.Scientific Computing Using Matlab

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Lecture 50

Matlab Code for Cubic Spline

Hello viewers, welcome back to the course on Scientific Computing using MATLAB. So today we will continue with the previous lecture and try to make the MATLAB code for cubic spline. So let us do that one.

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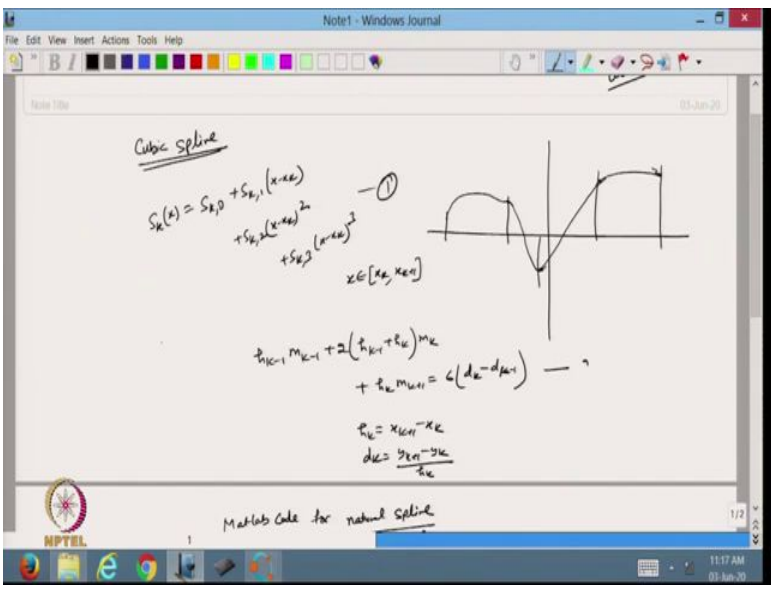

So we know that in the cubic spline, so in the cubic spline I know that we have the data points. So suppose this is my data points. And in all these data points, I am approximating with a spline which is satisfying few properties that its function should be smooth here, first derivative will be same, the second derivative be same at these points, so that this way we can define the cubic spline. So, in each of these sub intervals, so this is my basically sub interval.

So, I can define my, the cubic spline in any kth of the interval. So, that is given by

$$S_k(x) = S_{k,0} + S_{k,1}(x - x_k) + S_{k,2}(x - x_k)^2 + S_{k,3}(x - x_k)^3, x \in [x_k, x_{k+1}].$$

....(1).So, that is the cubic spline we are going to get and then I also know that these values we have to come across, so to solve this value they have to come across a system of equations.

So, that is represented by $h_{k-1}m_{k-1} + 2(h_{k-1} + h_k)m_k + h_k m_{k+1} = 6(d_k - d_{k-1})$....(2) where $h_k = x_{k+1} - x_k, d_k = \dfrac{y_{k+1} - y_k}{h_k}$ So, this is the way we have defined this one.

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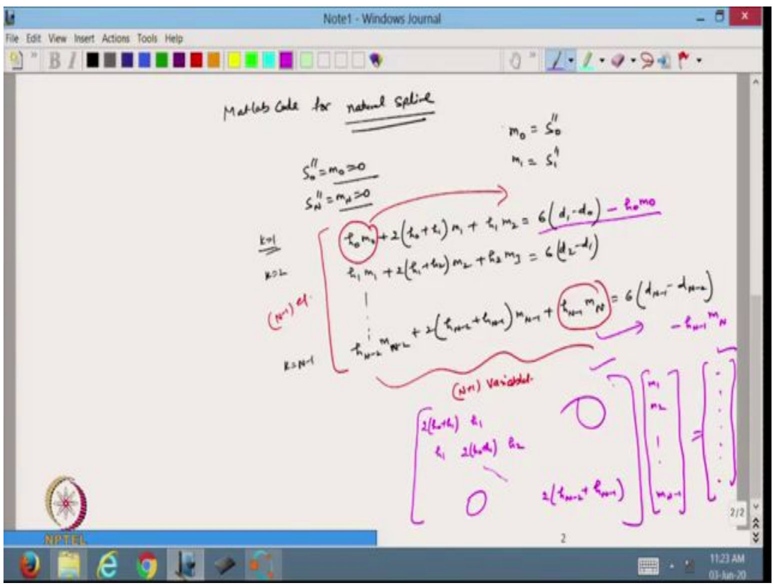

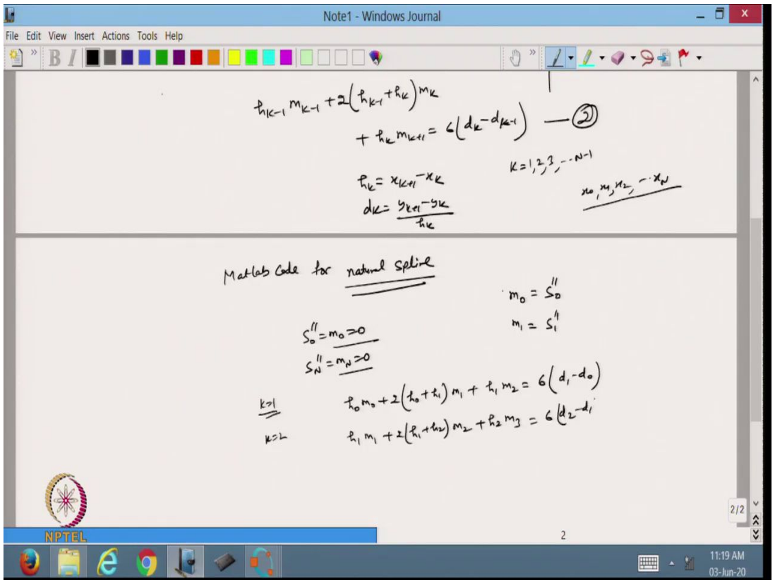

Today, today I am going to use the MATLAB code for natural splines. Now, if you see from here, this is my, I just call it equation number 1 and 2. Now, what I do is this m_k we know that this is if I put $m_0 = S_0''$, $m_1 = S_1''$ so this is the way we can find it. Now in the natural spline it is given to me that $S_0'' = m_0 = 0, S_N'' = m_N = 0$.

So, at these boundary points the value of $m_0 = 0, m_N = 0$, so this is given to me. Now from here, this is equal to $k = 1, 2, \cdots, N-1$. Suppose I have the points , so I will go from 1 to N-1. Because, if I go k = 1 I have to go m_0, so, that is the boundary point we are going to solve. Now, from here if you see then how we can make them, how we can solve this for the values of mk's. Now, if I put k = 1 I will get $h_0 m_0 + 2(h_0 + h_1)m_1 + h_1 m_2 = 6(d_1 - d_0)$.

So, that is, so, this is known to us what is the d_1 and d_0. I also know h_0, h_1. So, the only thing I need to find m_1 and m_2. Now, the next one will be k= 2. So, it will be $h_1 m_1 + 2(h_1 + h_2)m_2 + h_2 m_3 = 6(d_2 - d_1)$. So, if you see from here, I can continue with this one and in the end if I put N-1, so k =N-1. So, if you put N-1 here, so what I will get,

$$h_{N-2}m_{N-2} + 2(h_{N-2} + h_{N-1})m_{N-1} + h_{N-1}m_N = 6(d_{N-1} - d_{N-1})\,.$$

Now, if you see the system, then it gives you the first one. So the only problem comes here, that is the value of this, this and this because the boundary points are involved. If you see from here, then this is a system of N-1 equations, and the number of coefficients are $m_0, m_1 \cdots, m_N$ so that contains N+1 variables.

So, this is a N-1 equation and N+1 variables. So, now to solve this system, we have to find the value of these boundary points. So, this is the way we can define the natural spline. So, in this case what I can do is that, I will take this one on the right hand side and I will write it here - $h_0 m_0$. And the same way I can take this one on the right hand side and I will write $-h_{N-1}m_N$. Now, from here you can see that it will contain N-1 variables and N-1 equations are there. So, now it is a square matrix.

So, now based on what is the value of my m_0, m_N, I will put it here and I will solve this system. So, if you see then this system becomes a tridiagonal matrix with first is $2(h_0 + h_1)$ and then h_1, next is $h_1, 2(h_1 + h_2)$, then h_2 and so on. In the end I will get $2(h_{N-2} + h_{N-1})$.

So, this is a tridiagonal system. I will get $m_1, m_2, \cdots, m_{N-1}$. So, this I need to find and on the right side it is given that the first coefficient is given, this is the first one coming here, then all this here the value is coming and the last this is the value. So, that is known to me, this is known to me and this I need to find.

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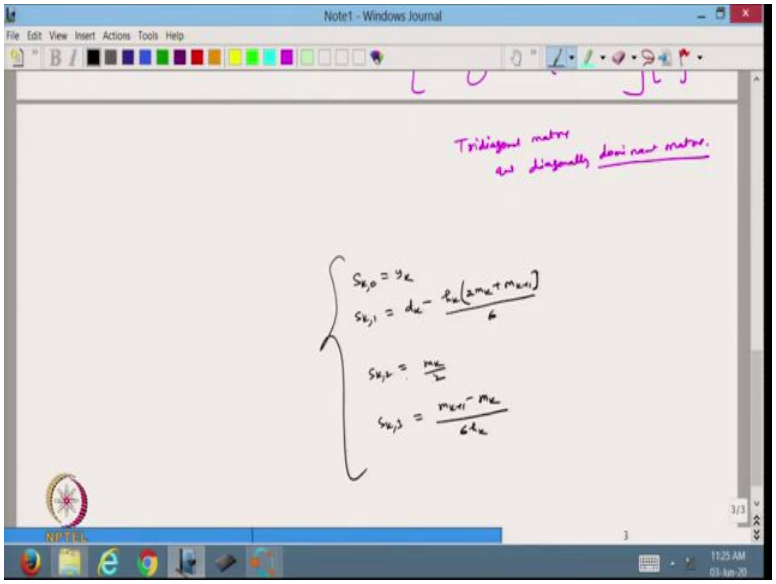

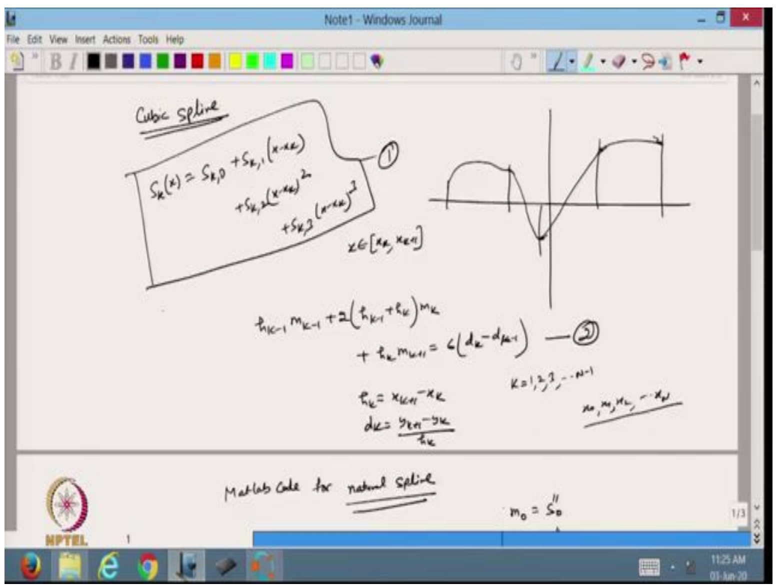

So, this matrix if you see then this is a tridiagonal matrix and you can see also this is diagonal dominant, and diagonal dominant matrix. So, this is, this system will have a unique solution and from that solution I can find the value of $m_1, m_2, \cdots, m_{N-1}$ directly from here. Once I know the value of this one, then I can find out my cubic spline. So, from here I find out that directly after solving this one, I can find my coefficient $S_{k,0}$.

So, that will be $S_{k,0} = y_k$,

$$S_{k,1} = d_k - \frac{h_k(2m_k + m_{k+1})}{6}, S_{k,2} = \frac{m_k}{2}, S_{k,3} = \frac{m_{k+1} - m_k}{6h_k}$$

. So I am able to find all this coefficient. So now I know the y value of yk is already given to me, mk is I found from this system, dk is given.

So now I can find the coefficient and based on this coefficient I can write my cubic spline. So this is my cubic spline I am able to write. Okay, so let us try to make the MATLAB code to solve a cubic spline problem. So let us try to find out.

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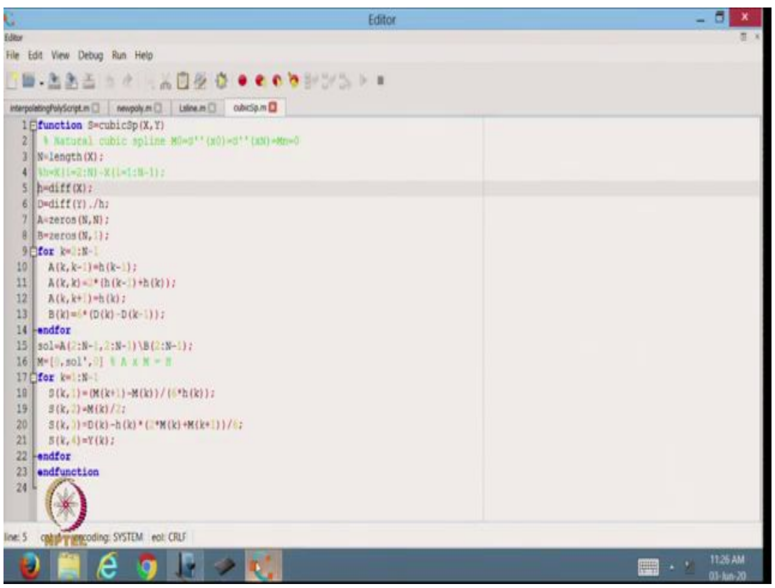

So here I have introduced the concept of the MATLAB code for cubic splines. So in the cubic spline I am passing X and Y, and I am getting back the value of S. So this is the splines we are going to get, the coefficients basically. Now from here, this is my N, I am introducing this N that is the length of X. So h is the difference. So this diff means the difference of X.

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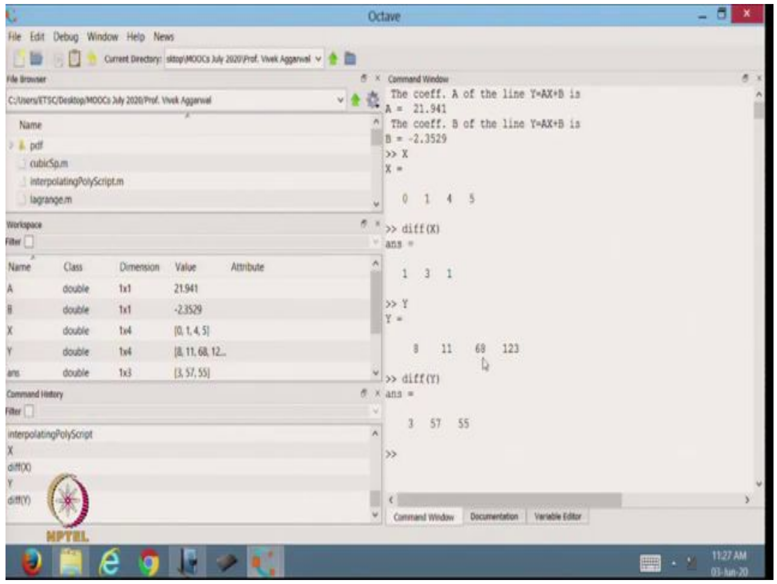
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A = 21.941
The coeff. B of the line Y=AX+B is
B = -2.3529
>> X
X =
0 1 4 5
>> diff(X)
ans =
1 3 1
>> Y
Y =
8 11 68 123
>> diff(Y)
ans =
3 57 55
>>
Workspace
Name Class Dimension Value Attribute
A double 1x1 21.941
B double 1x1 -2.3529
X double 1x4 [0, 1, 4, 5]
Y double 1x4 [8, 11, 68, 12...
ans double 1x3 [3, 57, 55]
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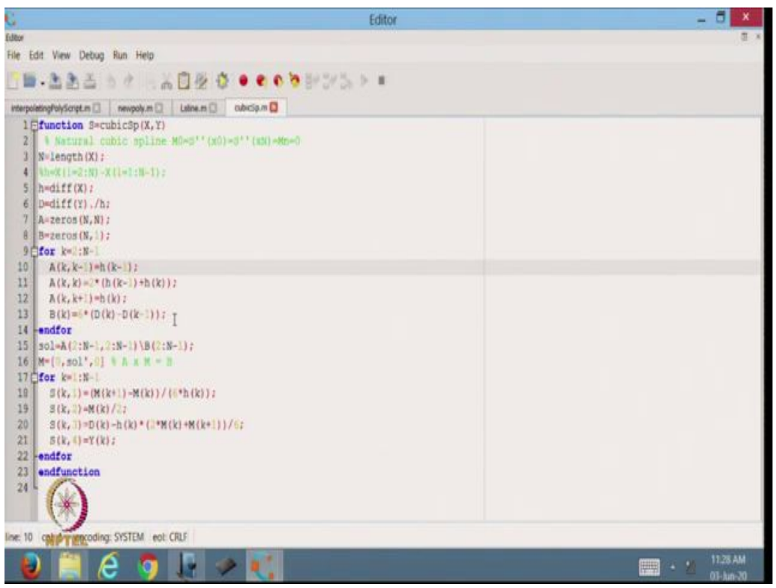
Editor
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interpolatingPolyScript.m newpoly.m Lsline.m cubicSp.m
function S=cubicSp(X,Y)
% Natural cubic spline M0=S''(x0)=S''(xN)=Mn=0
N=length(X);
%h=X(i=2:N)-X(i=1:N-1);
h=diff(X);
D=diff(Y)./h;
A=zeros(N,N);
B=zeros(N,1);
for k=2:N-1
A(k,k-1)=h(k-1);
A(k,k)=2*(h(k-1)+h(k));
A(k,k+1)=h(k);
B(k)=6*(D(k)-D(k-1));
endfor
sol=A(2:N-1,2:N-1)\B(2:N-1);
M=[0,sol',0] % A x M = B
for k=1:N-1
S(k,1)=(M(k+1)-M(k))/(6*h(k));
S(k,2)=M(k)/2;
S(k,3)=D(k)-h(k)*(2*M(k)+M(k+1))/6;
S(k,4)=Y(k);
endfor
endfunction
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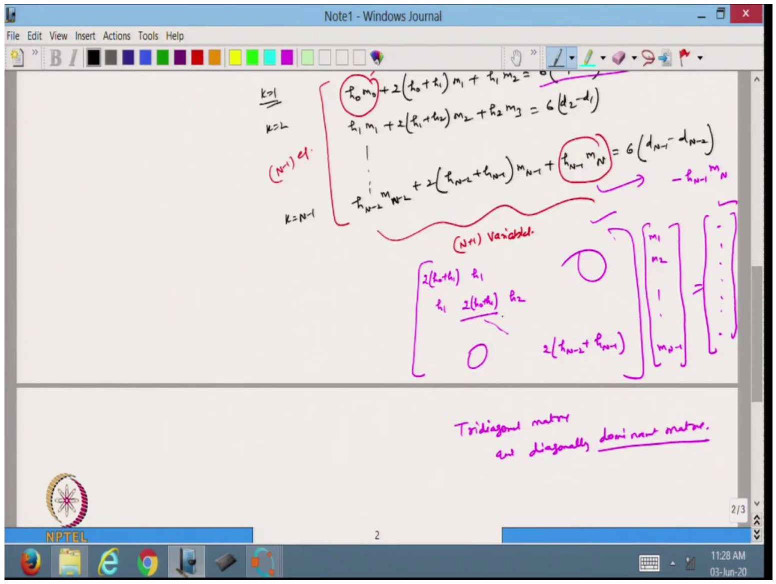

Like here, I have the value scores in the previous 1 I have X, so I will write diff(X). So diff(X) gives you 1-0= 1, 4-1 = 3, and 5-4 = 1. Similarly, I can find this is my Y and I can define the difference of Y. So that gives you 8 -11= - 3, and so on. So this way we can define the difference.

The capital D is the difference of Y divided by h. So this is the divided difference, A, I define the matrix with dimension N cross N and B is the vector I am defining that has the, that is a column vector of dimension N. Because here I am going to introduce the system AX = B. Now, I want to find the value the matrix A. So A matrix as we already defined. So this is going from 2 to N-1, because the length is from 1 to N, so I am finding in between, so this is 2 to N-1.

So this is the coefficient of A(k,k-1) = h(k-1) at the diagonal elements, the values 2(h(k-1)+h(k)) and the next one value will be h(k). So, that coefficient we have found and on the right hand side it is 6(D(k)-D(k-1)). So that we already know how we have defined this one in the case of, this way. So, this is the basically matrix I have written there. So, I have written this one matrix starting from here.

Only thing is that at the first row and the last row, we have to write it separately because here the first row and last row are having different forms as compared to this one. So, all other rows have the same form, that is the 3 points are there, that is why it is tri-diagonal, only the problem is coming to the first row and the last row. So, let us try to make this one.

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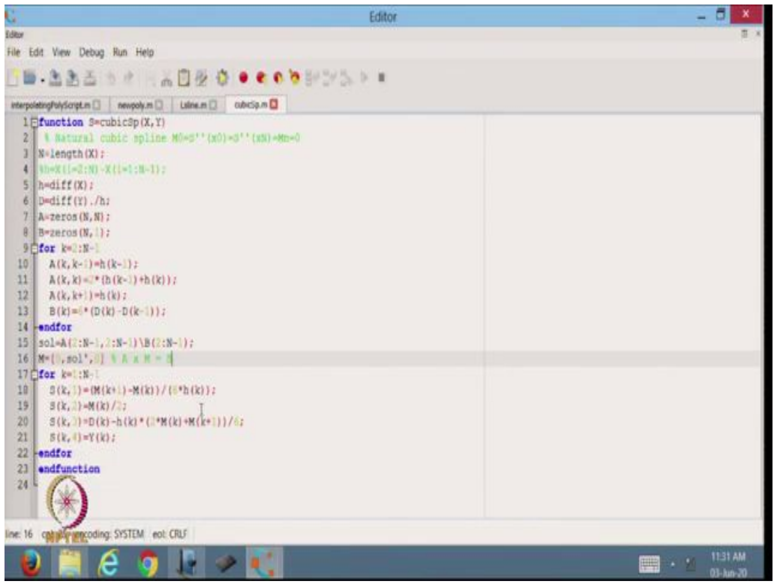

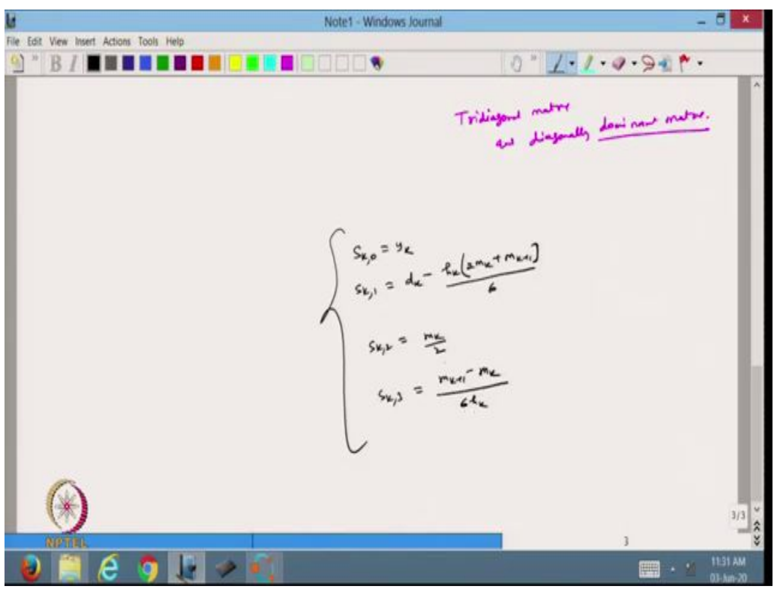

Now, in this case I have written these things for all the values of k, then I will what I do, I will take the, so this is my basically a matrix A, I have started with N cross N. Now in the N cross N from second row to N-1 row and from second column to N-1 column, I have written the coefficients of that matrix. Then what I do is that and this is the right hand side vector. Now I put the solution so, in this case I am taking the natural cubic spline, so, my $m_0 = 0, m_N = 0$.

So on the right hand side, if I take on the right hand side does not matter, because $m_0 = 0, m_N = 0$. So, my right hand side will be the same for all the values. So, this is the same value we are taking. Now, I find the solution. So, in the solution what I am going to do is that I am taking the matrix, the interior matrix starting from 2 to N-1 and 2 to N-1. So from here, you can see that I am taking the matrix that is lying in between, because here, taking the inner part of that matrix, so 2 to N-1 and 2 to N-1, then backslash and B on the right hand side, I am taking the points from 2 to N-1.

I am leaving the first and the last one. Now, from here, I find the solution of the interior matrix, then from here I will get the value of m's in the interior, so that is getting, I am getting the value of $m_1, m_2, \cdots, m_{N-1}$ and m_0, m_N is known to me. So, this is the value of m_0 and this is the value of m_N. So, I put in here 0 and in between the solution I am going to get. So this solution we are going to get will be the column vector. So I am taking a solution dash, so it is a row vector.

So, this is my m, we are going to get. So this m will contain all the second order derivatives of the cubic polynomials, cubic splines at the nodal values. Now from based on this one, I find the value of coefficient. So I find the value of coefficients from k from 1 to N-1, because I am introducing k+1. And so that is why I am taking the value 1 to N-1.

So as S(k,1) = (M(k+1)-M(k))/(6*h(k)), so I am going to introduce, going to write this coefficient. So I am writing, this is the coefficient of x^3 this is the x^2, this is the coefficient of $(x - x_k)^3$, $(x - x_k)^2, (x - x_k)$ and this is the last one, the constant one. So that coefficient I am calculating here.

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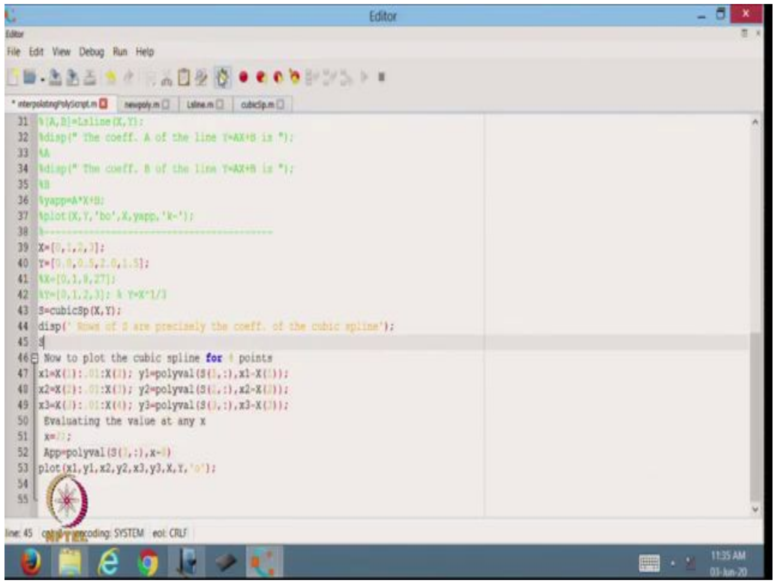

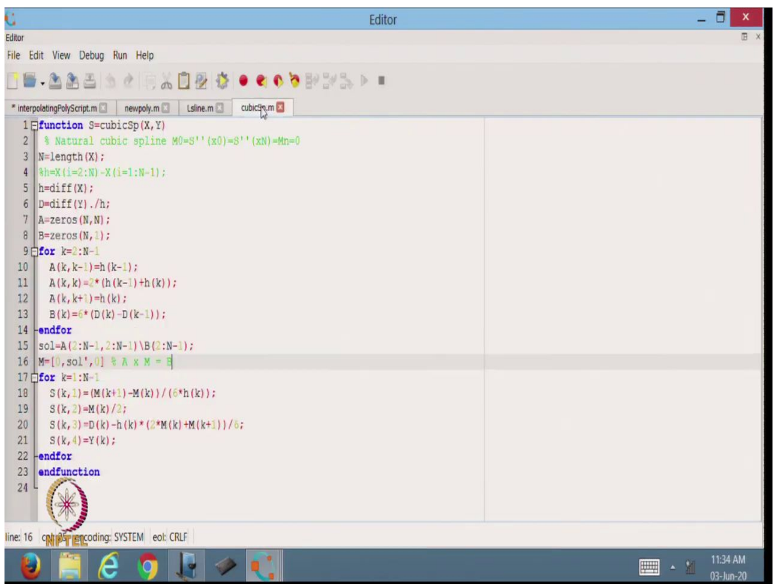

So S(k,1) is this, S(k,2) = M(k)/2, S(k,3) = D(k)-h(k)*(2(M(k)+M(k+1)))/6, S(k,4) = Y(k), whatever the value we are having here. So this is the end of this one and I am passing this as the coefficient as S here. So this one I saved. So this name is the cubicSp. So, let us try to run this one. Now, I will this value and then I can uncomment the data for the cubic spline.

So this is the data we can uncomment. So that is the data we received. So let us start with the X and Y. So let us take the first one. Now I am taking this X as 0, 1, 2, 3, 4 points and Y is this value. Now I call the function cubicSp(X,Y) then I get the value of S. So the rows of S are precisely the coefficient of the cubic spline. Because if you see in this case, a cubic spline if you see that this S(k), k moving from 1 to N-1.

So, I will get the matrix whose first column is this value, the second column in this value, third column in this value and fourth column in this value, and these are all the rows. So I get from here, the matrix of dimension N-1 cross 4, so that matrix we are going to get. So, let us try to run this one.

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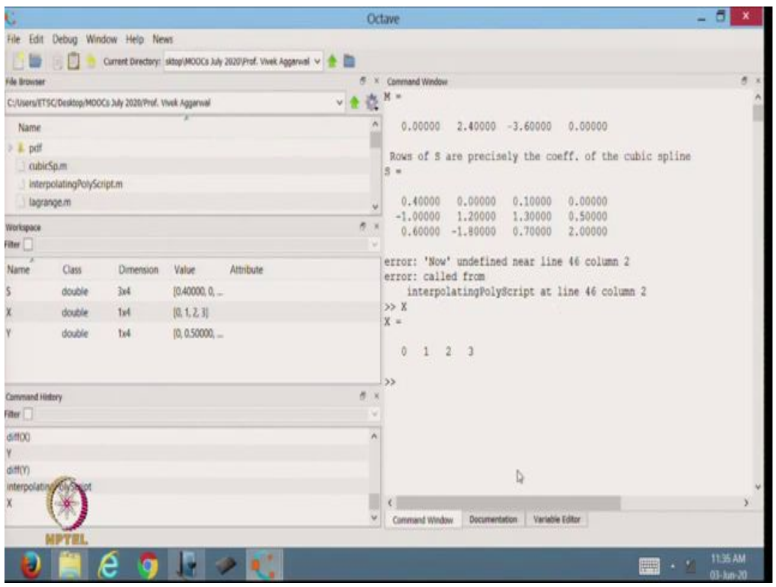

So, this is my M, the coefficient I got 0, 2.4, -3.6 and 0. So this is the endpoint that was already 0 and this is the in between points and these are rows of the coefficients that you explained. So that is 0.4 0 0.1 0. So this is the first cubic spline having so this is my X. So from 0 to 1, this is my cubic spline from 1 to 2, this is my cubic spline, its coefficients are there and from 2 to 3, this is my coefficient. So now we have to, what is this error now, maybe I have written somewhere, oh this is a comment, so that is why it is not coming.

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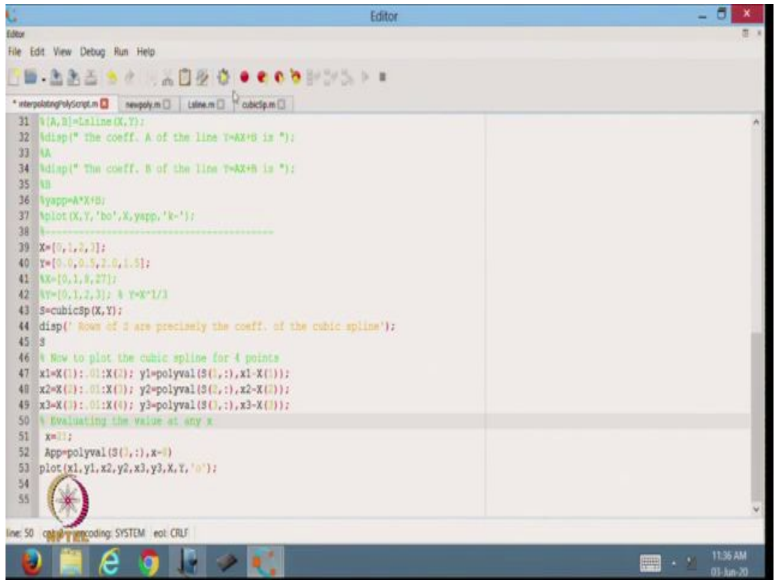

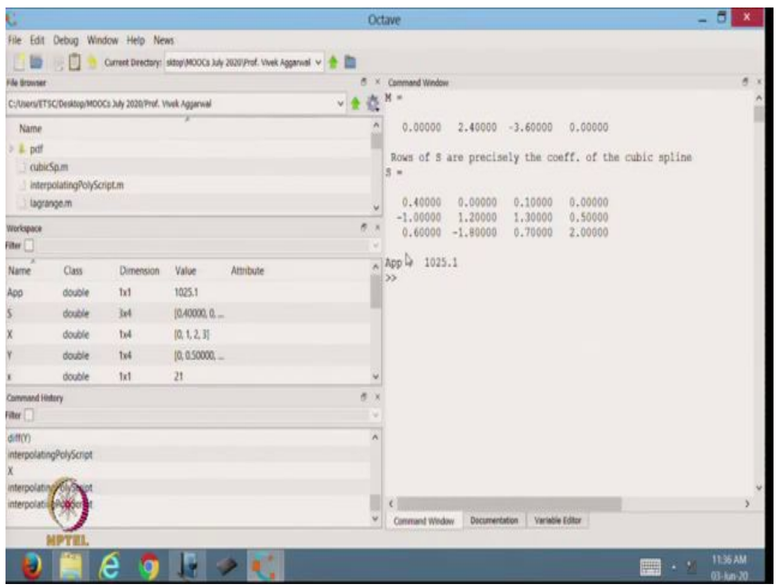

So, let us try to run it again. This is also I have to and that is it. So, now it should come. Now, from here I let us see what is the answer? So, this is the way we have found this one. Now I want to plot this.

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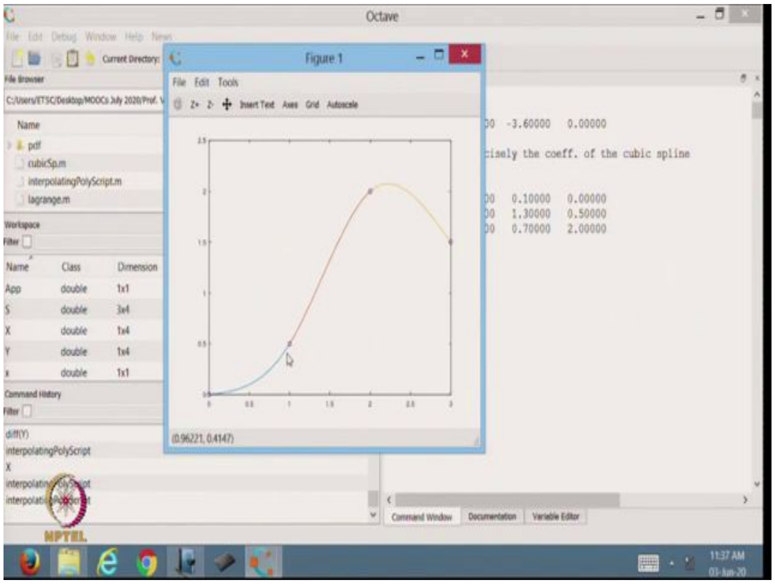

So, using this command, I try to plot this cubic spline. So you can see from here that from 0 to 1 that is my cubic spline, from 1 to 2, this is my cubic spline and 2 to 3 this is my cubic spline. So, from here you can see that this cubic spline is a smooth function, smoothly passing from 1 sub interval to another sub interval at this connecting node. So, that is why it is a smooth function. Now, the question is how we can find out.

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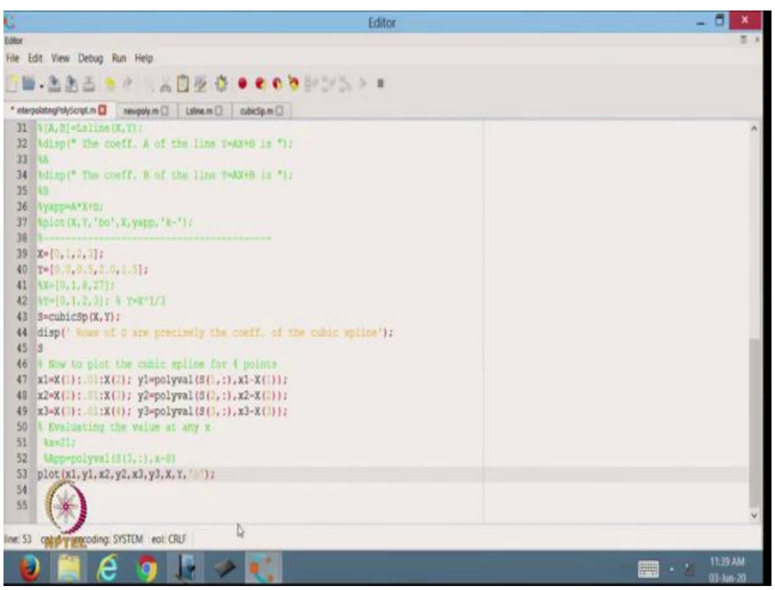

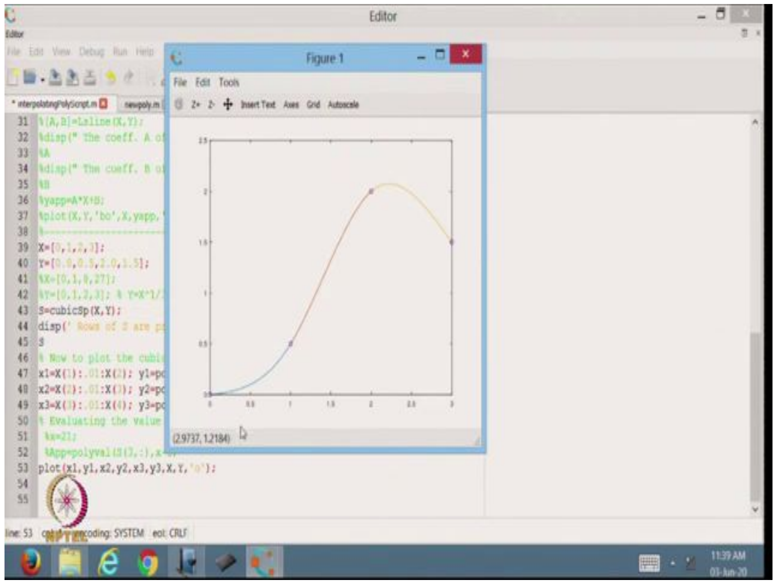

So this is the way we have done it. So, S, I have already displayed to you now. What I am going to do is that I am going to plot this function, so x1 to x2 that is 0 to 1, I am dividing this one into the hundred parts, so that I am able to plot the function nicely in this interval. And here I am putting this polyval. So polyval I told you that the first row represents the first cubic spline, so I am taking the first row and all the columns. So this is my first row. And I am finding these values at each x1. So whatever the value of x1- X(1) because the coefficient will be x minus xk.

So in each of the intervals, so x1 is 0, so that is why this value I am getting. Now for the next cubic spline, the same thing I am going to do, only thing is that it is now x2-X(2). So, x2 is basically 1 in this case and x3 is 2 in this case. So, based on this one I get this y1, y2, y3, three values for this value of x. So, this one is I have done only to test this function and then I plot all these together. So, x1 y1, x2 y2 and x3 y3 and this is the corresponding X and Y whatever we have started with. And I am plotting together with this the connecting side is the circle.

So, from here this is what we are going to get, so this is the first cubic spline, the second cubic spline and the third cubic spline. Now somebody asked me that we got this spline, so I want to find the value at 1.5. So, what is the value in 1.5 or what is the value at 0.5? So, in this case we have to choose, now suppose I want to find the value of this approximating the value at 0.5, it means I have to choose the first cubic spline. If I want to find the value at 1.5 I have to choose the second cubic spline.

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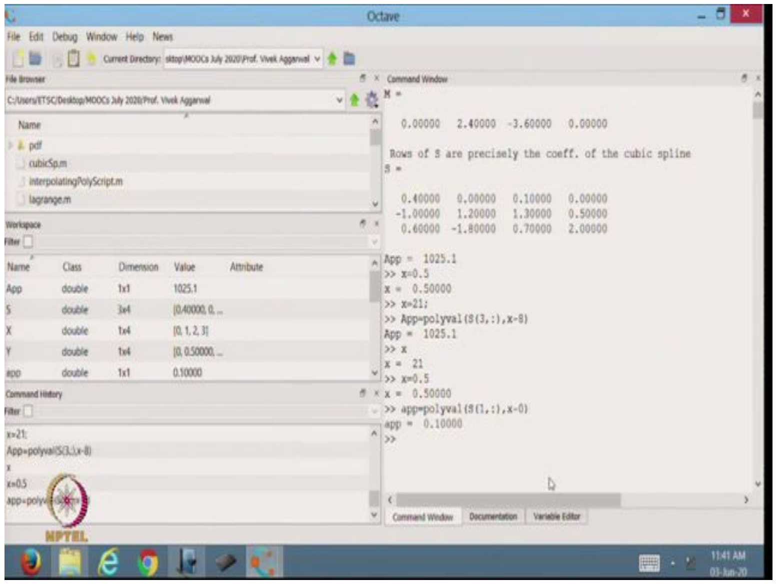

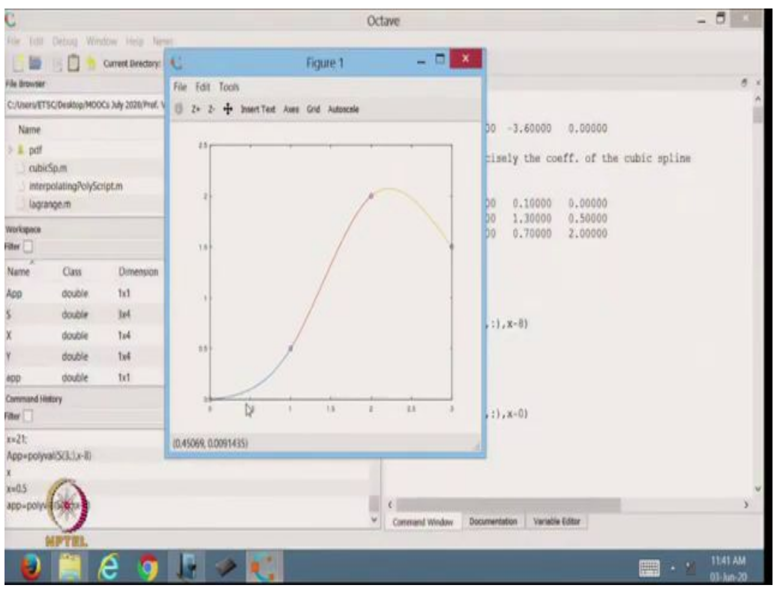

So, in this case, suppose I want to find the value of x at 0.5, so, I will let us, so suppose I want to find the value of x=0.5. Now, at this x I will write this value, no x = 0.5. So this is the value of x. Now I write this one approximate value. So I write an approximation equal to the polyval. Now I have to choose, in this case S(1,:). So S(1,:) I am going to choose because 0.5 belongs to this, so I have to choose the S(1,:).

So let us take the S(1,:) and this is the x we are going to introduce, x-0 because that cubic spline is this one having the coefficient 0.4 and 0.1. So this is the value we are going to have and that is the value 1 is coming. So from here I can approximate that the value of this is 1. So from here, 0.1, yeah, that is why I was a little bit. So this is the value I am going to get. So that is a 0.1. Now the same thing I want to verify for 1.5. So 1.5, it should be 1.4 or 1.3 somewhere. So let us try to find out.

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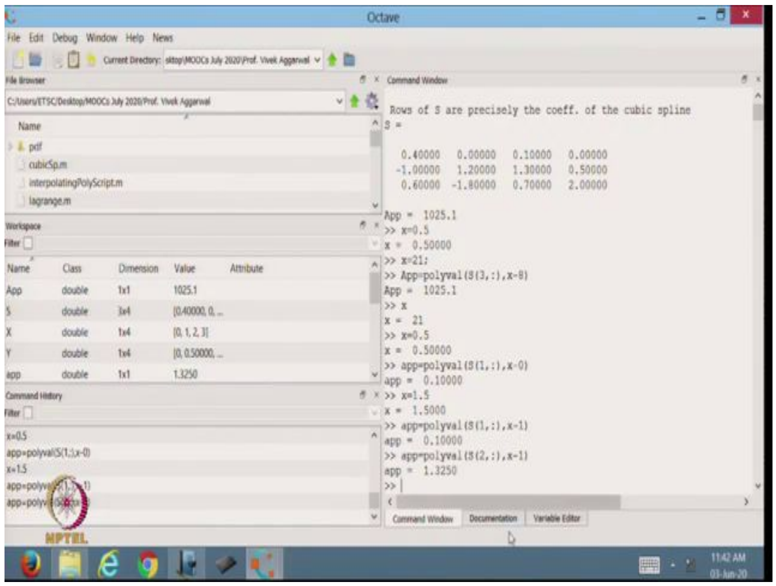

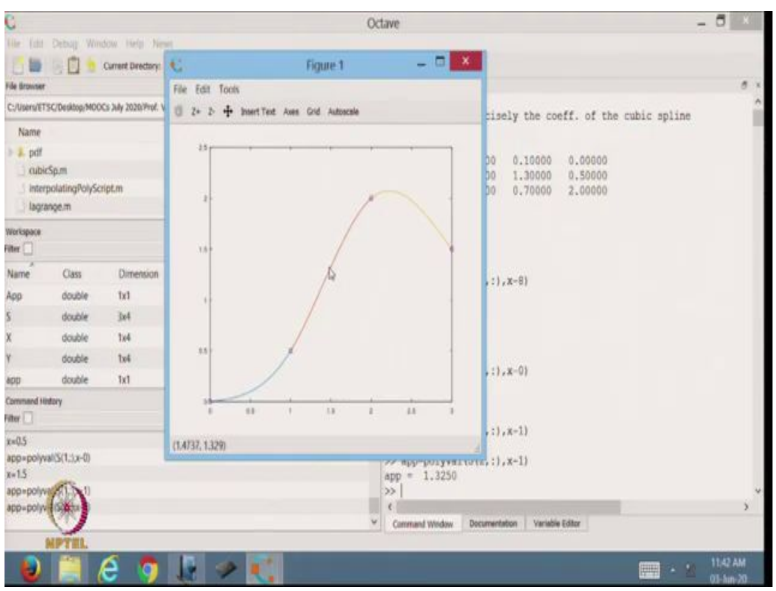

So x is equal to what I want to find at 1.5. So in this case, my x was because my coefficient capital X is 0 1, so it should be from 1 and that should be 2. So it is 1.325. So, you can verify also from this graph then it is coming 1.325, so it is 1.325. So, the same way we can define for 2.5 so it should be somewhere, so let us try to define the value at 2.5.

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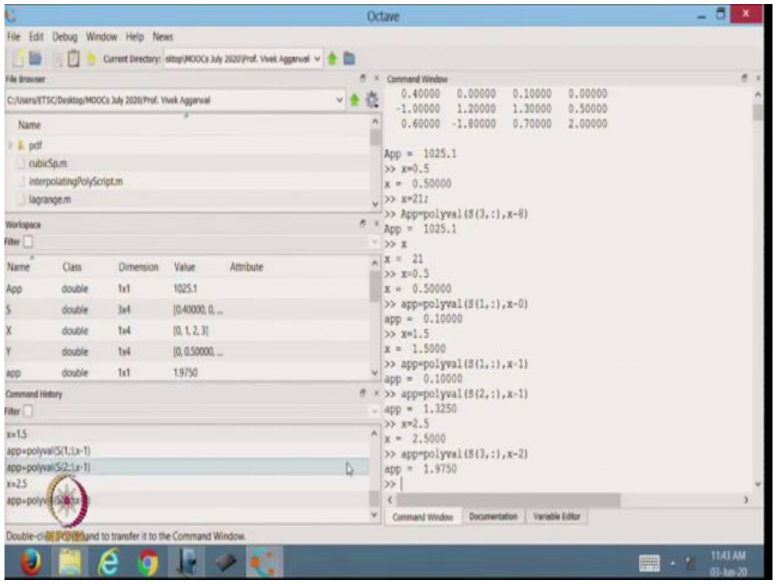

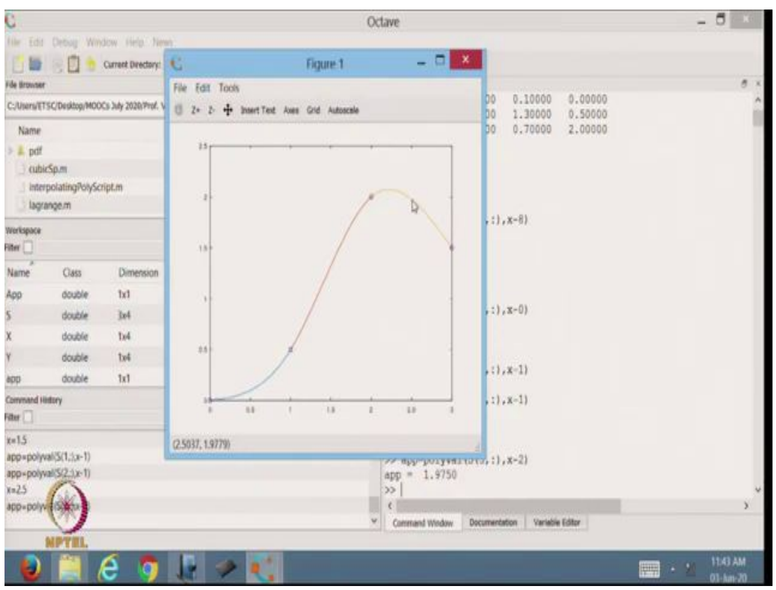

So, suppose I take x = 2.5 and then it is falling in the third polynomial cubic spline and the third is passing through the 2. So, it is 1.97, so it is coming 1.97. So, you can see that it is my 2 and coming at 1.97, close to 2, so it is 1.97, so that value is given. So this way we can find out all the cubic splines. And based on this one, let us try to take the other data.

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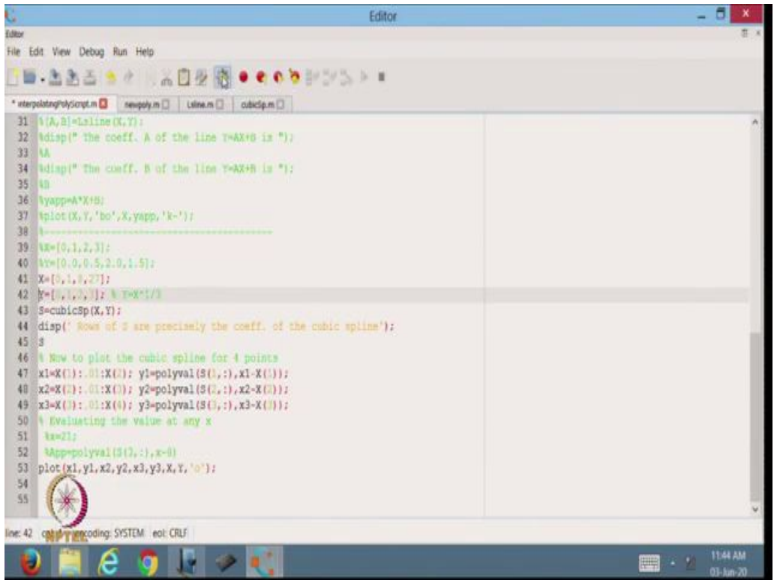

So let us take another data. So this is the function I am taking $x^{1/3}$. So the cube root of x I am taking here. So let us try to solve this one.

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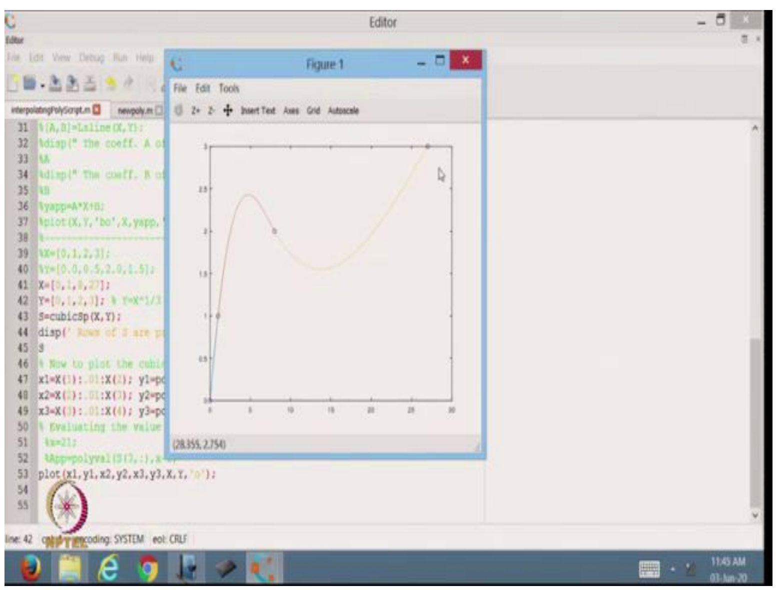

So this is a cubic spline we are going to get, so I have 4 points 0, 1, at 1, 2. So 0, 1, 8 and 27. So 0, 1, 8 at 27. The value of y is 0 at 0, 1 at 1, 8 at its value is 2 and this is 3. So this is my cubic spline we are getting for these values.

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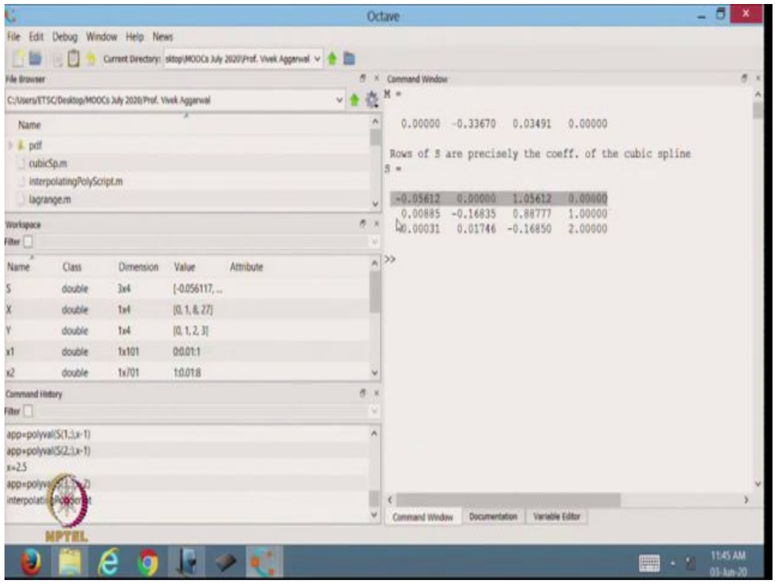

And from here if I find the value of S, so this is my first cubic spline, the coefficient of the first cubic spline, this is the second cubic spline and this is the third cubic spline. So based on this one, I can find the 3 cubic splines. The same thing you can define for any number of datas, suppose I just increase the number of points here.

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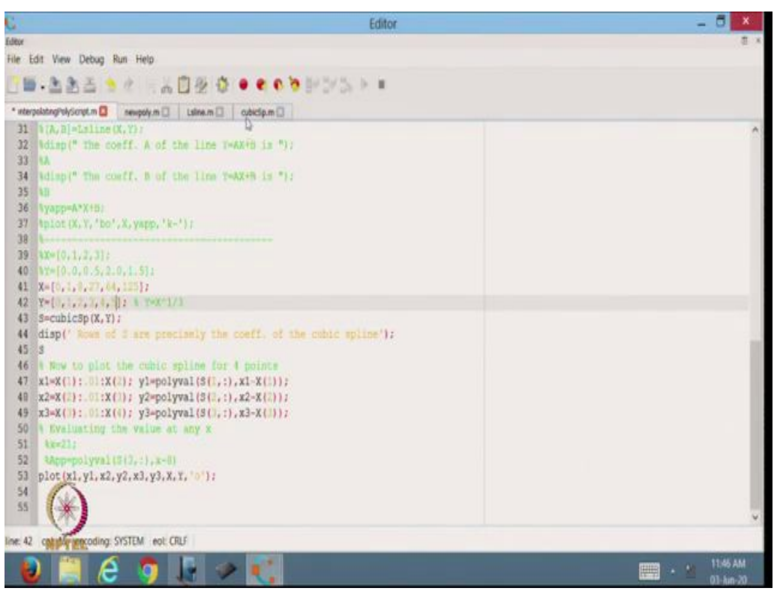

So, let us try to find out, so I take it 64 and 125, suppose I take this one and then I take it 4 and then 5. So let us increase the data, just I have increased 2 raises to power 3 is 8, 27, 64 and 5. So let us try to run this one.

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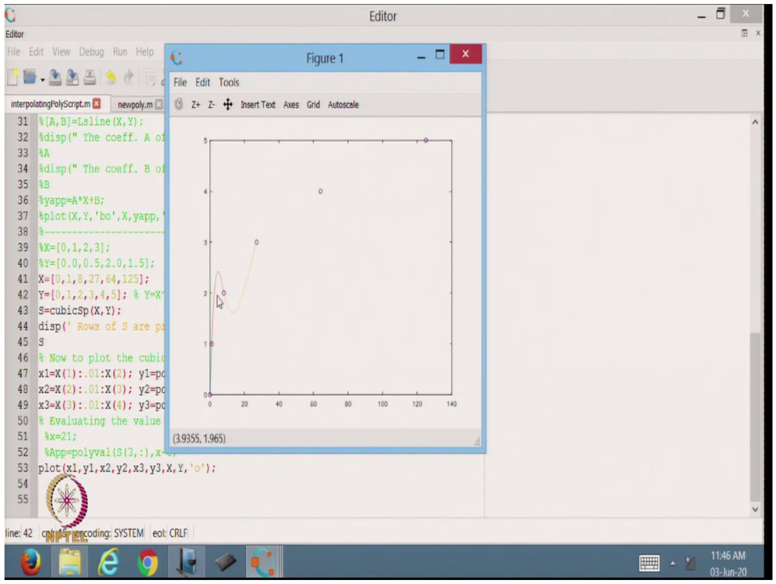

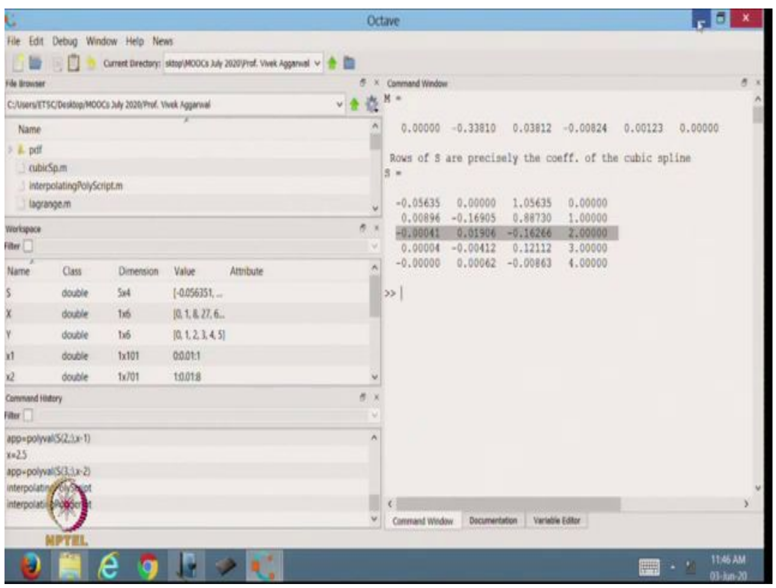

So this is from here you can see that this is the cubic spline we get and this is the last one. So I have to add this point there in this case, so I have to define the fifth degree polynomial also the same way and only then we can plot all this figure. But from here you can see that the matrix is a 5 cross 5 matrix. So these are the same. So these are the coefficients of the polynomial.

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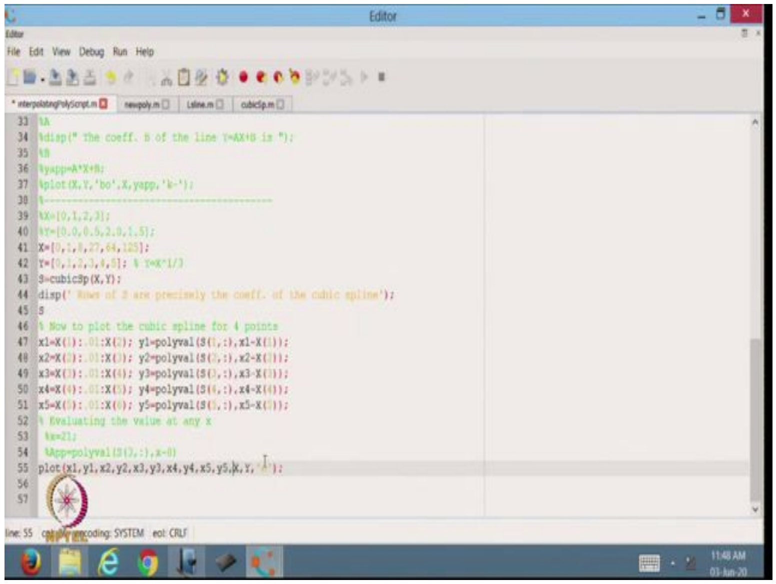

And from here, if I introduce the same way, x3 so let us try to do this one. So I will call it x4, x4 to x5 and this is y4, 4, 4, and then I define this 5, 5, 6, 5. And now based on this one, I can plot this. So here I can add x4, y4, x5 y5. So, let us try to, here.

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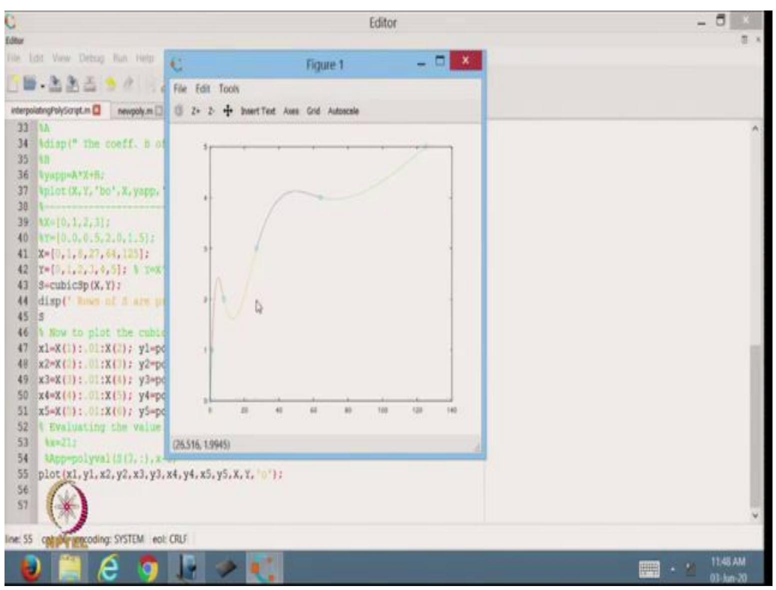

So, this is my now fifth degree spline. So, this is the first one, this is the second one, then the third one is this, fourth one is this and fifth one is this. So, based on this one, the number of points we have, we can plot all the cubic splines in each of the sub intervals with the help of this program. So, I have given you the only few examples that involve only a few data points, but you can extend this one for any number of data points. So, this is a way we can define the cubic spline function.

So, I will stop here today. So, today we have tried to solve, try to make the MATLAB code for the cubic spline and then we have plotted the cubic spline and found that the cubic spline is a smooth function in the given interval or in the for the given data set points. So thanks for watching this, thanks very much.