Scientific Computing using Matlab
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Lecture 43
Examples Based on Lagrange's and Newton's Divided Difference Interpolation

Hello viewers, welcome back to the course on scientific computing using MATLAB. So, in the previous lectures we have discussed how we can write the Lagrange interpolation polynomial or Newton divided difference polynomials. So, today we will discuss some examples based on this one. So, let us start with lecture 43, so let us solve some examples. Let us take 1 example.

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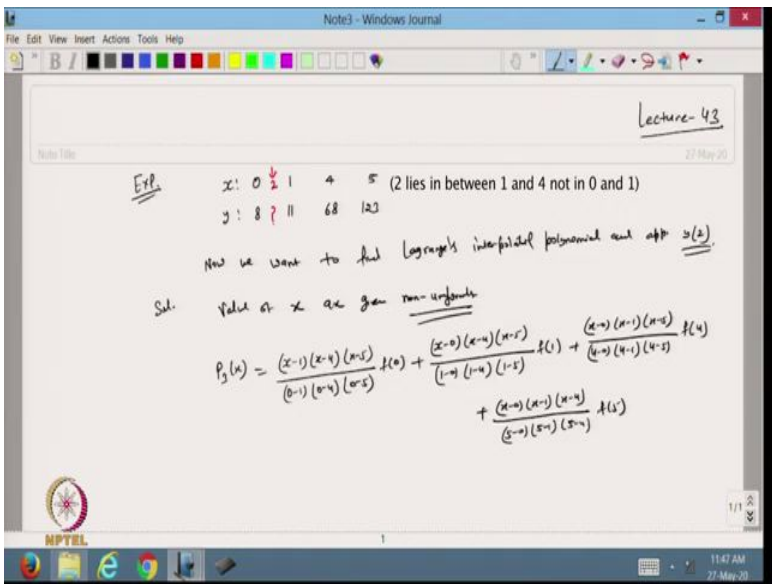

I have some data that is given to me. So, my x coordinates are given to me like 0 1 4 and 5 and the value of the function for the y is given to me that is 8 11 68 and 123. So, this is the value that is given to me. Now, we want to find a Lagrange interpolating polynomial and approximate y at 2, so this one I want approximate.

So, 2 lie here so at this value, I want to find what is the value of this. Now, in this case I want to define the, I want to use the Lagrange interpolating polynomial. So, you can see that from here the nodal points are not uniform. So, the values of x are given non uniformly that the difference between 1 and 0 is 1, then the difference is 3 and then difference is 1. So, this is a non equally spaced data that is given to me.

Now, we have 4 points of data. So, in this case, my polynomial will be cubic, because 4 values of the data that is given to me so I can write from here the $P_3(x)$. Now, I can define this should be equal to

$$P_3(x) = \frac{(x-1)(x-4)(x-5)}{(0-1)(0-4)(0-5)} f(0) + \frac{(x-0)(x-4)(x-5)}{(1-0)(1-4)(1-5)} f(1) + \frac{(x-0)(x-1)(x-5)}{(4-0)(4-1)(4-5)} f(4)$$

$$+ \frac{(x-0)(x-1)(x-4)}{(5-0)(5-1)(5-4)} f(5).$$

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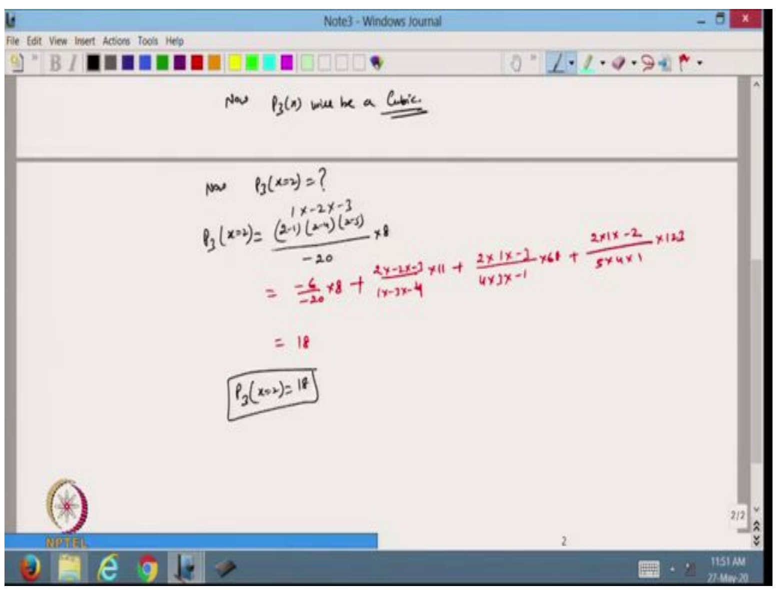

So, now if I calculate this value, then from here I can see that this is a cubic I am going to get and this is the value I am going to find out. Now, from here, if I do the calculation so now $P_3(x)$ will be a cubic. And then we have to do the calculation to find out the expression. Now in this case, if somebody asks me what the expression is, then we will calculate all this value and then we can write this in the form of an expression that is the cubic.

Now, we want to find what is the value of $P_3(2)$, so this one we want to find. So, I will directly put this one here. So, $P_3(2)$ will be what? So now I can write directly from here. So,

$$P_3(2) = -\frac{48}{20} + 11 + \frac{408}{12} - \frac{492}{20} = -2.4 + 11 + 34 - 24.6 = 18.$$

So, if I do all this calculation, the value is coming to 18.

So, from here I can see that the value, the interpolating value of $P_3(2) = 18$. So, here the value is coming 18, so that is the answer to this question. So, this way we can find the value of the interpolating polynomial at any points given in between the data. So, data is given from 0 to 5 and the value 2 is in between that one so that is why it is called the interpolating polynomial, interpolating equation.

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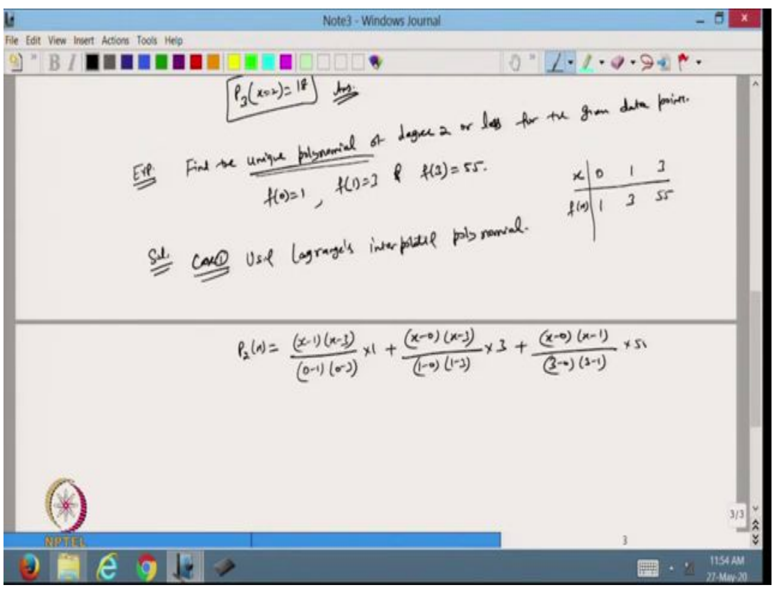

Another example I want to do, find the unique polynomial of degree 2 or less for the given data points so it is given $f(0) = 1, f(1) = 3$ and $f(3) = 55$, so this value is given to me, now let us do this one. So, in this case I want to find the unique polynomial. So, let us see, we already know what polynomial we are getting using the Lagrange or Newton divided difference. So, they are the unique polynomial so let us verify this one for this data.

So, let us solve this one, the case 1 I am taking using Lagrange. So, I have 3 points that are given to me. So, in this case, it is the quadratic so points are given to me like this one now, I can either point x and f at x, so it is 0, 1 and 3, so it is not, the spacing is not equal.

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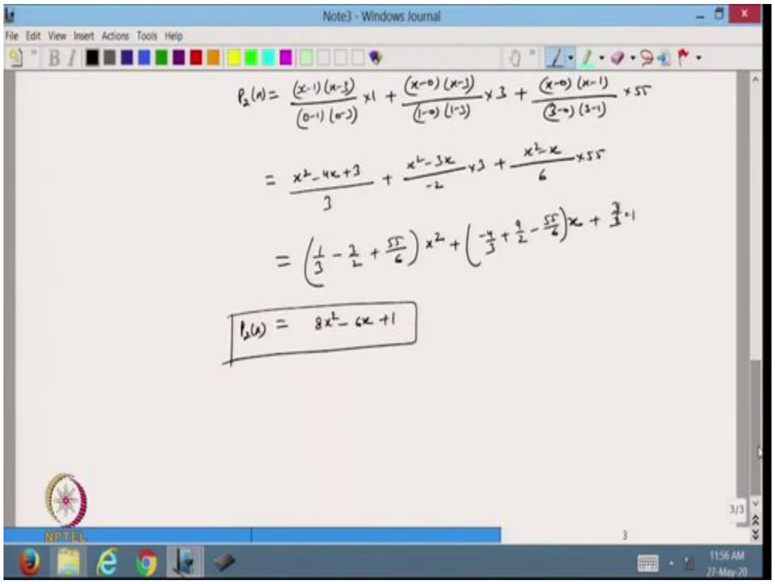

Here the spacing is 1, here it is 2, the value is given is 1, 3, and 55. So, 0 1 3 and 1 3 55 so that is given to me. Now, I will write from here expression so it is

$$P_2(x) = \frac{(x-1)(x-3)}{(0-1)(0-3)}1 + \frac{(x-0)(x-3)}{(1-0)(1-3)}3 + \frac{(x-0)(x-1)}{(3-0)(3-1)}55$$

$$= \frac{x^2-4x+3}{3} + \frac{x^2-3x}{-2}3 + \frac{x^2-x}{6}55$$

$$= \left(\frac{1}{3} - \frac{3}{2} + \frac{55}{6}\right)x^2 + \left(-\frac{4}{3} + \frac{9}{2} - \frac{55}{6}\right)x + 1 = 8x^2 - 6x + 1.$$

So, that is my $P_2(x)$. So, this is the quadratic polynomial we are able to find using these three data points. Now, this is what we have done with the help of Lagrange. So, let us do this one, case 2.

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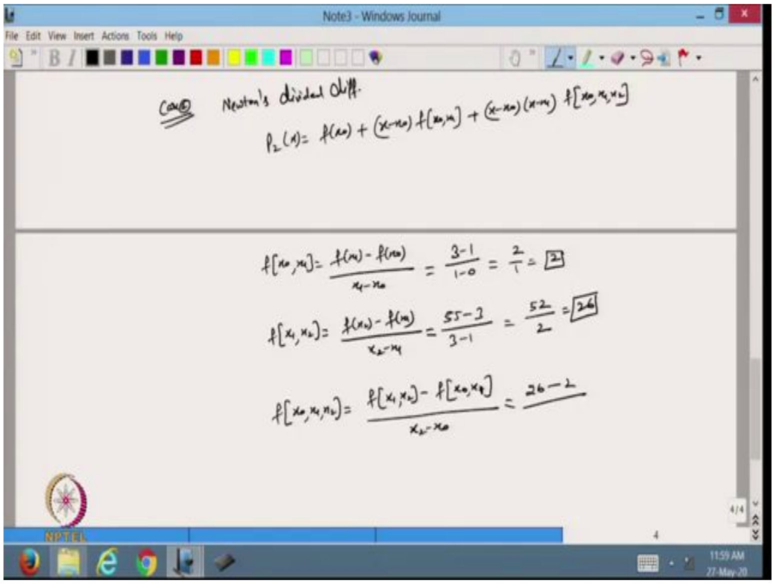

So, in case 2, I will use Newton's divided difference. So, Newton's divided difference now will be

$$P_2(x) = f(x_0) + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2].$$

Now i want to find first divide difference,

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{3-1}{1-0} = 2, f[x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{55-3}{3-1} = 26.$$

Now find second divided difference

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{26-2}{3-0} = 8.$$

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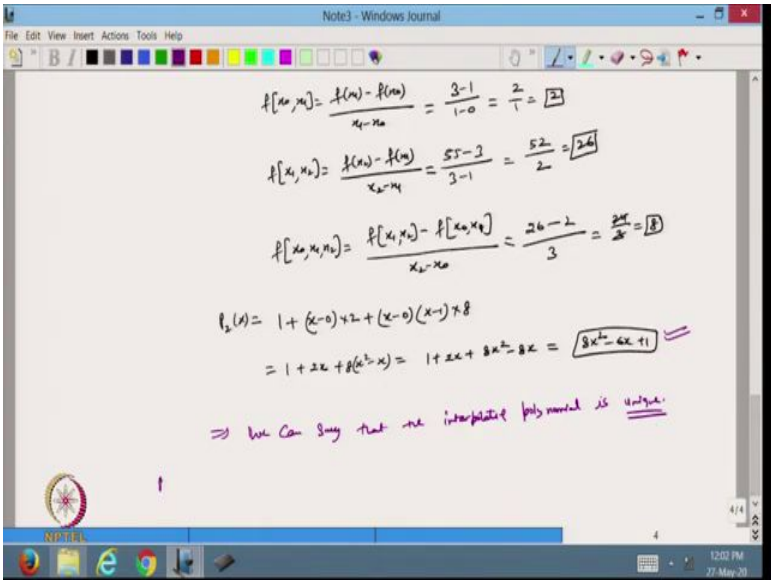

Now from here my $P_2(x)$ become

$P_2(x) = 1 + (x-0)2 + (x-0)(x-1)8 = 1 + 2x + (x^2 - x)8 = 1 + 2x + 8x^2 - 8x = 8x^2 - 6x + 1.$

And if you see from here, this is the same expression as we have found using the Lagrange interpolating polynomial. So, from here we can say that the interpolating polynomial is unique.

The only thing is that, the interpolating polynomial we are calculating using Lagrange interpolation and using the Newton's divided difference so, writing the interpolating polynomial using the Newton divided difference is much easier as compared to the Lagrange interpolating polynomial.

So, that is the way we can say, but we have already said that the interpolating polynomial will be unique. So, it has only 2 expressions in this form; either it will be of the type Lagrange or the other type it will be Newton divided difference, but this will be unique. So, that is the verification for what we have done in the previous lecture, the theorem about the interpolating polynomial for the non equi space data will be unique so, this is the verification for that.

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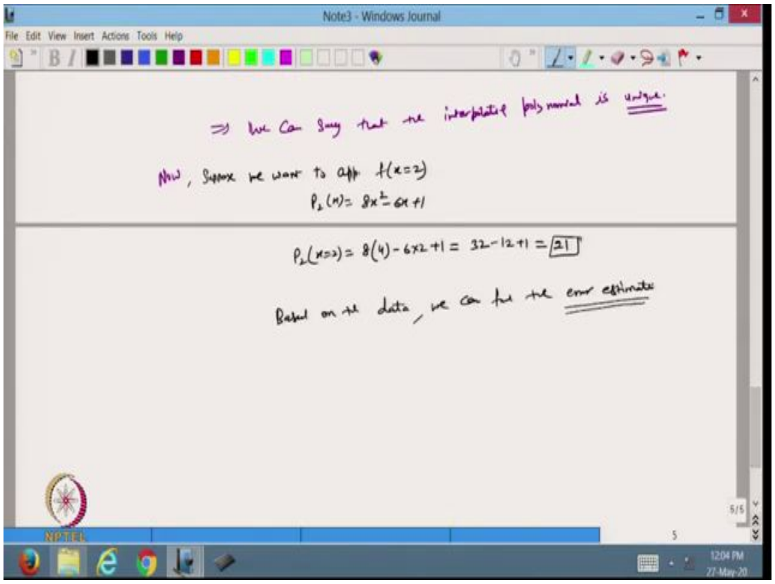

Now, suppose we want to approximate the value of the function at x = 2, so let us take x =2. So, in this case, they no need to rewrite again, I will do that. I know that my $P_2(x) = 8x^2 - 6x + 1$. So, in this case, we will just put the value at x = 2 here so it will be $P_2(2) = 32 - 12 + 1 = 21$.

So, the corresponding value at 2 will be 21 so this will be here somewhere. So, in this case, I can say that this function is an increasing function. Now, based on this one, so, from here I can find this one. So, based on the data we can find the maximum error estimate, but only thing is that I need to know that this is the values presented by some function.

So, if I know the value of the function in this case, then I should be able to get the error of how much error is there. So, $f(0) = 1, f(1) = 3, f(3) = 55$. So, there if the function is known to us, then only we are able to find the error estimate of this one. So, this is all about this type of data. Now, the question is that so, we know that in the Lagrange interpolating polynomial there were 2 types of drawbacks.

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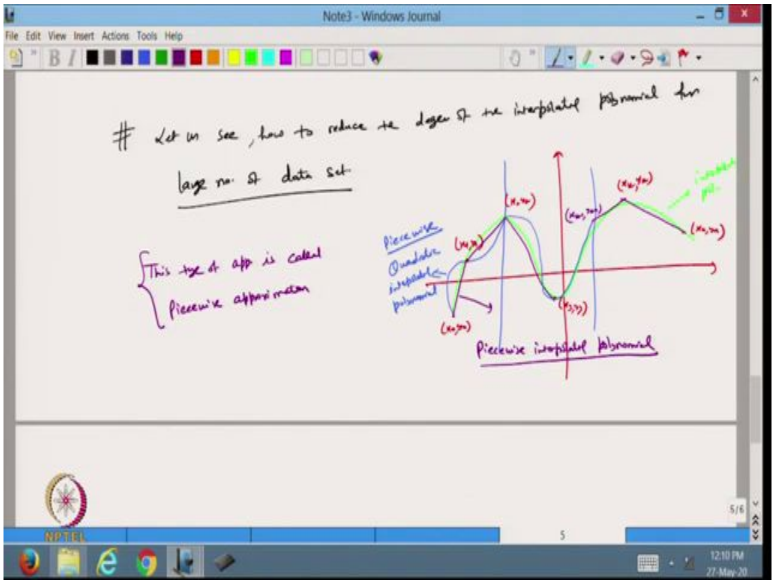

One drawback was that, so in the Lagrange interpolating polynomial there was a two drawbacks; one drawback was that we are going to get the higher degree polynomial for a bigger data and the second one was that we had 1 data point then we have to rewrite the equation again. So, the second drawback has been addressed by this Newton divided difference formula. Now, we will go for the first drawback, so let us see how to reduce the degree of the interpolating polynomial for a large number of data sets, so let us address this issue now.

So, in this case, what do we do? So, let us take this one. So, suppose I have some data, this is suppose I have some data, this is some data value $(x_0, y_0), (x_1, y_1), \cdots, (x_k, y_k), \cdots, (x_n, y_n)$ Now, suppose I want to, so if I find out the interpolating polynomial, the interpolating polynomial will be like this one, so, this is my interpolating polynomial.

Now, but I do not want to apply the interpolating polynomial because the degree will be high. So, what I do in this case, I will do the piecewise approximation, so, in the piecewise approximation, what I do? Suppose, I want to approximate these with the linear interpolation, so what I do is that I approximate this by this linear function, then this by this linear function, this by this linear function and this by this linear function and this is suppose the previous one is this one, so this by this linear function if I take this as x k minus 1, y k minus 1 and so on.

So, this type of interpolation is called piecewise interpolation, interpolating polynomials. So, in this case we have the data and this data we split into the pieces, and then we approximate the pieces based on which one we want. So, here suppose we have 2 points, then we will approximate with the linear data. Now, what I do is that instead of taking the 1, 1 point, suppose I split the data into 3 points.

So, suppose I split the data into 3 points 3 point this 1, 2, 3, another 3 point or these like this one. So, what I do now I approximate this with a quadratic function like this one. I approximate these 3 points with a quadratic function and so on. And so on this is I call it quadratic interpolating polynomial.

So, earlier I have done with the piecewise interpolating polynomial that is linear function, and now I am doing the same with the quadratic interpolating polynomial, but this is also piecewise. So, in this way we are able to approximate the data with a small degree of polynomial, it does not matter how big data is this one, so this type of method is called piecewise approximation. So, in this case we will discuss, we will define the piecewise approximation and for that one in our course will go for Splines.

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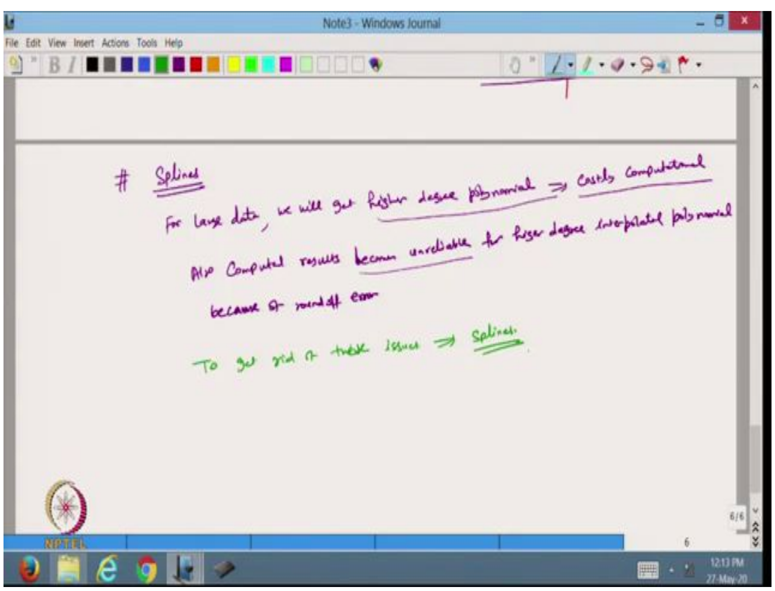

So, this is basically how we are heading towards the Splines and how we can discuss that Splines. So, in this case I can write down that for large data we will get higher degree polynomials which implies costly computational. So, for this one also, computed results become unreliable for higher degree polynomials because of round effect. So, for the higher degree polynomial, the two things are there, it is computationally very costly. And also the result becomes unreliable because of the round off error.

So, to get rid of this one, these issues we apply for Splines, so that we will discuss in the coming lecture. So, stop today, so today we have discussed the examples based on the Lagrangian interpolating polynomial and the Newton divided difference interpolating polynomial.

And we have seen that whatever the method we are going to apply, we are going to get the unique interpolating polynomial, and then for the larger number of data we are going to get the higher order polynomial. So, to get rid of this one we are going to start with the piecewise approximation and that is we are going to discuss in the coming lectures that is called the spline. So, I hope you have enjoyed this one. So, thanks for watching this, thanks very much.