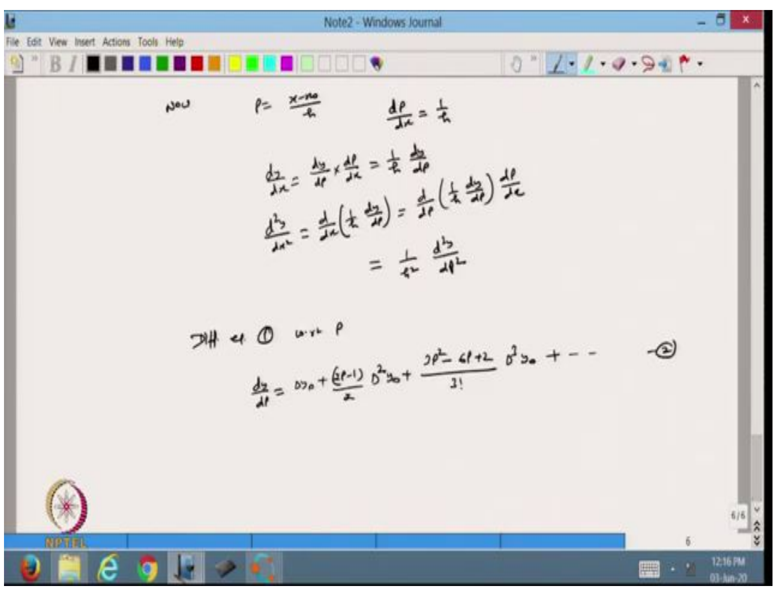

Now, I can write my $p = \frac{x - x_0}{h}$. So, from here I can find the derivative $\frac{dp}{dx}$. So, $\frac{dp}{dx} = \frac{1}{h}$. Now, I can find what is $\frac{dy}{dx}$. So, I can find $\frac{dy}{dx} = \frac{dy}{dp}\frac{dp}{dx} = \frac{1}{h}\frac{dy}{dp}$. Similarly, I can define a second derivative.

So, from here I can define my $\frac{d^2 y}{dx^2} = \frac{d}{dx}\left(\frac{1}{h}\frac{dy}{dp}\right) = \frac{d}{dp}\left(\frac{1}{h}\frac{dy}{dp}\right)\frac{dp}{dx} = \frac{1}{h^2}\frac{d^2 y}{dp^2}$. So this way we can find any derivative. Now from here differentiate equation 1 with respect to p. So that we can find out we can differentiate the equation 1 with respect to p.

So now I will get my

$$\frac{dy}{dp} = \Delta y_0 + \frac{2p-1}{2}\Delta^2 y_0 + \frac{3p^2 - 6p + 2}{3!}\Delta^3 y_0 + \ldots \cdots (2)$$

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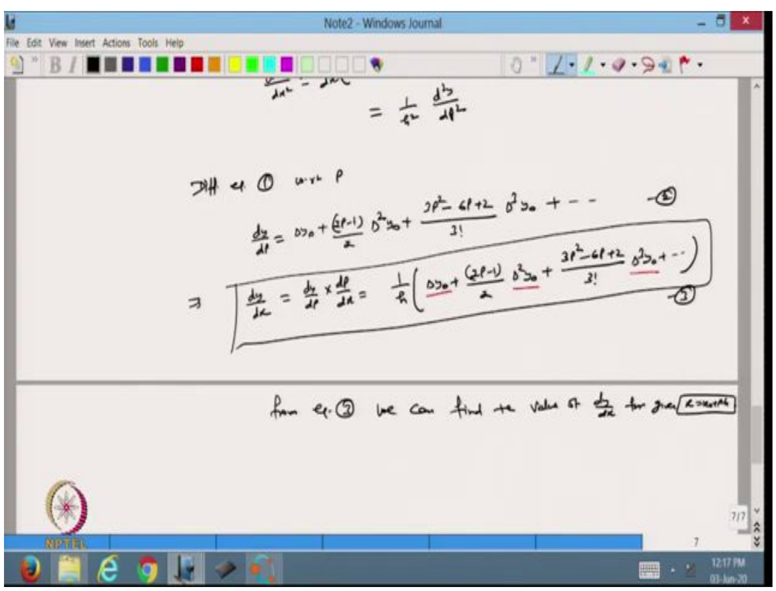

So, based on this one I can find what is my $\frac{dy}{dx}$ So, at the value of x, so this can be written as my

$$\frac{dy}{dx} = \frac{dy}{dp}\frac{dp}{dx} = \frac{1}{h}\left(\Delta y_0 + \frac{2p-1}{2}\Delta^2 y_0 + \frac{3p^2-6p+2}{3!}\Delta^3 y_0 + \dots\right) \cdots (3)$$

So, from equation 3 we can find the value of derivative $\frac{dy}{dx}$ for given $x = x_0 + ph$. So then using this one we Newton forward methods we can find out this one.

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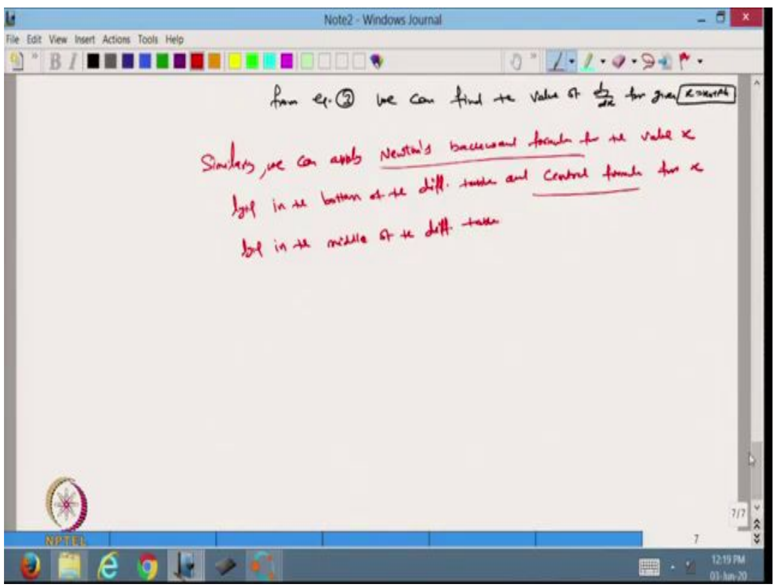

So the same way we can apply. Similarly, we can apply Newton's backward formula for the value x lying in the bottom of the difference table and the central formula, Sterling central formula for x lying in the middle of the difference table. So, and that we already know how to apply this Newton backward formula or the Sterling central formula. So, once we find this Newton backward or central formula, we can take the derivative corresponding that interpolating polynomial, and then we put the value of p in this one and then we can find the value of the derivative there.

So, in this case, we have to repeat the process for finding out the interpolating polynomial and then we can take the derivative just to get the approximate value. So, in this case, we are not going to repeat this again and again. So, this is what we have done for the case when the nodal values were equi-spaced.

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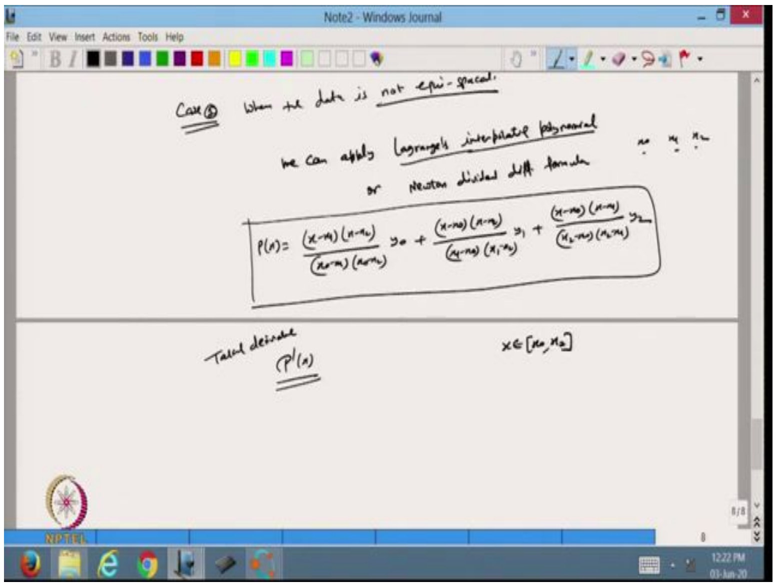

Then the next case is case 2 when the data is not equi-spaced. So, if in this case if the data is not equi-spaced, then I apply, then we know that we can apply Lagrange interpolating polynomial or Newton divided difference formula. So, in this case, suppose I apply the given data, I apply the Lagrangian interpolating polynomial. So, I know that suppose I have 3 points, $(x_0, y_0), (x_1, y_1), (x_2, y_2)$ so I have the polynomial like

$$P(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2$$

So I will get this polynomial and that is a polynomial in x, then I can take the derivative of this one, taking the derivative. So, that will be the $P'(x)$. So, from here I can approximate the value of the derivative for any x lying between x_0 to x_2.

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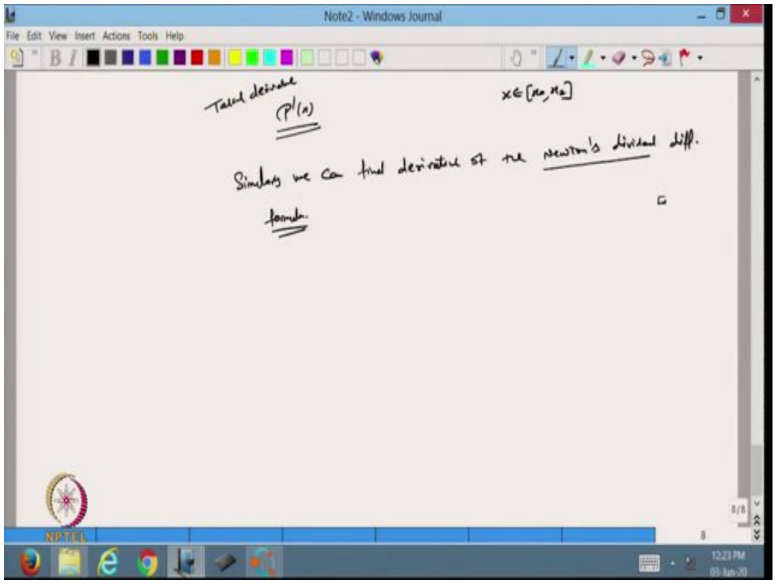

Or the same way similarly, we can find derivatives of Newton's divided difference formula. So, in that case also we are getting the interpolating polynomial and then I can take the derivative with respect to x, and then we can approximate any value of x in between x_0 to x_n to find out the value of the derivative for the given x. So, this way we can find out with the help of the interpolating polynomial that we have discussed in the previous lecture, we can find out the value of the derivatives, first derivative or the second derivative of any value of x lying in the given values of x. So, that is x from x_0 to x_n.

So, I will stop it here, stop today. So, today we have started with how we can apply the standard formula for the least square lines. And then we started with the numerical differentiation. So with the help of the interpolating polynomial, we can find out the value of the derivative for any x, find any x lying between the datas so there is no need to do the, to repeat this again and again. So we will continue with this one in the next lecture. So thanks for watching. Thanks very much.