Scientific Computing Using Matlab
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Lecture 46
Continued

Hello viewers, welcome back to the course on Scientific Computing Using Matlab so, today we will continue with the previous lecture on the cubic spline.

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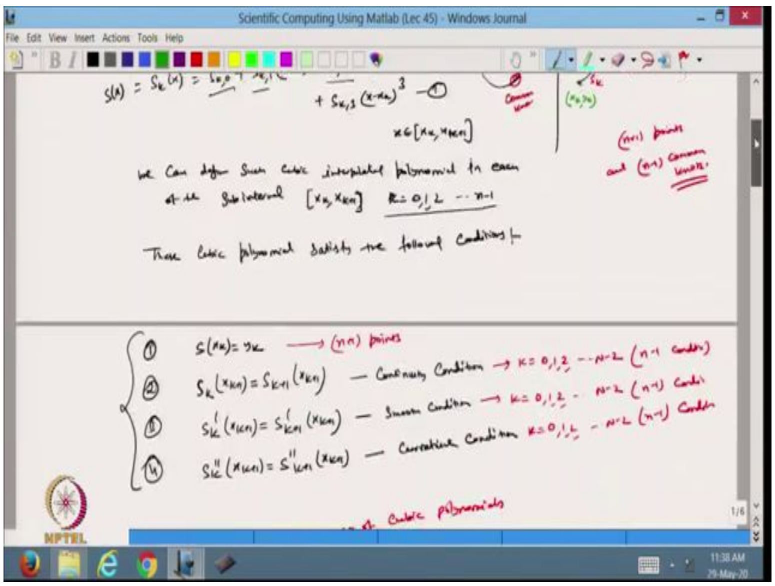

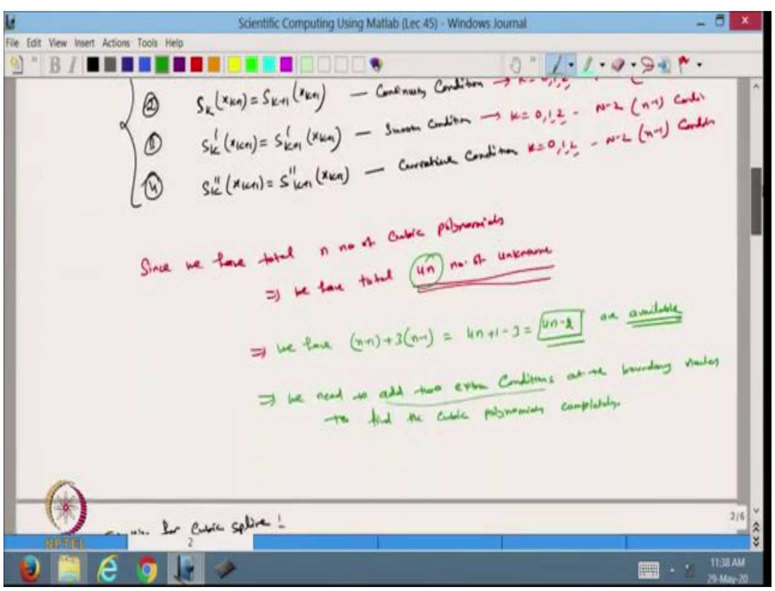

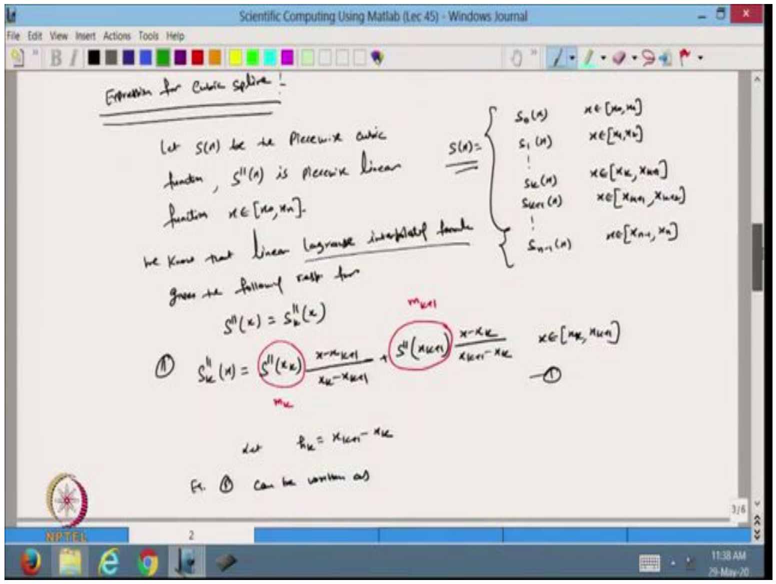

So, in the previous lecture, we have discussed how we can find the cubic spline for a given interval.

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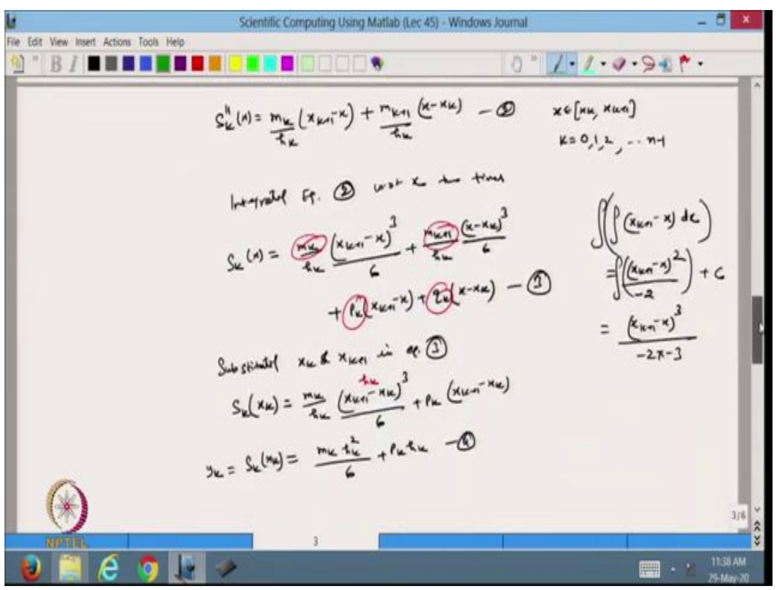

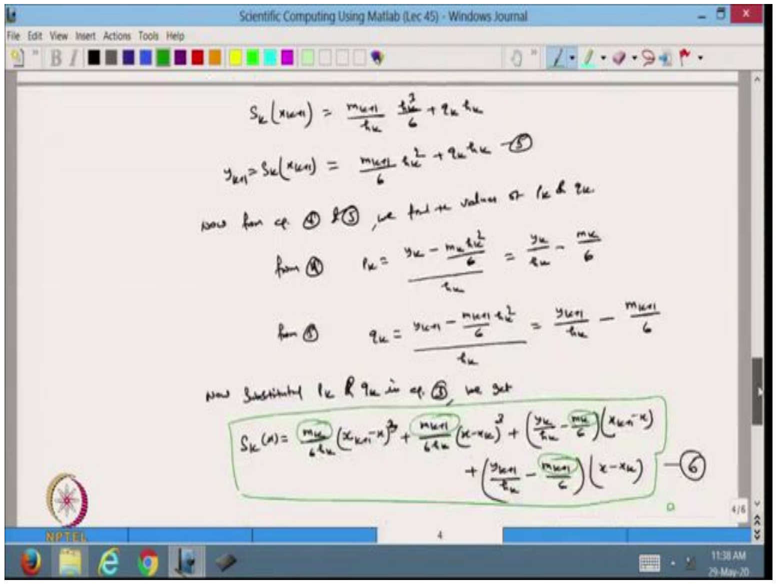

And then we have started with the expression that how we can express the cubic spline in each of the some interval and then in the end we will, we came with the equation number 6 so, that is the given polynomial that is the cubic polynomial we have defining and now we have to find out what is the value of this m_k the second derivative in the, for the given cubic value, cubic spline. So, now we have to find this value, so let us continue with this one.

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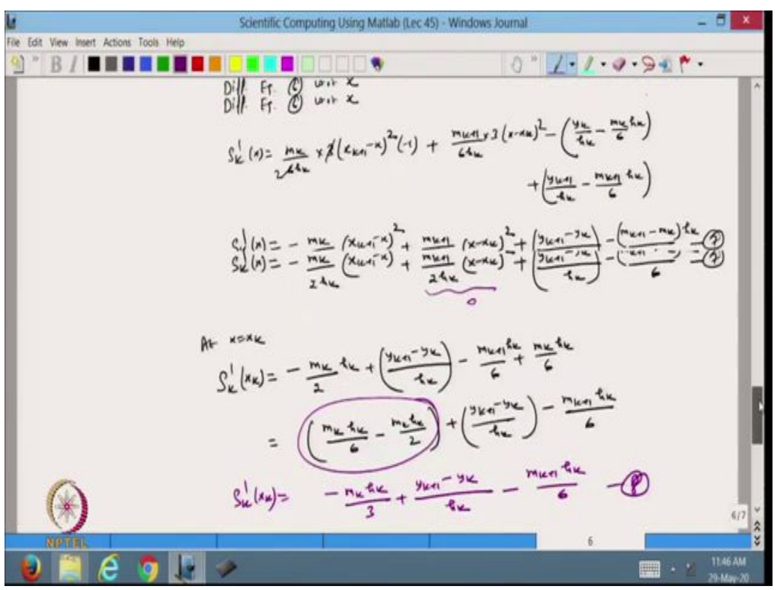
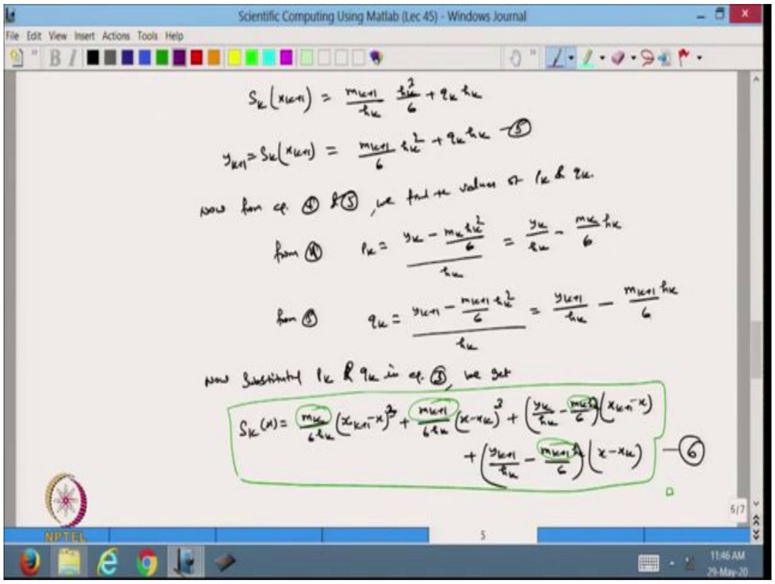
- 



Now, from equation 6, differentiate equation 6 with respect to x, so I will get

$$S_k'(x) = \frac{m_k}{6h_k}3(x_{k+1} - x)^2(-1) + \frac{m_{k+1}}{6h_k}3(x - x_k)^2 - \left(\frac{y_k}{h_k} - \frac{m_k h_k}{6}\right) + \left(\frac{y_{k+1}}{h_k} - \frac{m_{k+1} h_k}{6}\right)$$

$$S_k'(x) = -\frac{m_k}{2h_k}(x_{k+1} - x)^2 + \frac{m_{k+1}}{2h_k}(x - x_k)^2 + \frac{y_{k+1} - y_k}{h_k} - \frac{(m_{k+1} - m_k)h_k}{6} \cdots (7)$$

So, this is a function of x now, I find out the value at x_k so, what about at $x = x_k$? So,

$$S_k'(x) = -\frac{m_k h_k}{2} + 0 + \left(\frac{y_{k+1} - y_k}{h_k}\right) - \frac{(m_{k+1} - m_k)h_k}{6} = \left(\frac{m_k h_k}{6} - \frac{m_k h_k}{2}\right) + \left(\frac{y_{k+1} - y_k}{h_k}\right) - \frac{m_{k+1} h_k}{6}$$

so,

$$S_k'(x_k) = -\frac{m_k h_k}{3} + \frac{y_{k+1} - y_k}{h_k} - \frac{m_{k+1} h_k}{6} \cdots (8).$$

Now we rewrite equation (7) for (k-1) subinterval

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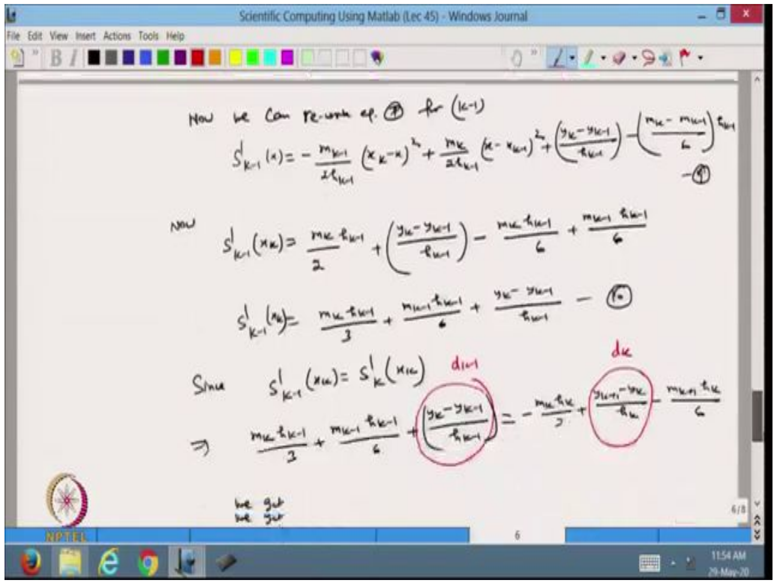

$$S'_{k-1}(x) = -\frac{m_{k-1}}{h_{k-1}}(x_k - x)^2 + \frac{m_k}{2h_{k-1}}(x - x_{k-1})^2 + \frac{y_k - y_{k-1}}{h_{k-1}} - \frac{(m_k - m_{k-1})h_{k-1}}{6} \cdots (9)$$

Now,

$$S'_{k-1}(x_k) = \frac{m_k h_{k-1}}{3} + \frac{m_{k-1} h_{k-1}}{6} + \frac{y_k - y_{k-1}}{h_{k-1}} \cdots (10)$$

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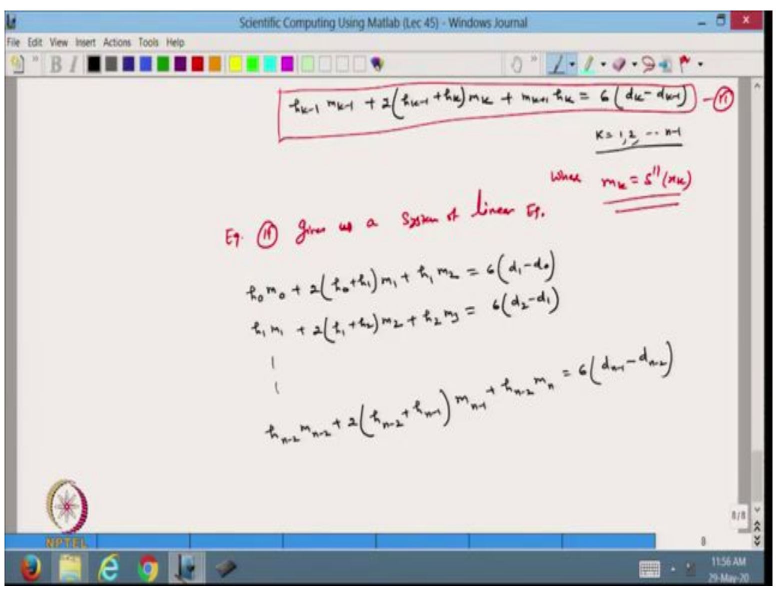

Since $S'_{k-1}(x_k) = S'_k(x_k)$ implies

$$\frac{m_k h_{k-1}}{3} + \frac{m_{k-1} h_{k-1}}{6} + \frac{y_k - y_{k-1}}{h_{k-1}} = -\frac{m_k h_k}{3} - \frac{m_{k+1} h_k}{6} + \frac{y_{k+1} - y_k}{h_k}$$

Since $d_{k-1} = \frac{y_k - y_{k-1}}{h_{k-1}}, d_k = \frac{y_{k+1} - y_k}{h_k}$ We get

$$h_{k-1} m_{k-1} + 2(h_{k-1} + h_k) m_k + m_{k+1} h_k = 6(d_k - d_{k-1}), \cdots (11)$$

Where $m_k = S''(x_k)$ and $k = 1, 2, \cdots, n-1.$

Now from here we will see that equation 11, gives us a system of linear equations how? So, let us start with k = 1.

So, if I take k = 1 it will be $h_0 m_0 + 2(h_0 + h_1) m_1 + m_2 h_1 = 6(d_1 - d_0)$ If I take k = 2 , I will get

$$h_1 m_1 + 2(h_1 + h_2) m_2 + m_3 h_2 = 6(d_2 - d_1)$$

$$\vdots \qquad\qquad \vdots \qquad\qquad\qquad \vdots$$

$$h_{n-2} m_{n-2} + 2(h_{n-2} + h_{n-1}) m_{n-1} + m_n h_{n-1} = 6(d_{n-1} - d_{n-2})$$

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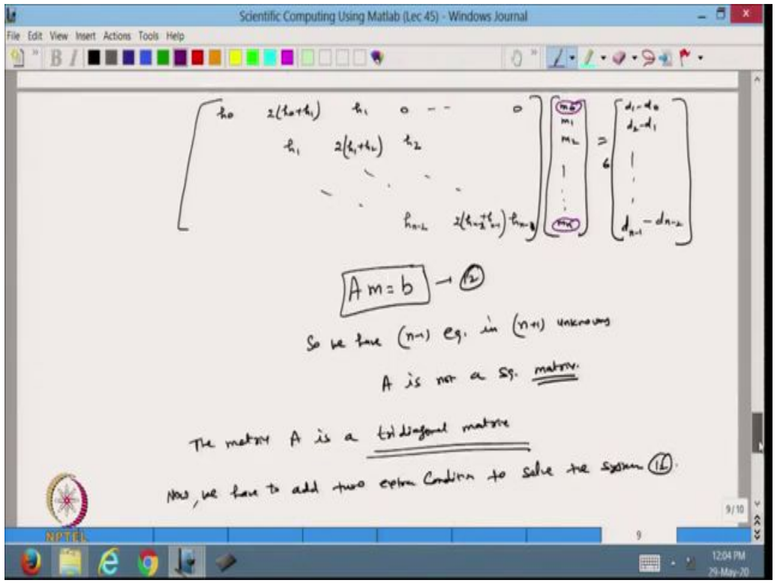

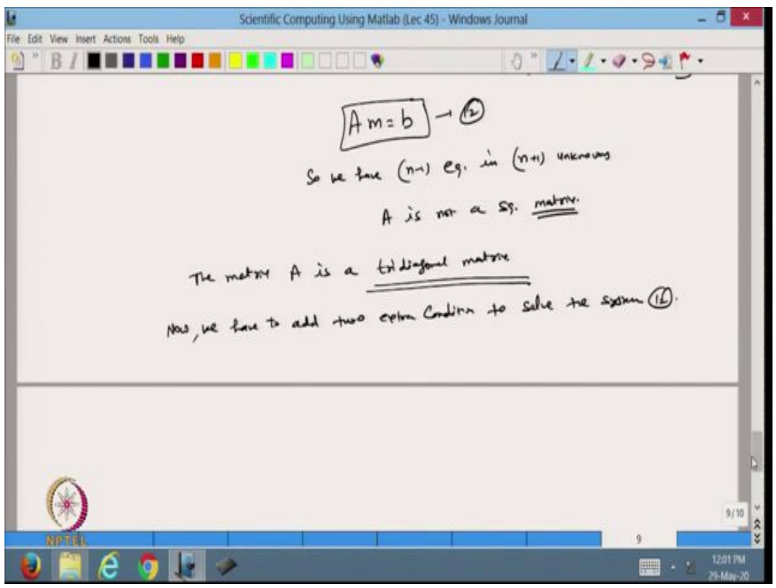

If I will write this expression in matrix form I will set

$$\begin{pmatrix} h_0 & 2(h_0+h_1) & h_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & h_1 & 2(h_1+h_2) & h_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & h_{n-2} & 2(h_{n-2}+h_{n-1}) & h_{n-1} \end{pmatrix} \begin{pmatrix} m_0 \\ m_1 \\ m_2 \\ \vdots \\ m_n \end{pmatrix} = 6 \begin{pmatrix} d_1 - d_0 \\ d_2 - d_1 \\ \vdots \\ d_{n-1} - d_{n-2} \end{pmatrix}$$

So, now from here, if you see we will get this expression so we will get so, suppose I call it this matrix Am = b ....(12) now, this expression if you see now, so we have an n-1 equation in n+1 unknown. Because, I need to find this $m_0, m_1, \cdots, m_n$ So, these n+1 unknowns are there and the total number of equations depends upon how many conditions we have, so that is n-1.

So, we have an n-1 equation in n+1 unknowns. So, this is a rectangle so, from here we can say that A is not a square matrix. So, how I will find this equation so, that depends upon the extra two conditions we have to add so, from here I will now, to solve I call it now, the matrix A is a tri diagonal matrix. This is a tri diagonal matrix basically, now we have to add two extra conditions to solve the system 12 so, we have to add two extra conditions so, this is the way we can define so, this is called some extra conditions.

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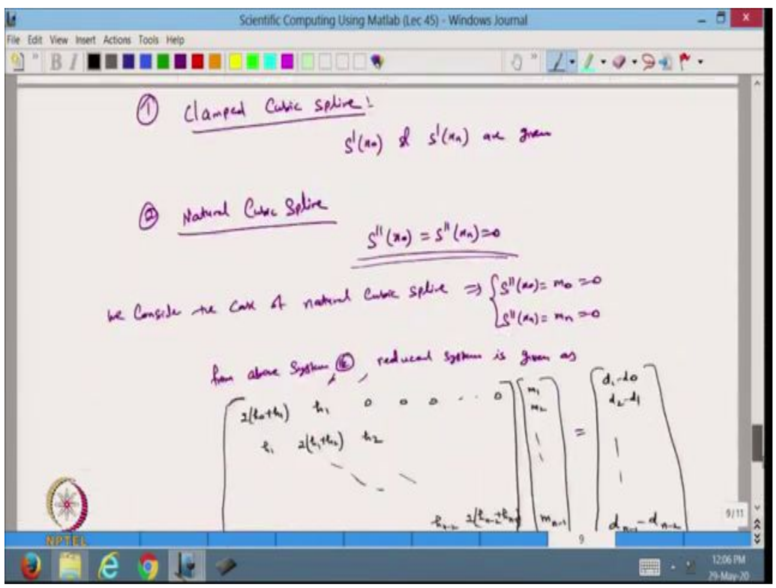

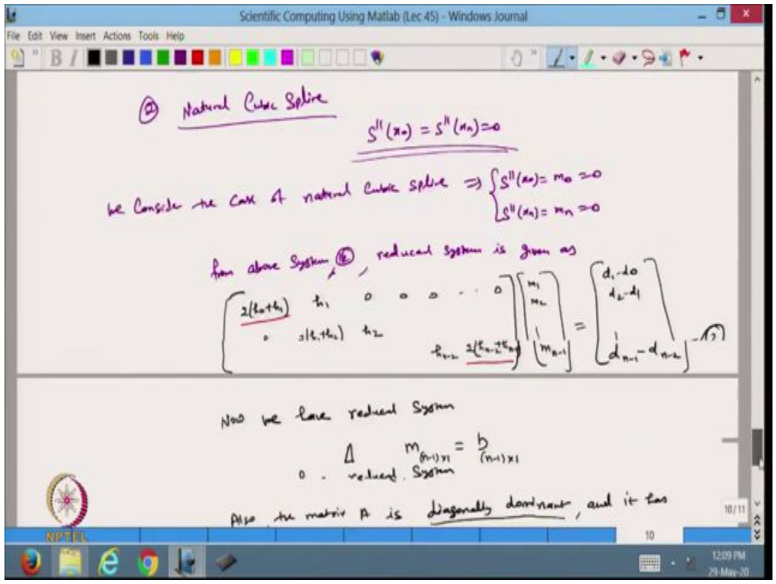

So, the case 1, we call it clamped cubic spline so, clamped cubic spline means we define that our $S'(x_0)$ The left boundary condition at the left boundary and $S'(x_n)$ are given so, these values are given to us that is called the clamped cubic spline because, the derivative is given the value the derivative at the extreme left and extreme right that is given to me.

The second one is that natural spline, natural cubic spline so, in the natural cubic spline, we have the condition that $S''(x_0) = S''(x_n) = 0$ so, this is given that the curvature, the value of the curvature at the extreme left and extreme right, is 0. So, that is called the natural cubic spline, so in this case depending upon the, what condition we will want to find? We can solve the system.

So, let us take this one so, we consider the case of natural cubic spline which implies that $S''(x_0) = m_0 = 0$ and $S''(x_n) = m_n = 0$ From the above system (12), the reduced system is given as

$$\begin{pmatrix} 2(h_0+h_1) & h_1 & 0 & \cdots & 0 & 0 \\ h_1 & 2(h_1+h_2) & h_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & h_{n-2} & 2(h_{n-2}+h_{n-1}) \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ \vdots \\ m_{n-1} \end{pmatrix} = 6 \begin{pmatrix} d_1 - d_0 \\ d_2 - d_1 \\ \vdots \\ d_{n-1} - d_{n-2} \end{pmatrix} \cdots (13)$$

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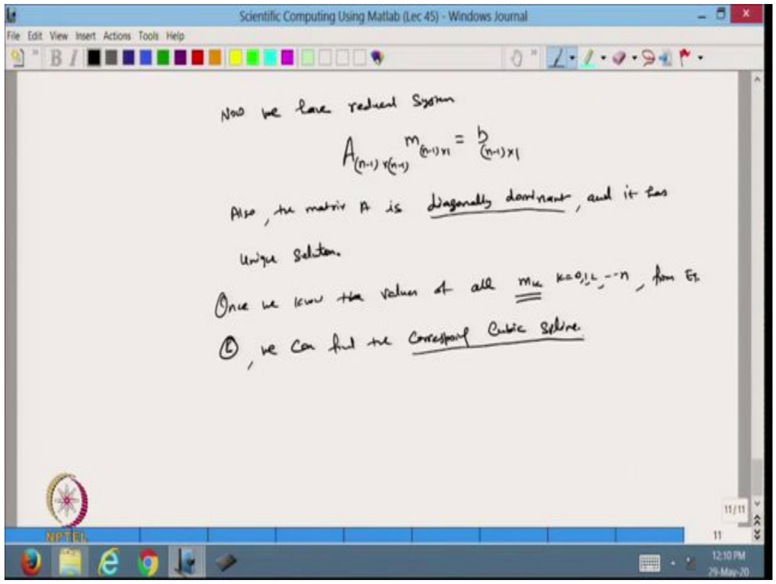

Now, we have reduced the system so, this is my $A_{(n-1)\times(n-1)} m_{(n-1)\times 1} = b_{(n-1)\times 1}$ so, this is now a system of linear equations and also the matrix A is diagonal dominant. Why is it diagonal dominant? If you see from here that this is some of the previous one in the next one, it is definitely great, this value is greater than h_1. This is greater than h_1 and h_2 so, I am taking the sum and 2 times of this one.

The same thing we are doing here is that 2 times of this one, so this matrix is a diagonal dominant matrix and it has a unique solution. So, I can solve this system this is known to me I will take, I can apply my Gauss seidel or Gauss jacobi iterative methods to find out this one, because this matrix is the diagonal dominant matrix so, I know that for the iterative methods the diagonal dominant are the sufficient condition so, if I have apply the Gauss seidel or Gauss jacobi methods to find out the solution then definitely we are going to get the solution and the solution will be unique.

So, based on this one we are able to find the cubic spline, so that is the solution for the cubic spline once I get the value of this $m_0, m_1, \cdots, m_n$, I will substitute in the given equation here. So, once I get the value of all m_i I will put the values here and then from here I will get the cubic spline.

So, once we know the values of all $m_k, k = 0, 1, 2 \cdots, n$ from equation 6, we can find the corresponding cubic spline. So, from the expression 6 I just substitute the values and I will get the, h_k is known to us only the thing that we need to find is m_k so, that is known to us now, so I will substitute the value and get these cubic splines to function. So, this is the, we will get the corresponding cubic spline function.

So, this is all about that how we can express the cubic spline so, we will stop today so, today we have started with the expression of a how to write the expression for the cubic spline and the we have discussed that we get the matrix and then how we can introduce the two extra conditions to solve that matrix to find out the cubic spline to the given data? So, I hope you have enjoyed this lecture so, thanks for watching this, Thanks Very Much.