Scientific Computing Using Matlab
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Lecture 48
Quadratic Polynomial Fitting and Code for Lagrange's Interpolating Polynomial Using Octave

Hello viewers, welcome back to the course on Scientific Computing Using MATLAB. So, today we will continue with the least square method to find out maybe today how we can do the quadratic fit and then we also make some MATLAB codes in octave just to verify whether we can find out the different polynomials using data or not. So, let us start doing this one.

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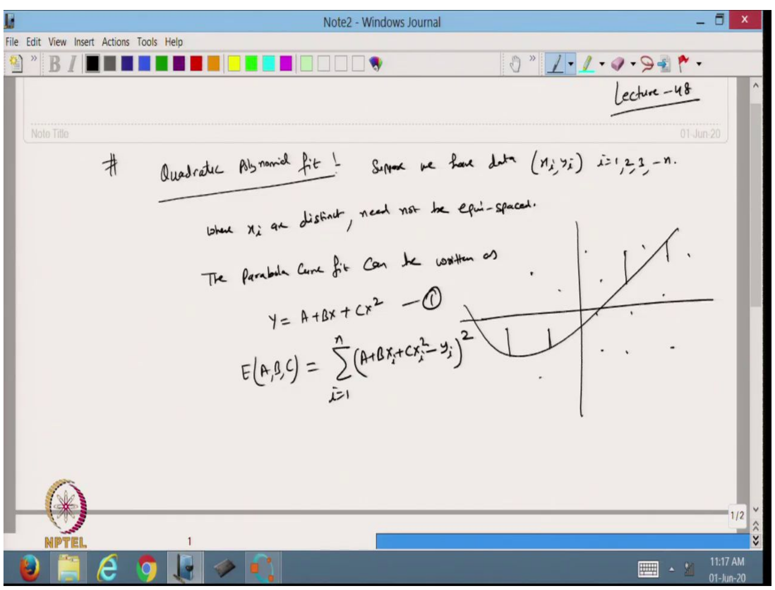

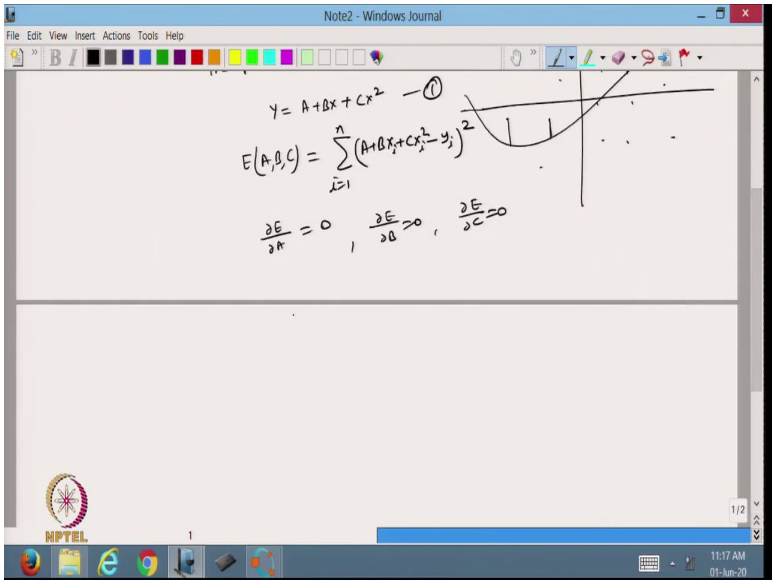

So, today I will give you that how we can do the least or I can call it quadratic polynomial fit, so suppose we have data, so that data is given to me as points $(X_i, Y_i), i = 1, 2, \cdots, n.$ so I am taking the n number of points, where all X_i are distinct and need not be equi-spaced. So, what do I want to do? I want to apply the quadratic polynomial fit or the parabola fit. So, the I can call it parabola also, parabola curve fit can be written as, so I will call it Y that is equal to suppose I write as $Y = A + BX + CX^2 \cdots (1)$, so this is the quadratic polynomial I am fitting and I want to find the value of A, B and C.

Now I can go with the same methods that suppose I have this and from here I can take this, so now I take the parabola like this one, so suppose I have this parabola something in the quadratic function and my data points are given to us in this way. So, that is the data points, then I can find the error of this one, from this one, from this one, that we have already discussed in the previous lecture, so then I can define my error function as E(A,B,C).

So, this one I can define as $E(A, B, C) = \sum_{i=1}^{n} (A + BX_i + CX_i^2 - Y_i)^2$. Then I can find out the value of this one by, so I want to find the minimum error, so I will define

$\frac{\partial E}{\partial A}=0, \frac{\partial E}{\partial B}=0, \frac{\partial E}{\partial C}=0.$ So, from here I can have the three equations. So, based on these three equations I can find the value of A, B and C, and then you can find out the given parabola.

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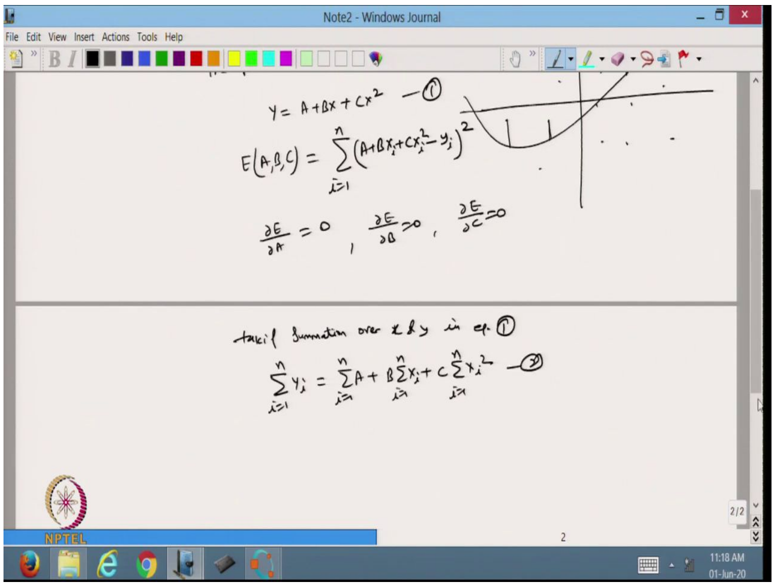

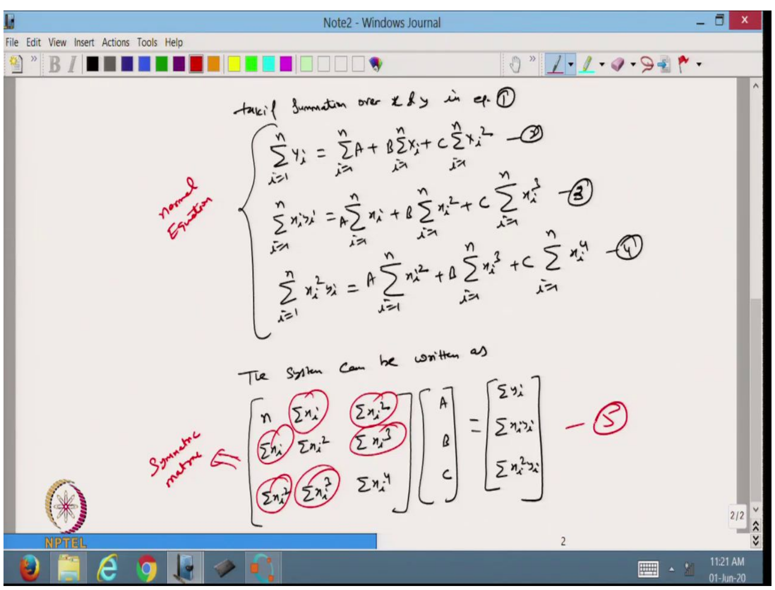

But I can write the direct normal equation also so from equation number 1, taking summation over the over X and Y in equation number 1, so I take the summation

$$\sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} A + B\sum_{i=1}^{n} X_i + C\sum_{i=1}^{n} X_i^2 \cdots (2)$$

So, this is my normal equation, first normal equation, so the second one I can write

$$\sum_{i=1}^{n} Y_i X_i = A\sum_{i=1}^{n} X_i + B\sum_{i=1}^{n} X_i^2 + C\sum_{i=1}^{n} X_i^3 \cdots (3)$$

then multiplying one more time also with the X_i, so I can have

$$\sum_{i=1}^{n} Y_i X_i^2 = A\sum_{i=1}^{n} X_i^2 + B\sum_{i=1}^{n} X_i^3 + C\sum_{i=1}^{n} X_i^4 \cdots (4)$$

So, based on this one, this is my normal equation, so based on this normal equation I can find the system.

So, from here the system can be written as so from here I can write this system as

$$\begin{pmatrix} n & \sum_{i=1}^{n} X_i & \sum_{i=1}^{n} X_i^2 \\ \sum_{i=1}^{n} X_i & \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 \\ \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} Y_i \\ \sum_{i=1}^{n} Y_i X_i \\ \sum_{i=1}^{n} Y_i X_i^2 \end{pmatrix} \cdots (5)$$

So, from here I will get this system, so this system from here you can see that this is equal to this, this is equal to this and this is the same as this one, so from here I can say that this matrix is a symmetric matrix. So, based on this one I can take my iterative methods or other methods to solve the system, so I call it five. So, now based on this one, I can find out the value of A B C and then we are able to find the parabola fit.

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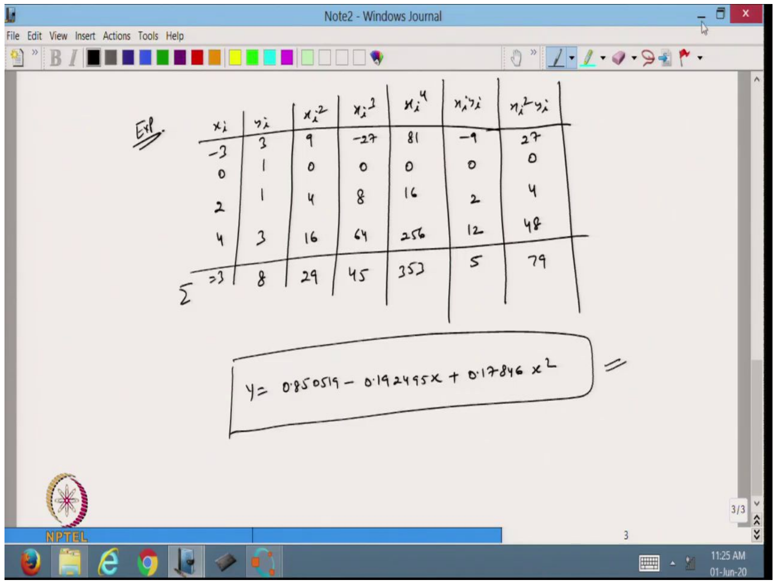

So, let us take one example, suppose I have given data, so this in my x_i and this is y_i, that is given to me, so $x_i = -3, 0, 2, 4$ and $y_i = 3, 1, 1, 3$, now I take x_i^2, so it will be $x_i^2 = 9, 0, 4, 16$, x_i^3 also needed, so it will be $x_i^3 = -27, 0, 8, 64$, x_i^4 is also needed, so it will be $x_i^4 = 81, 0, 16, 256$, on the right hand side I need $x_i y_i$ and $x_i^2 y_i$. So, $x_i y_i = -9, 0, 2, 12$ and $x_i^2 y_i = 27, 0, 4, 48$.

Now, I just take the summation also, so based on this one I just take the summation, so $\sum x_i = 3, \sum y_i = 8, \sum x_i^2 = 29, \sum x_i^3 = 45, \sum x_i^4 = 353, \sum x_i y_i = 5, \sum x_i^2 y_i = 79.$. So, once I have this data, then I can find the value A B C and then we should be able to get the curve fitting.

So, if I solve this one then my y we will get, so this is $y = 0.850519 - 0.192415x + 0.17546x^2$, that is my second order polynomial fit for the given data. So, this is the way we can define the polynomial fit for the given data. So, let us do this one with the help of MATLAB code. So, let us start doing the MATLAB code. So, today we will make the codes with the help of octave.

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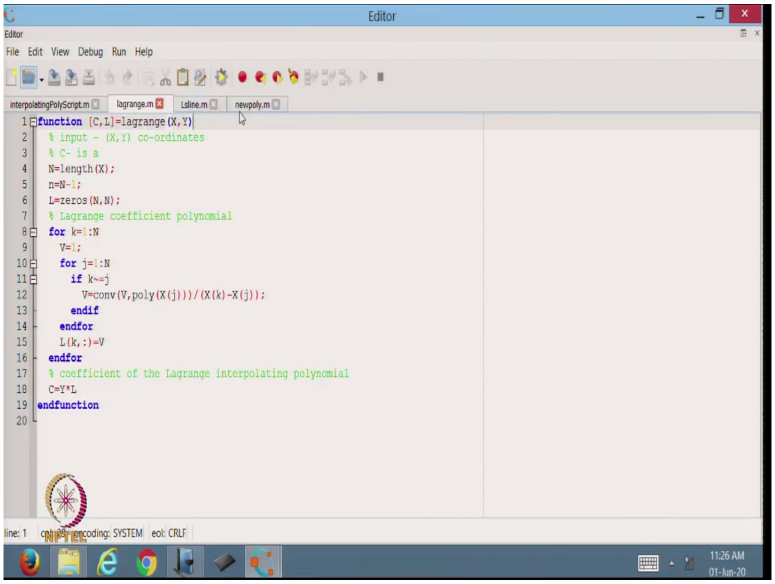

So, I start with the first one, I will start with the Lagrange methods, so in the Lagrange methods, I know that suppose I have the data, so that data is given to me, so this is my X and Y, so that X and Y is given to me now from here I will give the input as X Y, so this code is I already made for you, so here I can directly discuss this one, so I will call it the Lagrange and in this case the input value is X and Y and the output is C and L. So, what is the L? L is the Lagrange coefficient I know and C is the coefficient of the polynomial.

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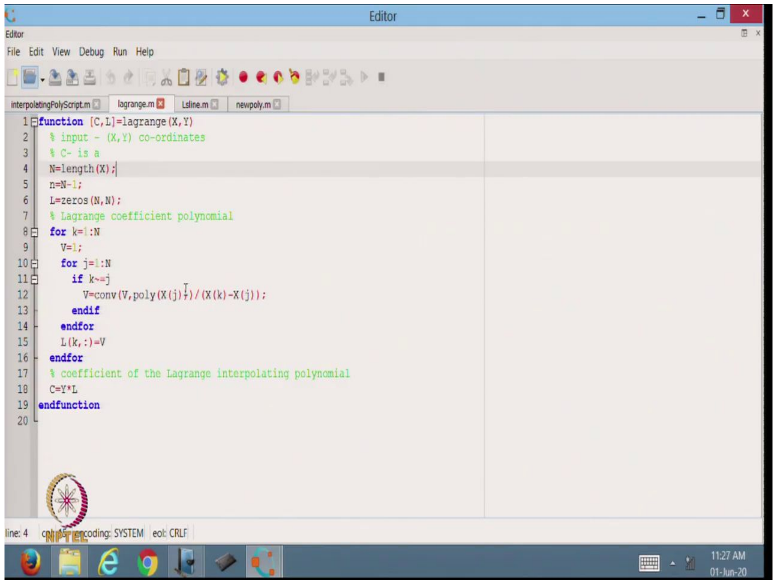

Now, in this case I will take the length, so this gives me the length of the X, so I call it capital N and n = N-1, so I call it small n. Now, the L is zeros I take that is of N cross N, so now in this case I will find the Lagrangian coefficient polynomials. So, what do I want to find? I want to find the Lagrangian coefficient. So, in this case what I will do? I will use two functions. The first one is the poly, polynomial.

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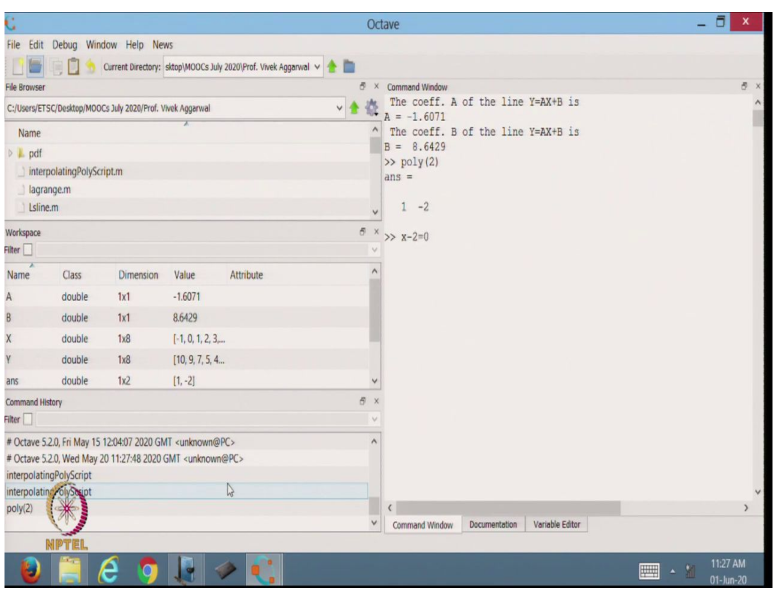

So, let us discuss what this means so I call it poly, so let us define poly 2. So, poly(2) means it gives me the polynomial whose root is 2, so this is equal to basically x-2, so if I put x-2= 0, so in this case I get the polynomial whose root is 2.

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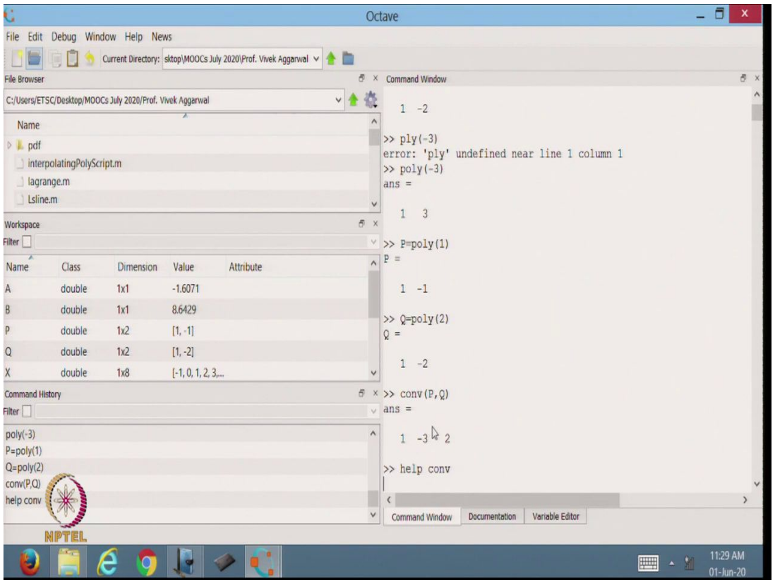
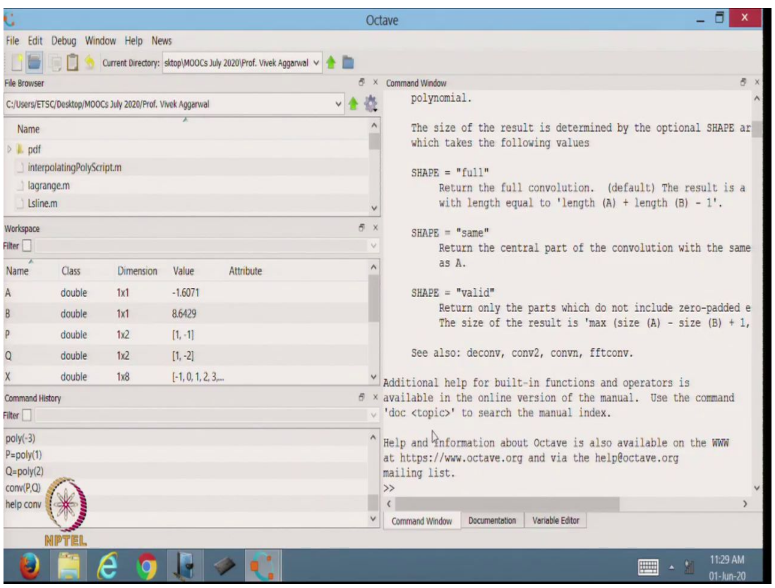
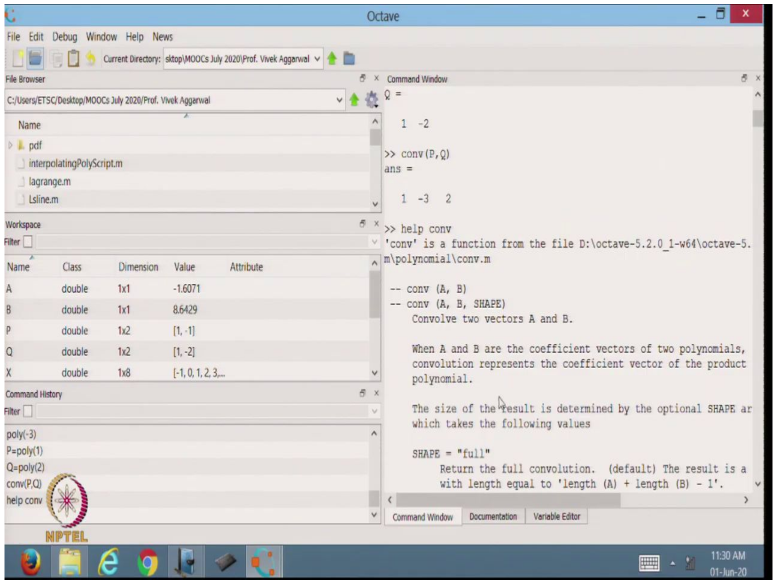
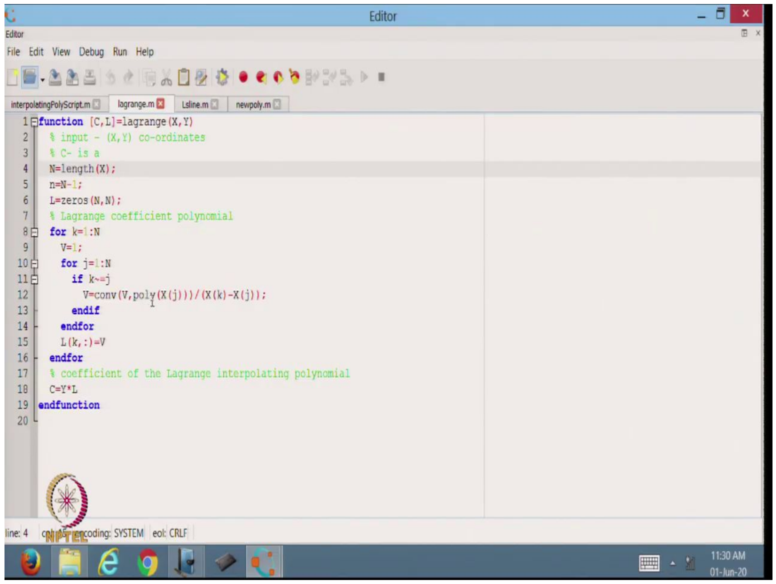


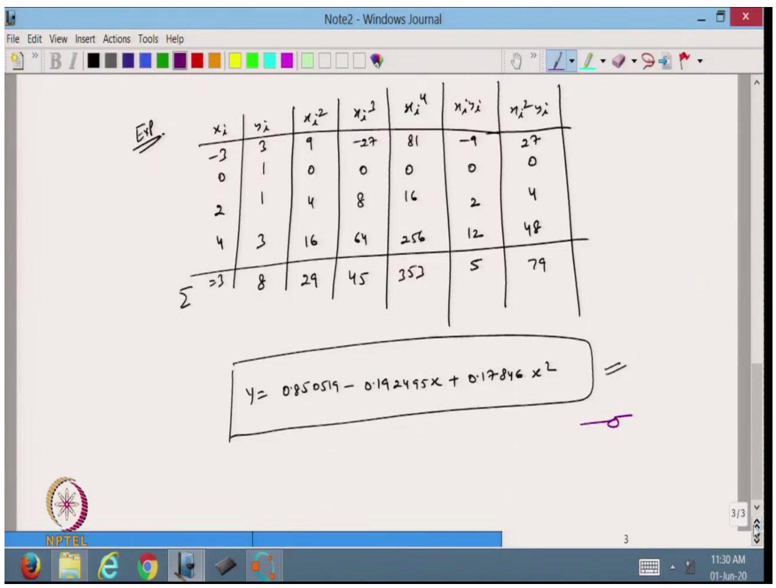




Similarly, I can define poly(3), or maybe poly(-3), so that is the polynomial whose root is -3. So, this is what I am finding here. So, let us call it a P= poly(1), I call it another polynomial that is Q = poly(2), now I apply convolution of P and Q, so this is the convolution and it gives me the, so basically what it is doing? So, it is multiplying x-2, x-1, so this is what I am doing. So, now from here I can see that this will be x^2 , so the coefficient of x^2 is 1, now x-2 and -1, so -3x, so this is -3 and in the end -1 into -2 it is 2.

So, it gives me the coefficients of the quadratic having the coefficient 1 , -3 and 2. So, that is the function convolution, I can write that help, so from here I can just delete this one and I can write help, so it gives you the documentation of the function. So, when A and B are the coefficient vectors of two polynomials, convolution represents the coefficient vector of the product polynomial. So, convolution of A and B. So, this is the way we can define. So, let us go to the. (Refer Slide Time: 16:19)





So, now in this case what do I do? I will define the for loop k = 1 to n, V I just take as 1, now for j = 1 to n, when $k \neq j$ I will apply this function, so what is this meaning? First you will find out poly(X(j)) because I know that. So, after doing this one, I just take the next.

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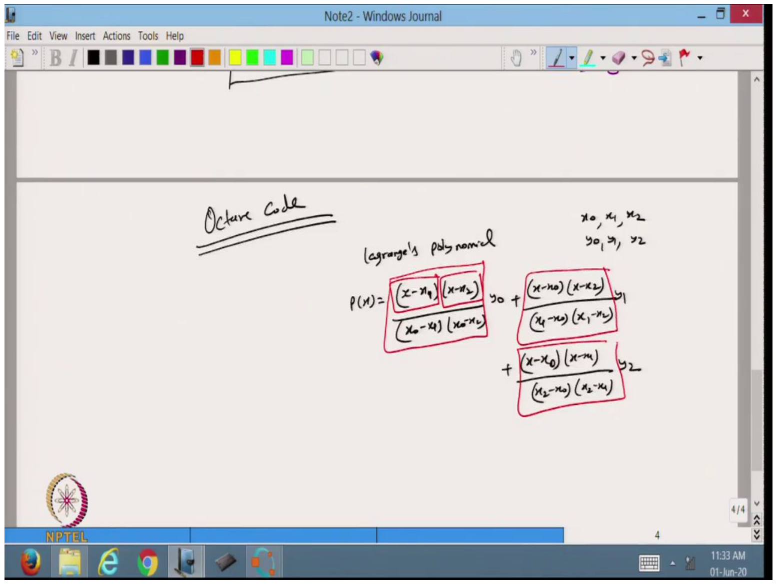

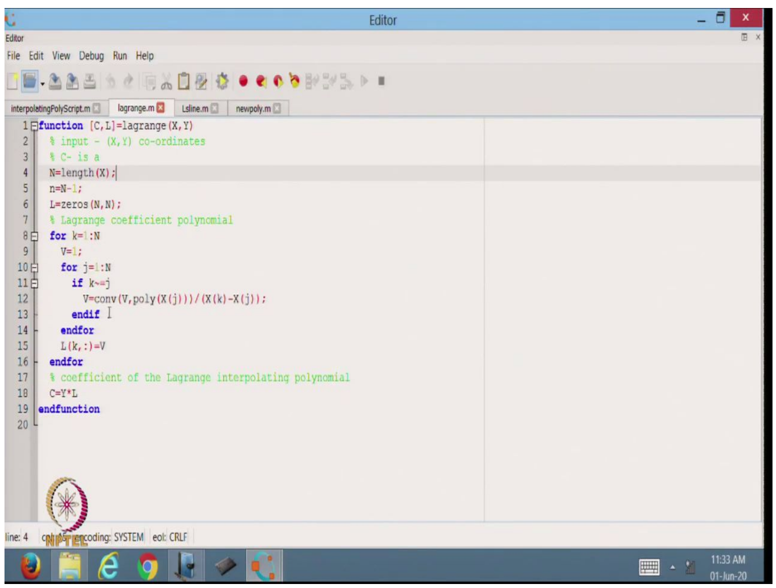

So, in the Lagrange polynomial, octave code, so I know that in the Lagrange polynomial I have the polynomial suppose I have P(x), so in this case, I know that suppose I have the point $x_0, x_1, x_2, y_0, y_1, y_2$ so I will get

$$P(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

So, from here you can see that these here I will put the points, so this is a constant value, but here you can say that this is a polynomial whose root is x_1 multiplied with the polynomial whose root is x_2.

So, I am using this one here, I am defining the polynomial whose root is X(j), when k is not equal to j and I am multiplying this one with a V, I started with the V = 1, divided by X(k)- X(j), so I am doing this one so this is a multiplication I am doing divided by X(j)-X(k), so that product is also I am taking, so with the help of this one, I am able to find all this Lagrangian coefficients, Lagrangian polynomials, whatever we are defining, so what I am going to do? I am going to find out what is the value of this one, what is the value of this one and what is the value of this one. And this one I can do for the all values of the data that is given to me. So, this is what I can define from here.

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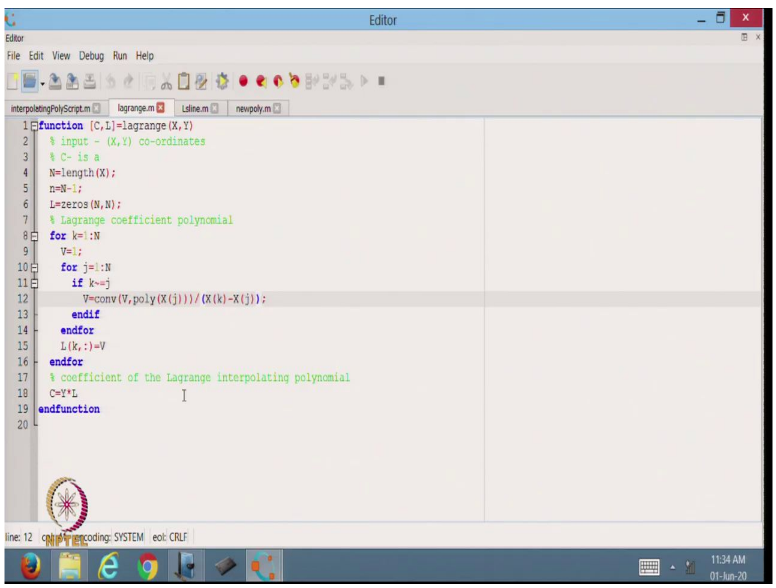

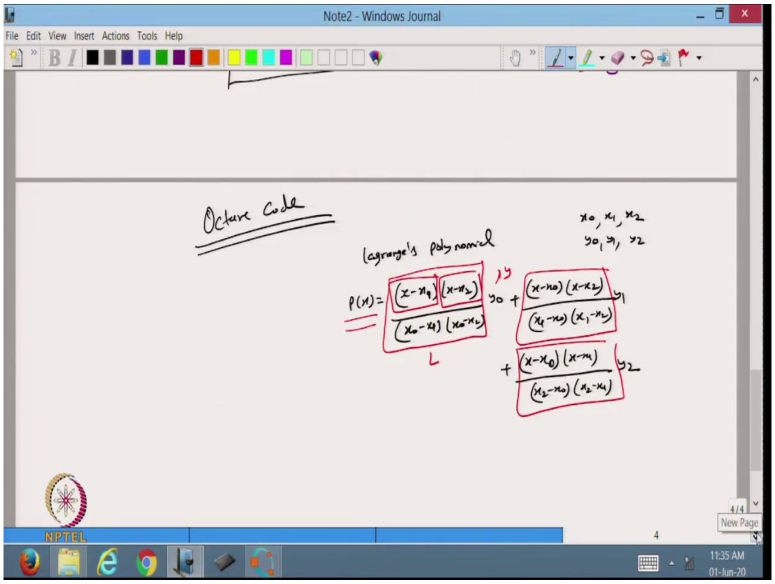

And based on this one, so based on this one first I will do the convolution and then I multiply by all these factors whatever k is not equal to j, other factors I will take and I will all this will be saved in the value of V, so this V is I am going to have, so now this if loop is over and then this for loop is over and then I will, what I do? I will write this value of V as a 0 and n, it means I have defined this as a matrix, so in the matrix in the if I take Lk and all the columns, so I will get the value of V.

So, I am saving this value in the kth row, so this is you can see that k =1, so the first value will be saved in the first row, then the second row, then the third row, then the fourth row, so I have total n number of the Lagrangian fundamental polynomial and that the value of this will be saved in the L function, L matrix. So, once I have done this one, then I can find the coefficient of the Lagrangian interpolating polynomial, that is y into L. So, what is y into L? So, this is y into L. so, this is my y and this is my L, so I am taking this as a L and this is my y, so from here I just take the multiplication and then my polynomial will come, so let us take a one example of this one.

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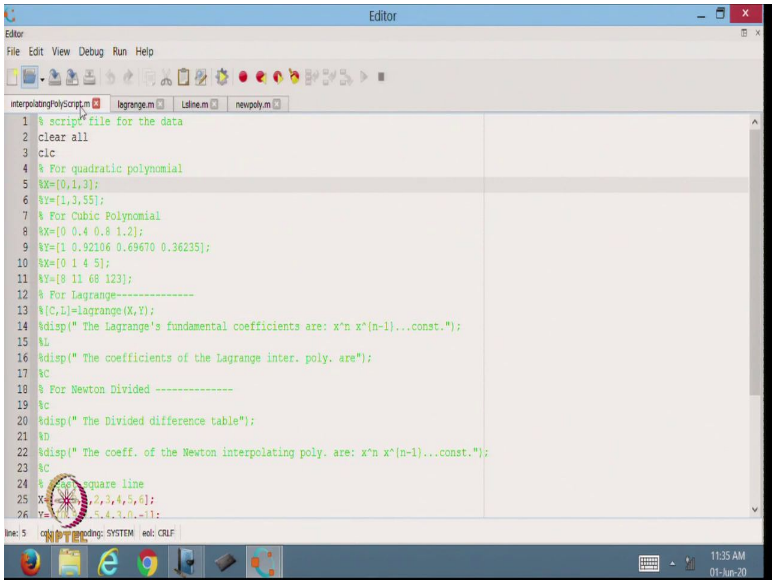

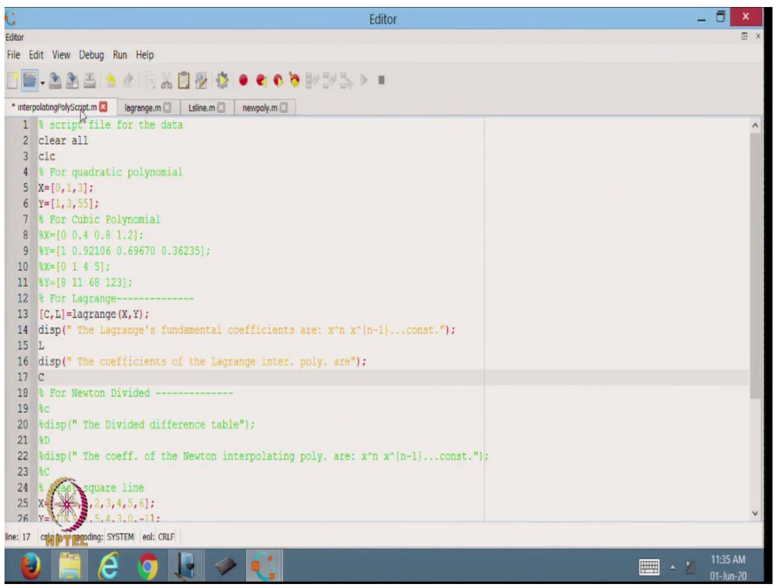

So, let us take this script file, so I will take first with the quadratic polynomial, so let us this one I am doing and for the Lagrange I will do this one L, this one, so generally I make the script files, so that I should be able to this one. So, I have done the comment. So, I have the value of x and this y and then I want to find this one. So, let us run this one.

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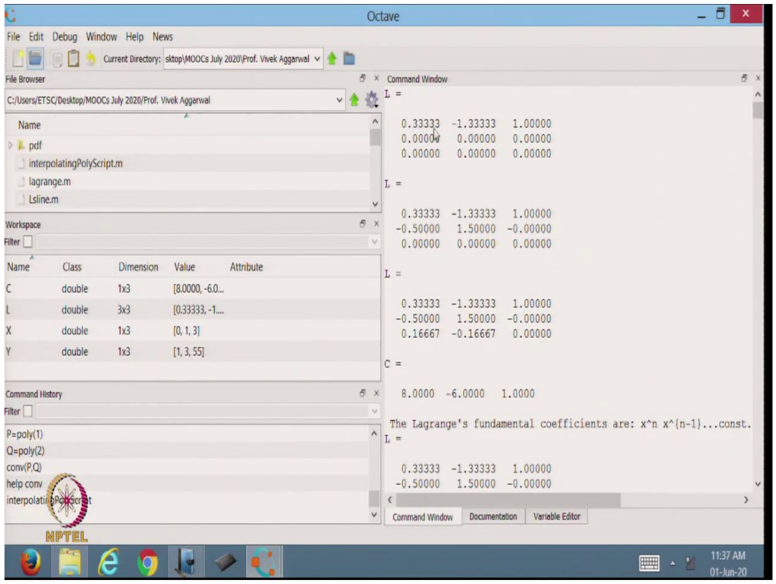
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L double 3x3 [0.33333, -1....
X double 1x3 [0, 1, 3]
Y double 1x3 [1, 3, 55]
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Q=poly(2)
conv(P,Q)
help conv
Command Window
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0.00000 0.00000 0.00000
L =
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0.16667 -0.16667 0.00000
C =
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The Lagrange's fundamental coefficients are: x^n x^{n-1}...const.
L =
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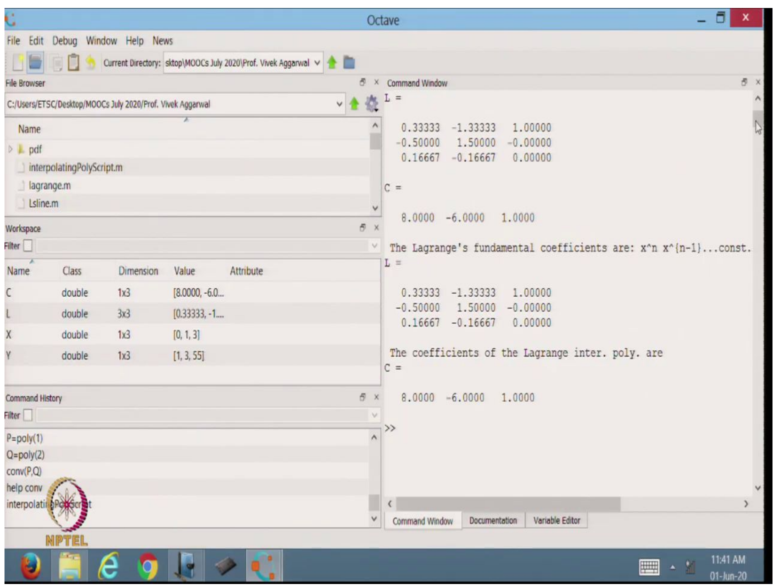
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L double 3x3 [0.33333, -1....
X double 1x3 [0, 1, 3]
Y double 1x3 [1, 3, 55]
Command History
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Q=poly(2)
conv(P,Q)
help conv
Command Window
L =
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0.16667 -0.16667 0.00000
C =
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The Lagrange's fundamental coefficients are: x^n x^{n-1}...const.
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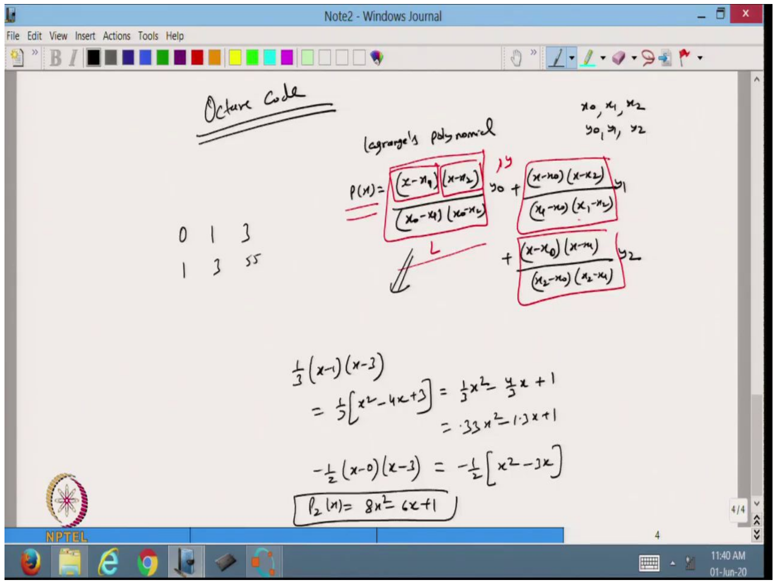

So, this is the answer I am going to get. Now, the first because I am printing this one for the whole loop, so if I go for the first one, so this is the values I am getting 0.33 -1.3 and 1, so this is the coefficients of the first Lagrangian polynomial that fundamental polynomial, this is the second one and this is the third one.

So, I know that I will get only three values, so this is if you see from here, the first one is saved here, because this is a quadratic, so I need for the quadratic I have three coefficients so that coefficients there, then this and then this. So, based on this one, so that is the value I get and then this is the C, so that is the coefficient of the corresponding polynomial, the Lagrange polynomial.

So, based on this one I will get the value of 8 -6 and 1. So, if you remember then in the previous examples we have solved this one, so I have solved this one in the previous lectures and if you see then I will got this value, so I will get this polynomial as (⅓)(x-1)(x-3), because I have the data 0 1 and 3, 1 3 and 55.

So, I got this one, so from here, you can just see it will be equal to $\frac{1}{3}x^2 - \frac{4}{3}x + 1 = 0.33 - 1.3x + 1$ so the coefficient is 0.33 -1.3 1. So, this is the coefficient I am getting. The second one is from here I am getting

$$-\frac{1}{2}(x-0)(x-3) = -\frac{1}{2}(x^2-3x)$$

, so the coefficient is from here you can see that it is -0.5 and 1.5. So, -0.5 and 1.5 and other values 0. So, similarly, I get this one.

So, based on this one, if I find out the Lagrange polynomial, so the finally I get this data, so the final my $P_2(x)$ the Lagrangian polynomial is coming $8x^2-6x+1$ and that is the answer and from here also, I am getting this answer. So, that is the coefficient of $8x^2-6x+1$. So, using this one I can find the corresponding Lagrange polynomial for any type of data. So, here we have taken just three data points, but we can have many as many data points as we wish and I can run the same code for that type of data.

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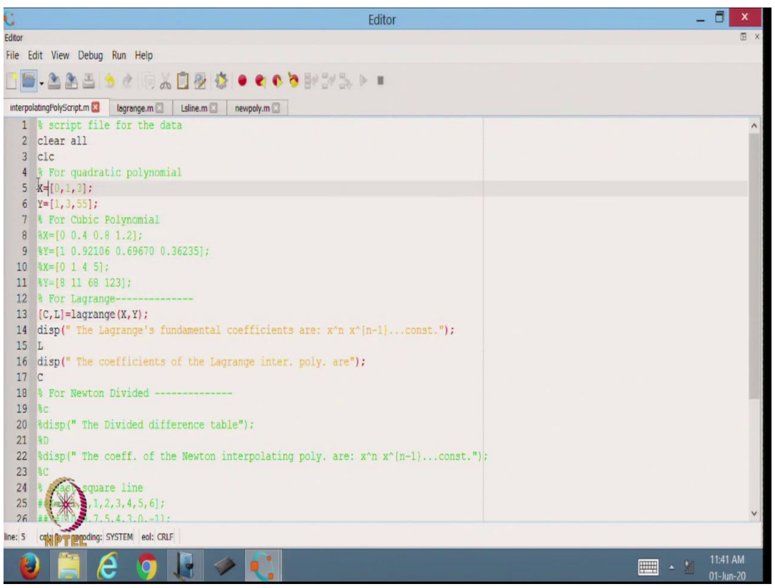

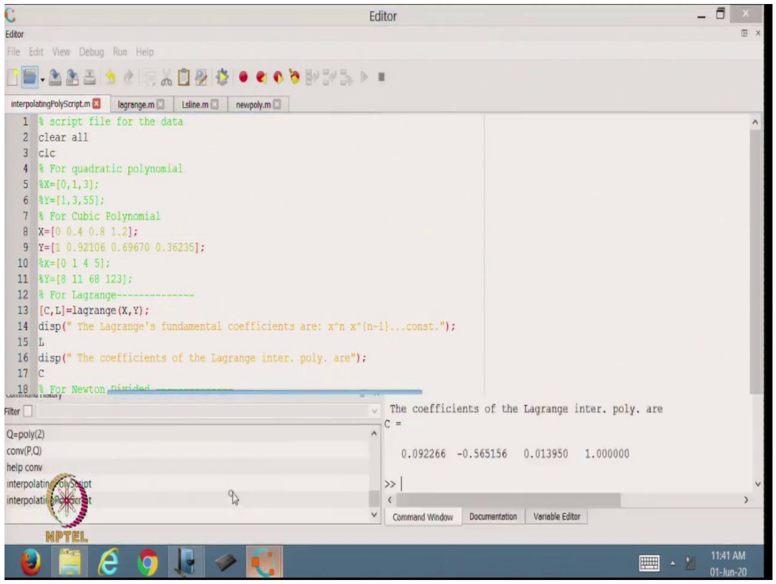

So, this is already there now the same thing I have done, so let us again try to find out for another type of data. So, let us take cubic polynomials. So let us take this data. So, this data is the four points I have taken and I run this one.

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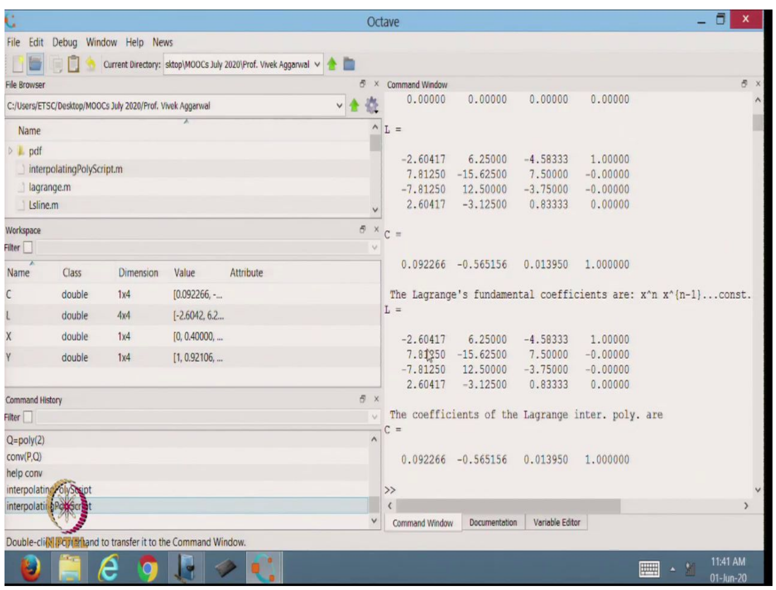

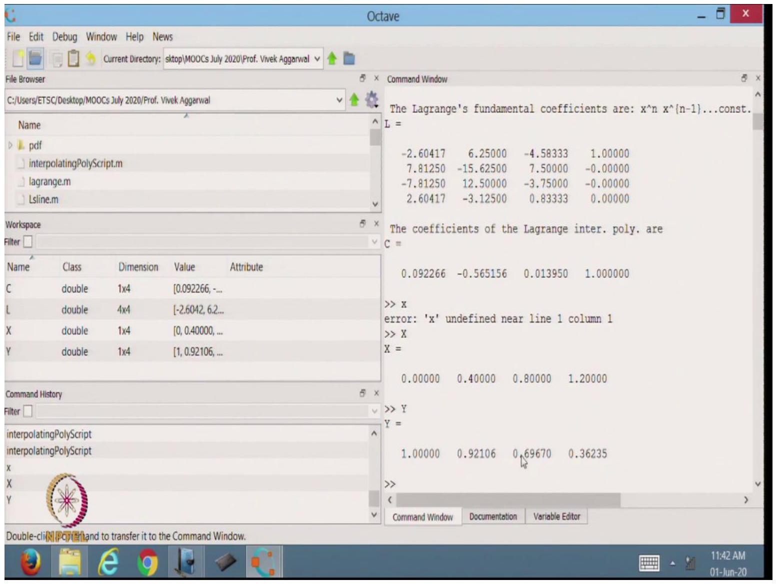

So, I get this value. So, this is a 4 by 4 matrix I am going to have, so this is the coefficient of the first fundamental Lagrangian polynomial, then second the third and fourth and the coefficient is this one, so it means the $0.092266x^3 - 0.565156x^2 + 0.013950x + 1$. So, that is the corresponding cubic Lagrangian interpolating polynomial passing through the given points. So, this is the, my given point x is this one, so it is a capital X and this is capital Y. So, that is the point I have chosen that passes through these points.

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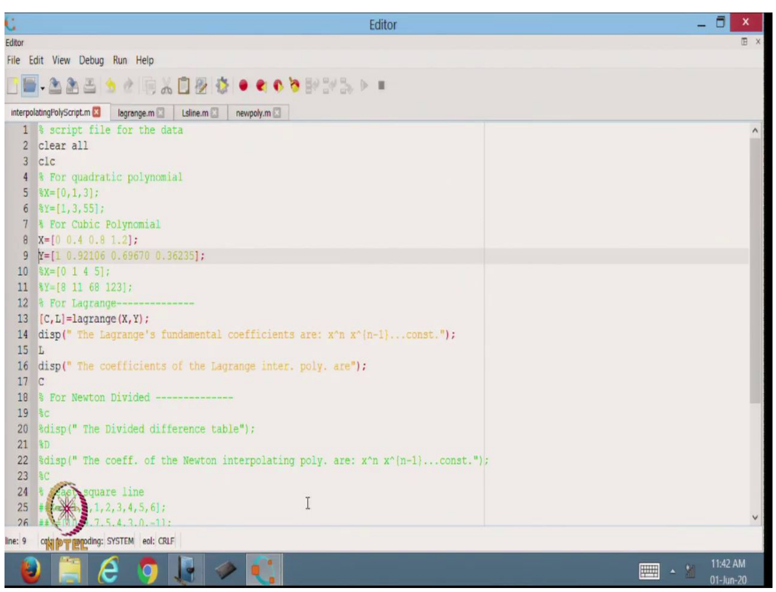

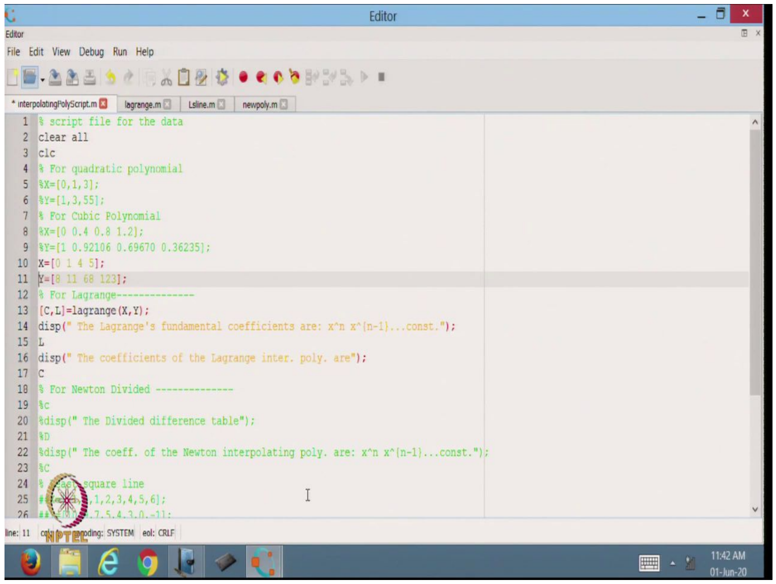

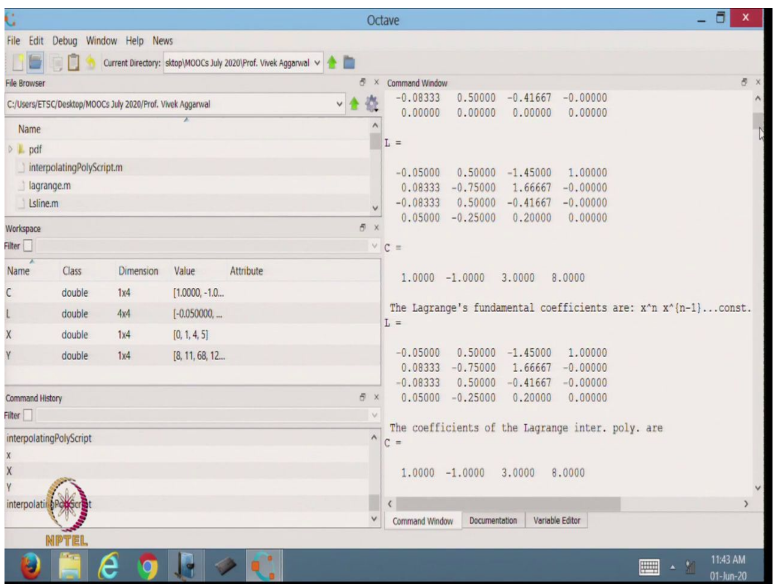

So, the same way I can define with the other data this is the corresponding so simple data I have chosen, so in this case, this is the corresponding fundamental coefficient of the fundamental Lagrangian polynomial that is the cubic here. And in the end we will get this value. So, it means that this is $x^3 - x^2 + 3x + 8$, so that is a polynomial.

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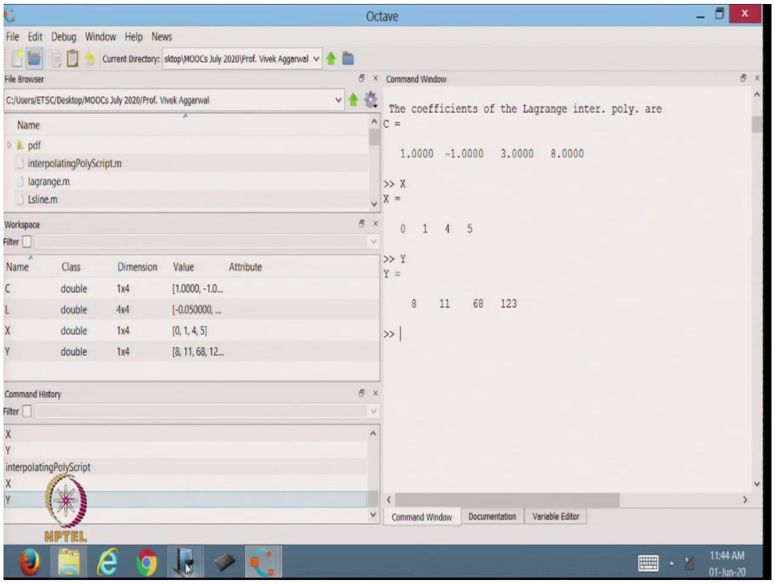

So, after that I can see that what is my x and what is my y, so based on this x and y, this is my cubic polynomial Lagrangian polynomial passing through these points. So, I am defining this with the cubic function. So, let me stop today here. So, today we have started with the parabolic fit and then we have done one example, and then we have started with the MATLAB code or the octave code for doing such types of problems numerically with the help of MATLAB. So we will continue with this one, thanks for watching, thanks very much.