Scientific Computing Using Matlab

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Lecture 47
Curve Fitting

Hello viewers. Welcome back to the course on Scientific Computing using Matlab. So in the previous lecture we have completed that linear spline and cubic spline. So today we will continue with the curve fitting. So today we will start with the curve fitting.

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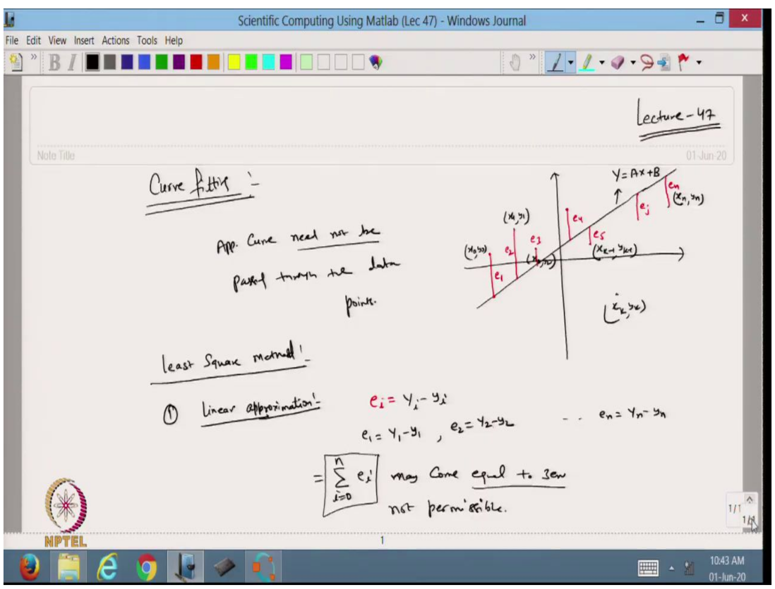

So now, suppose I have the data. And this data is distributed in the given domain like this one. So suppose this is my $(x_0, y_0), (x_1, y_1), \cdots, (x_k, y_k), \cdots, (x_n, y_n)$. So in this case, I know that if I pass any polynomial passing through all these points, then it is called the interpolating polynomial. That we have already discussed. Then we have discussed that if I want to approximate this with the piecewise polynomial, then we have the splines function. So we have discussed linear splines and cubic spline.

Now the next thing is that, suppose I want my curve, what I want is that our approximating, approximating curve need not be passing through the data points. So that is our condition that in the previous one all the polynomials, approximating polynomials, were passing through the points. But in this case, polynomials need not be passing through the points. So maybe I will take, I will start with a linear function. So suppose I take a line like this one, and this line is, in this case it is not passing through any of the points, because the points are distributed like this one and I want to approximate this with a line.

And this is my line, so I call it $y = ax + b$. So in this case, our function or the approximating polynomial or approximating curve that is not passing through any of this point. So, in this case how can we find this one? So I will start with this one. So the first one I will start. So the method for finding this is the least square method. So the least square method, I start with the first one, that is the linear approximation. So what is the meaning of linear approximation? So in this case let us, I start with this one. So these are my points and this is the line I am passing, that line I have wanted to approximate.

Now what I do, now these are the points that are already given to me. Now what I want to find, I will try to find the error between this. So suppose at this point, or at this point, or at this point, the value of y_0 is this one. And the line I am going to approximate, so based on this one, the value of y is this one. So this is my y_0 value, that is the exact value that has been given to me. And this is the value if I put the x_0 in the given linear equation, that is the equation of a line, then I get this value. So the difference between this is, I call it e_1.

So that is the error between these two values. Now the same way I will do for x_1. So I call it e_2, then I will e_3, this way I will have e_4, e_5 and in the this is my suppose e_j anyone and in the last, suppose this is my e_n. So that I call it the $e_i = Y_i - y_i$, I will be represented by Y = Ax+B, so this one. So based on this one, my $e_1 = Y_1 - y_1$, my $e_2 = Y_2 - y_2, \cdots, e_n = Y_n - y_n$.

So this difference I find. Now suppose I add all these values, so suppose I add i.e. $\sum_{i=0}^{n} e_i$. So in this case it may happen that some e_i are positive and some e_i are negative. So if I take the sum of this one, it may come equal to 0. So if it comes equal to 0, so it may, if it comes equal to 0, then I say that the, I will say that the error is 0, but in reality the error is not equal to 0. So that is why, I cannot directly add these errors and then I will take the total error. So this is not possible. This is not permissible.

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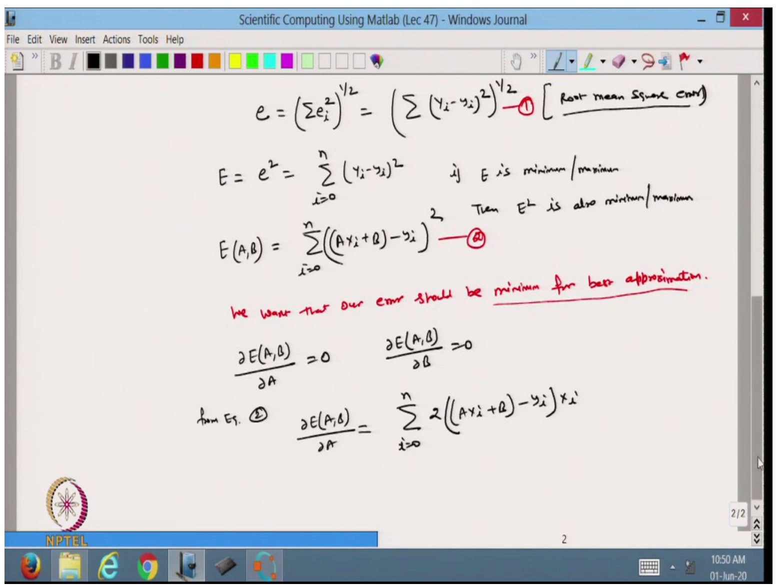

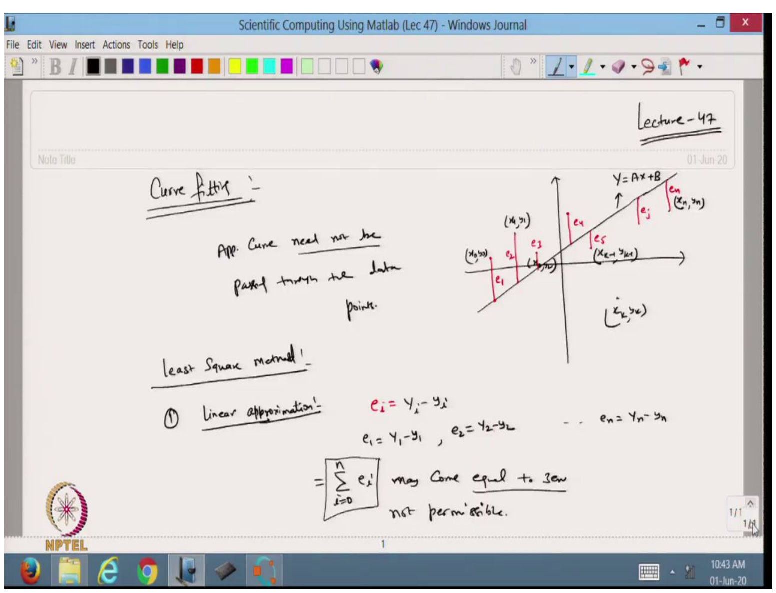

So in this case, what I will do is take the help of the least square method. So in the least square method, what I do is show that first I will take the square. So what I do is, I will take the error e_i, doing the square of this one, adding all these errors and then taking the square root. So it is at least, root means square error. So I am taking the root means square. So I am taking the square, then mean and the root. So this call it as e, so that is my error I am taking.

Now in this case what is happening,

$$e = \Big(\sum_{i=0}^{n} e_i^2)^{1/2} \Big) = \Big(\sum_{i=0}^{n} (Y_i - y_i)^2 \Big)^2 \cdots (1)$$

So this is called root mean square error. So this is basically the norm I am defining. So I am taking this one. Now if I substitute this value, so from here, I can call it as

$$E = e^2 = \sum_{i=0}^{n} (Y_i - y_i)^2$$

Now I know that if e is minimum or maximum, then e^2 is also minimum or maximum.

So instead of under root, I just take this term and then try to find out. So this is my e^2. So let us call it the error, so I will call it an error. So just the e^2, I am calling error. Now from here, I can write that this error can be written as

$$E(A, B) = \sum_{i=0}^{n} (AX_i + B - y_i)^2 \cdots (2)$$

Now this is a function.

So from here I can say that we want our error to be minimum for best approximation. Because this is the line passing through, this is the line, so I want that all errors term should be very, very small so this line passes through all the points. If this line passes through all the points, then that is the best approximation we can have for the given data. So our main concern is how we can make this error minimum of all. So this is our requirement.

Now from the mathematical point of view, we know that this is a function. Now, it is a function of two variables A and B, that we need to find out to find out the equation of a line. So from here I can say that if we want to find out the minimum of this one, then I take the partial derivative of this, to find out the critical point.

So from here, if I take the partial derivative with respect to A and B then it should be equal to 0

i.e. $\frac{\partial E(A,B)}{\partial A} = 0, \frac{\partial E(A,B)}{\partial B} = 0$ So from here equation (2),

$\frac{\partial E(A,B)}{\partial A} = \sum_{i=0}^{n} 2(Ax_i + B - y_i)x_i$ So from here

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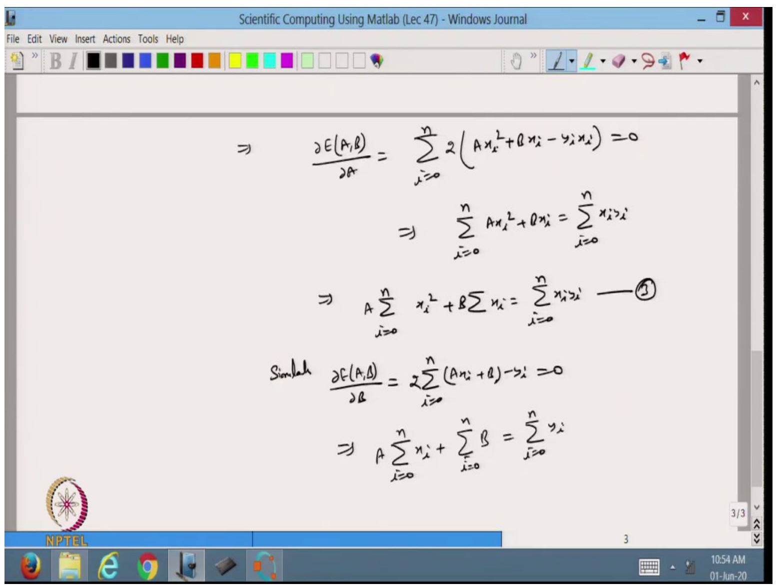

$$\frac{\partial E(A,B)}{\partial A} = \sum_{i=0}^{n} 2(Ax_i^2 + Bx_i - y_i x_i) = 0$$

It implies that

$\sum_{i=0}^{n}(Ax_i^2 + Bx_i) = \sum_{i=0}^{n} x_i y_i$ So $A\sum_{i=0}^{n} x_i^2 + B\sum_{i=0}^{n} x_i = \sum_{i=0}^{n} x_i y_i \cdots (3)$

Similarly,

$$\frac{\partial E(A,B)}{\partial B} = 2\sum_{i=0}^{n}(Ax_i + B - y_i) = 0$$

It can be written as

$$A\sum_{i=0}^{n} x_i + \sum_{i=0}^{n} B = \sum_{i=0}^{n} y_i \cdots (4)$$

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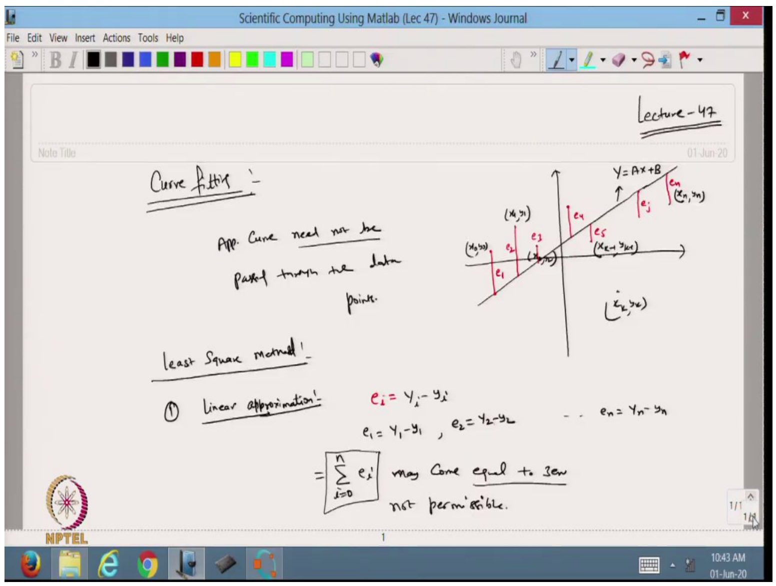

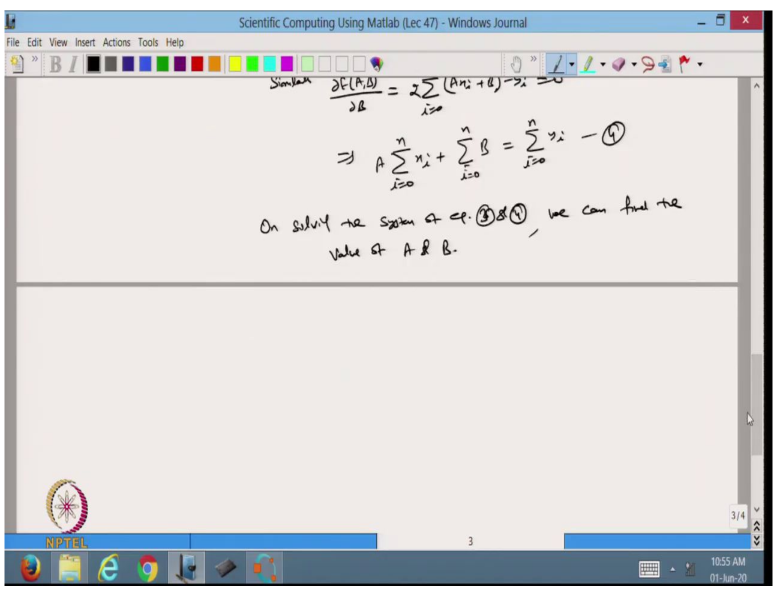

So now we have a system, so now on solving the system of equations, we can find the value of A and B. So once I know the value of A and B, from here I can find the value of this linear function, this line. And that will be my best approximation, or the curve fitting passing through these points. So that is the way we can define the, the equation of a line. So let us do that so we can find the minimum value. So let us do one example. So whatever the equation I am getting, the equation 3 and 4, so I can write from here also.

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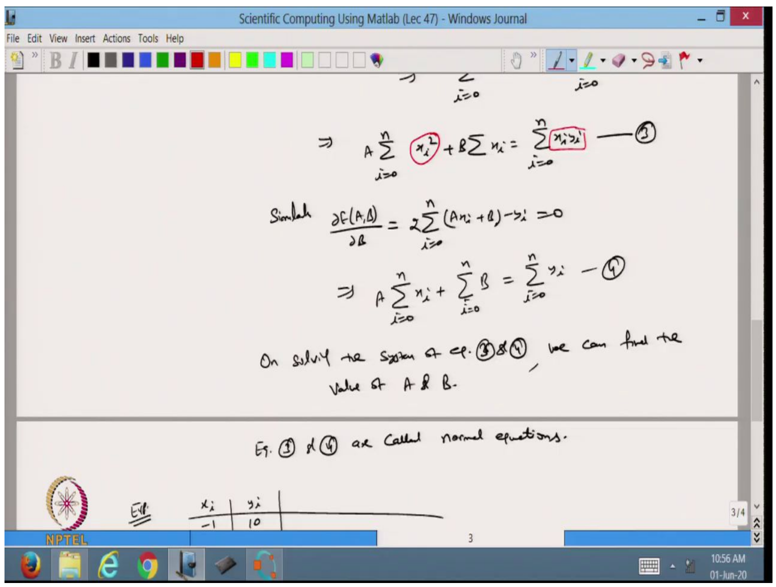

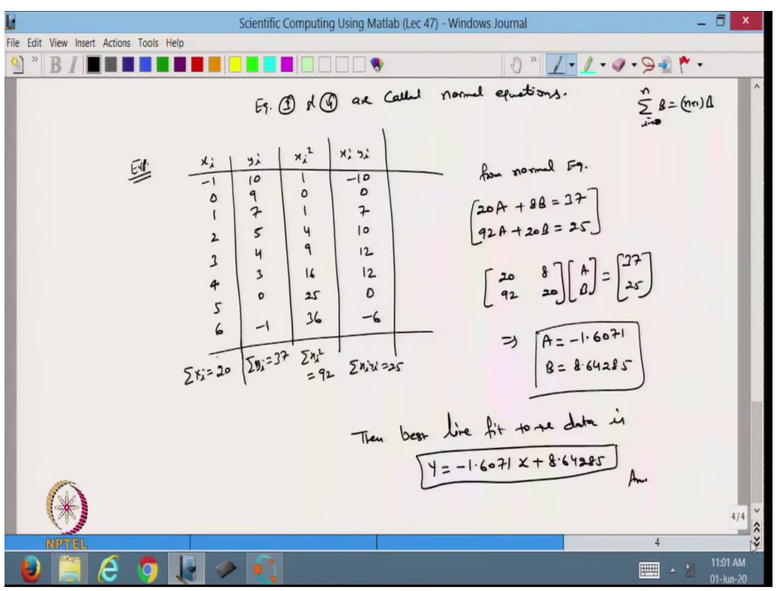

Equations 3 and 4 are called normal equations. So these are called the normal equations. So using this normal equation, I can find the value of A and B, and then we can find the equation of the line. So let us do one example. So in this case, I have the data. So let us my x_i and $y_i =$ are given to me, $x_i = -1, 0, 1, 2, 3, 4, 5, 6$ and $y_i = 10, 9, 7, 5, 4, 3, 0, -1$ Now if you see from here, if I want to solve the system then what I need it, I need the, $x_i^2, x_i y_i, \sum_{i=0}^{n} x_i$ and $\sum_{i=0}^{n} y_i$.

So it will be $x_i^2 = 1, 0, 1, 4, 9, 16, 25, 36$ and $x_i y_i = -10, 0, 7, 10, 12, 12, 0, -6$ And I need the sum also. So from here, I can find the summation. So from here $\sum_{i=0}^{7} x_i = 20, \sum_{i=0}^{7} y_i = 37, \sum_{i=0}^{7} x_i^2 = 92$ and $\sum_{i=0}^{7} x_i y_i = 25$. So this value is now, we have calculated. Now from the normal equation, so from normal equations, so normal equation is this, so first I will write 20A+8B = 37 this is my first equation.

The second equation is 92A+20B = 25. So this is the system of equations I am going to get. And if I solve this system with the help of the methods, we have already done it, but it is a simple 2 by 2 system, so I can solve it with the help of any method. So if I find the solution to this one, so I get the value of A = -1.6071 and B = 8.64285. So this is the value of A and B I am getting.

And now once I know the value, then the best line fit to the data is -1.6071x+ 8.64285. So this is the equation of the line and that is my answer to this one. So based on this one, I am able to fit a, so I tell, I will call, that I am able to fit a line for the given data. And this is the best fit we can have. So this is the way we can find out the linear fit for this one. Now the second one is, what about the other function? So based on this one, this is a linear fit we have taken. Now we will want to define the polynomial fit.

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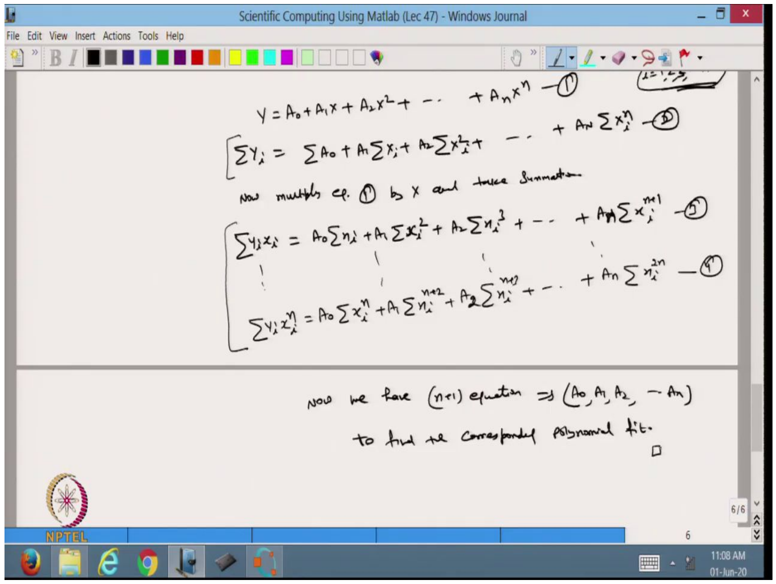

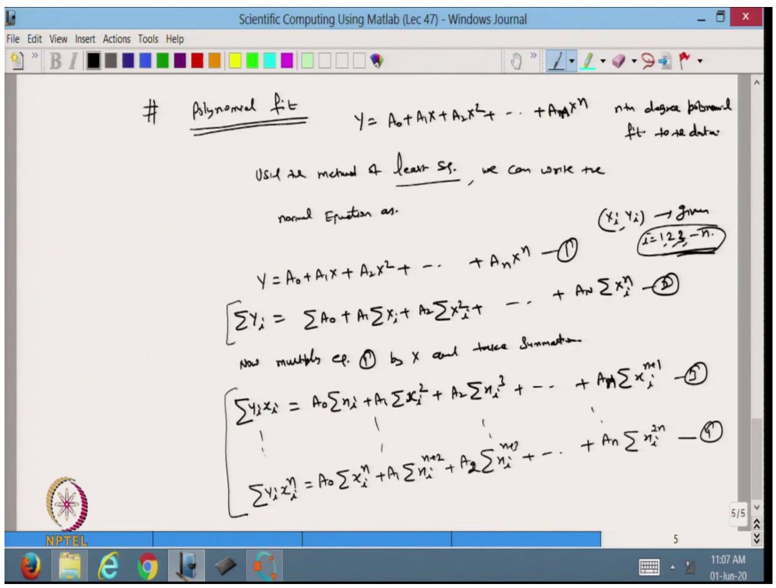

So polynomials fit that, I have a polynomial. Suppose I take the polynomial as $Y = A_0 + A_1 X + A_2 X^2 + \cdots + A_n X^n$. So this is the nth degree, polynomial, polynomial fit to the data. Now, in this case, I need an n+1 number of equations to find out the value of $A_0, A_1, \cdots, A_n$, so I can, I will tell you how we can write the normal equation directly and $(X_i, Y_i), i = 0, 1, 2, \cdots, n$ So using the method of least squares we can find, we can write, we can write the normal equations as, so what I do is, first I have write the equation in term of i

$Y_i = A_0 + A_1 X_i + A_2 X_i^2 + \cdots + A_n X_i^n \cdots (5)$ Now I take the summation of both sides. So

$$\sum_{i=0}^{n} Y_i = \sum_{i=0}^{n} A_0 + A_1 \sum_{i=0}^{n} X_i + A_2 \sum_{i=0}^{n} X_i^2 + \cdots + A_n \sum_{i=0}^{n} X_i^n \cdots (6)$$

. Now multiply equation (5) by X_i and taking the summation

$$\sum_{i=0}^{n} Y_i X_i = A_0 \sum_{i=0}^{n} X_i + A_1 \sum_{i=0}^{n} X_i^2 + A_2 \sum_{i=0}^{n} X_i^3 + \cdots + A_n \sum_{i=0}^{n} X_i^{n+1} \cdots (7)$$

Similarly,

$$\sum_{i=0}^{n} Y_i X_i^n = A_0 \sum_{i=0}^{n} X_i^n + A_1 \sum_{i=0}^{n} X_i^{n+2} + A_2 \sum_{i=0}^{n} X_i^{n+3} + \cdots + A_n \sum_{i=0}^{n} X_i^{2n} \cdots (8)$$

So based on this one, I get the total n+1 equations which gives me the values of $A_0, A_1, \cdots, A_n$, to find the corresponding polynomial fit.

So this is all about today. I will stop here. So today we have started with the curve fitting and then we discuss for a given data how we can fit a linear function. And also we have discussed how a curve fitting, polynomial fit can be done. So we will continue in this, with this way in the next lecture. And we also try to make some Matlab codes. So thanks for watching this, thanks very much.