Scientific Computing Using Matlab
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Lecture No. 42
Interpolating Polynomial Using Newton's Divided Difference Formula

Hello viewers, welcome back to the course on scientific computing using Matlab. So, in the previous lecture we have started with the Lagrange's interpolating polynomial and then we have discussed the drawback of that polynomial that if we have to add one extra point, then we have to repeat the whole process to compute that Lagrange's interpolating polynomial. So, to get rid of this one, we start with another polynomial and that is called a Newton's divided difference formula.

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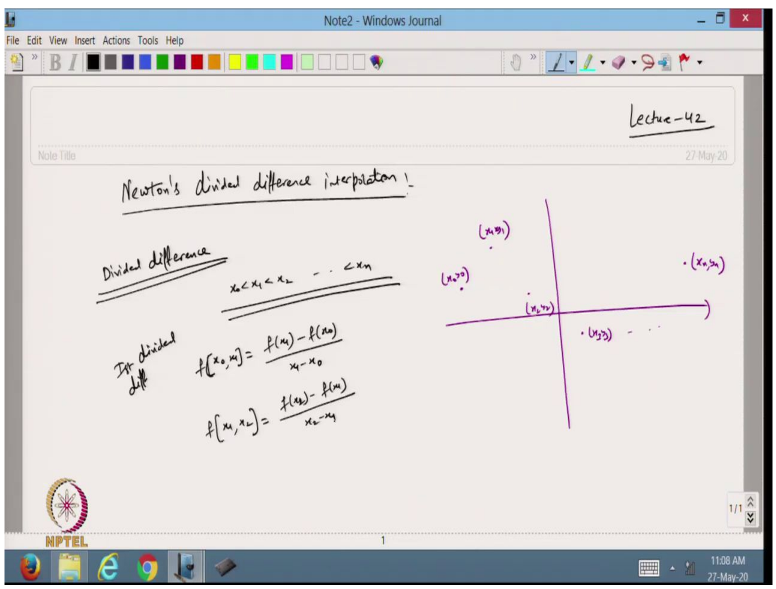

So, today we will discuss how we can define Newton's divided difference formula or interpolation. So, in this case again I have the points. So, I have some points that is $(x_0, y_0), (x_1, y_1), \cdots, (x_n, y_n)$ so, these points are given to me. Now, I define what is the meaning by the divided difference table. So, first of all I will define divided differences.

So, in this case suppose I, so these points are given to me and let I rearrange these points and I write the points in the form of $x_0 < x_1 < \cdots < x_n$ so this is the ascending order I am taking. Now, I define the first divided difference. So, first of all divided difference involving x_0, x_1 I represent by $f[x_0, x_1]$. So, this is given by $f[x_0, x_1] = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$ Similarly, I can define $f[x_1, x_2] = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$.

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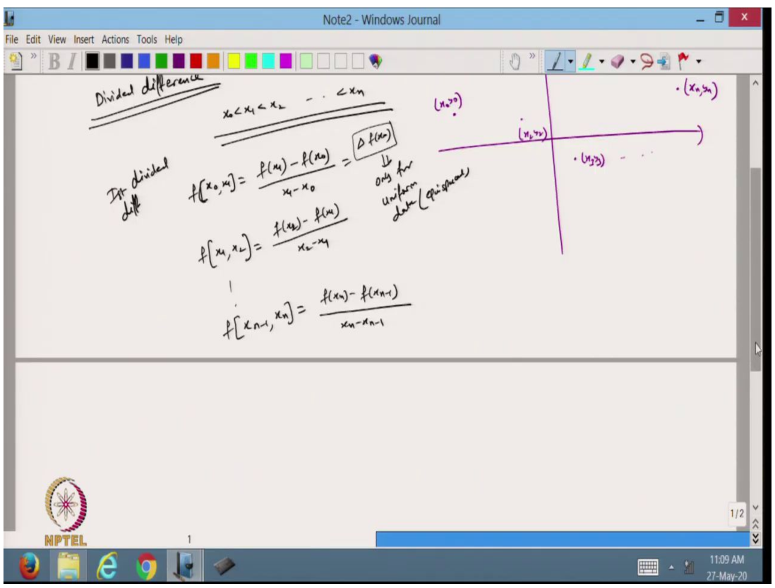

So, the same way we can define all the 1st order So, $f[x_{n-1}, x_n] = \dfrac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$ And if you remember from the previous one that, $f(x_1) - f(x_0) = \Delta f(x_0)$, the only condition was that in that case we know that, so this h was uniform in that case. So, that is why $x_1 - x_0 = h$. So, we are not divided by h there.

So, it is similar to that one, only thing is that, that this we have defined for uniform spaced nodal points and this is for any, it may be a uniform and non uniform. So, this is only for uniform data that is equi-spaced. So, this is the first divided difference we have discussed.

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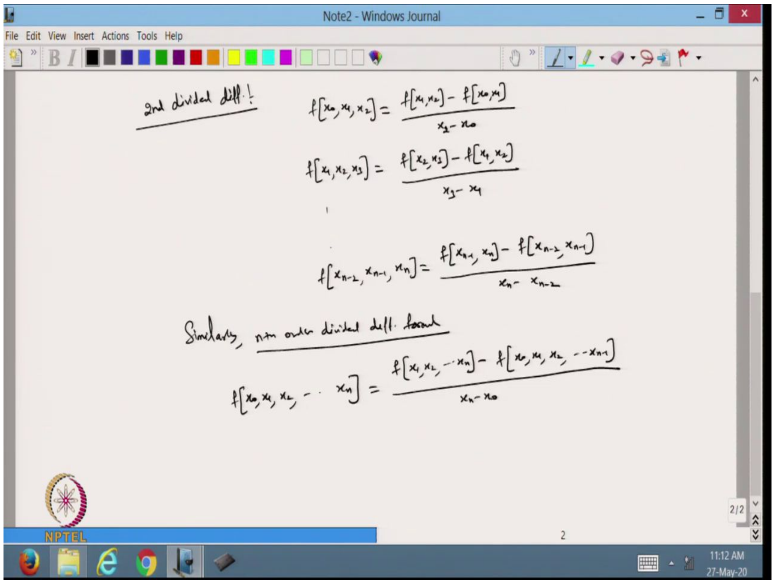

Similarly, I can define the second divided difference. So, in the second divided difference, suppose I have 3 points, so points are given to me, so suppose I will write $f[x_0, x_1, x_2]$. So this will be can be written as $f[x_0, x_1, x_2] = \dfrac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$. Similarly, I can define my $f[x_1, x_2, x_3]$, so this is equal to $f[x_1, x_2, x_3] = \dfrac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1}$.

So, in this way I can define the, all the second order divided difference formula. So, in the end, I will get $f[x_{n-2}, x_{n-1}, x_n]$. So, that can be written as $f[x_{n-2}, x_{n-1}, x_n] = \dfrac{f[x_{n-1}, x_n] - f[x_{n-2}, x_{n-1}]}{x_n - x_{n-2}}$ So, this is the way we can define the second order, second divided difference formula.

So, similarly we can define the nth order divided difference formula. So in that case, it will be I am writing that $f[x_0, x_1, x_2, \cdots, x_n]$ So, that can be written as $f[x_0, x_1, x_2, \cdots, x_n] = \dfrac{f[x_1, x_2, \cdots, x_n] - f[x_0, x_1, \cdots, x_{n-1}]}{x_n - x_0}$

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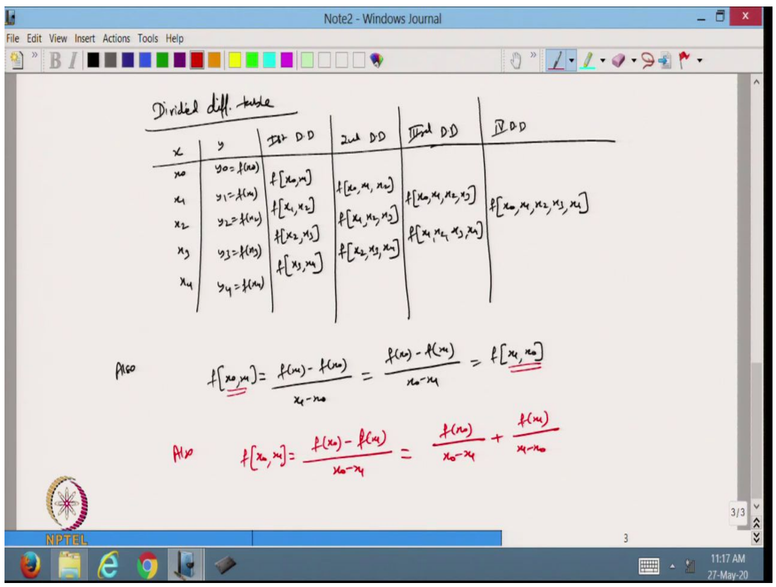

So, based on this one, I can define all these divided difference formulas. So, now let us, we can write the, so based on this one we can make the table for this one. So, divided difference table, so divided difference table we can write same as the finite difference table. So, I have the value of x and this is the value of y that is given to me, so x_0, x_1, x_2, x_3, x_4 suppose, these 5 values are given to me.

And this is y_0, y_1, y_2, y_3, y_4. So, the first divided difference DD, so that can be written as here. So, this is y1 minus, so I can write instead of formula, I can write the expression, so this will be, so this is y_0 is the value of f at x_0 that is $y_0 = f(x_0)$, this is $y_1 = f(x_1)$, this is a $y_2 = f(x_2)$, because we know that these data points are given to us and that is based on some function. So, the first divided differences are

$f[x_0, x_1], f[x_1, x_2], f[x_2, x_3], f[x_3, x_4]$.

The second divided difference is, so from here I can write this as $f[x_0, x_1, x_2], f[x_1, x_2, x_3], f[x_2, x_3, x_4]$ so I can write the third one. So, this is $f[x_0, x_1, x_2, x_3], f[x_1, x_2, x_3, x_4]$ and the last one is the fourth one, so that will be the only one value, the constant value. So, the fourth divided difference will be $f[x_0, x_1, x_2, x_3, x_4]$ using all this value together. So, this is the way we can write the divided difference table.

Also, if I write $f[x_0, x_1]$, so this is equal to $f[x_0, x_1] = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{f(x_0) - f(x_1)}{x_0 - x_1} = f[x_1, x_0]$ So, in this case I can say that the ordering of these coefficients or the values of x_0 and x_1, the indexing, so that does not matter. It does not matter if we write x_0 first and then x_1 or x_1 first, then x_0, the value of the divided difference will be the same.

So that is there, in this case, also. So, this way written also we can write from here that $f[x_0, x_1] = \dfrac{f(x_0) - f(x_1)}{x_0 - x_1} = \dfrac{f(x_0)}{x_0 - x_1} + \dfrac{f(x_1)}{x_1 - x_0}$.

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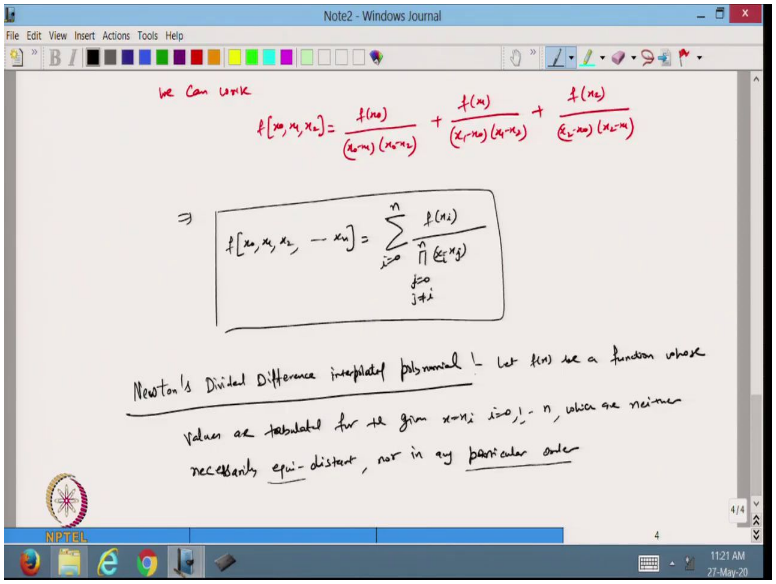

So now, from here I can write, we can write $f[x_0, x_1, x_2]$ directly from here, so this can be written as

$$f[x_0, x_1, x_2] = \frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2)} + \frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2)} + \frac{f(x_2)}{(x_2 - x_0)(x_2 - x_1)}$$

So, the third final difference can be directly written in this form.

And from here we can see that the interchanging of these arguments or the value of xi's does not matter, it will be the same. So, from here I can write the, the directly the nth order divided difference. So, this will be the

$$f[x_0, x_1, x_2, \cdots, x_n] = \sum_{i=0}^{n} \frac{f(x_i)}{\prod_{j=0, j\neq i}^{n}(x_i - x_j)}$$

so then, so this is the expression for the nth divided difference formula. Now, based on this divided difference, now we want to find out what is the interpolating polynomial in this case.

So, I want to define now Newton's divided difference interpolating polynomial, so this one we want to write. So, let the function f(x) be a function whose values are tabulated for the given

$x = x_i, i = 0, 1, 2, \cdots, n.$, which are not necessarily equidistant. So which are neither, should I write here, neither necessarily equidistant nor in any order. So, this is not necessary in any particular order, so nor, no ordering is given to me and they are not also equidistant.

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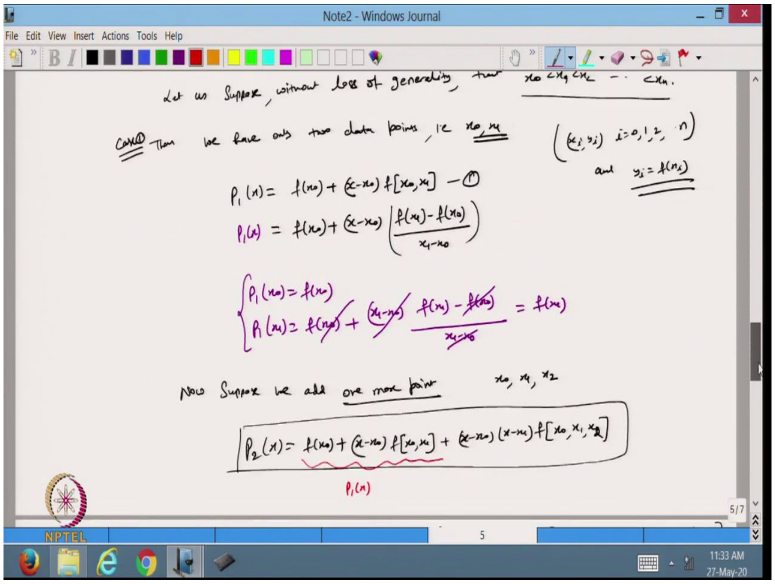

So, in that case what we do, so we put them in the order. Now, let us suppose, without loss of generality that the ordering is this one $x_0 < x_1 < \cdots < x_n$ so, we put that in the ordering, the ascending ordering then suppose I, suppose we have only two points, suppose we took case 1, we have only two data points that is x_0 and x_1, So, I have only two data points. So, in that case I will write my interpolating polynomial that is $P_1(x)$.

So in the $P_1(x)$ What do we do? We write, so this value is given to me. So, I will write

$$P_1(x) = f(x_0) + (x - x_0) f[x_0, x_1] \cdots (1)$$

Now let us check this. So, now from here, I know that if I passing through two points it should be a linear, so it is linear interpolating polynomial and if I put $x = x_0$,

$$P_1(x_0) = f(x_0)$$

$P_1(x)$ at x_1, so this is

$P_1(x_1) = f(x_0) + (x_1 - x_0)\dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = f(x_1)$ So, based on this one $P_1(x_0) = f(x_0)$, $P_1(x_1) = f(x_1)$. So, it is the interpolating passing through these two points, but this is a linear interpolating polynomial $P_1(x)$.

Now, what I do now, suppose we add one more point. So now, suppose I have x_0, x_1 and I have added another point that is x_2, so 3 points are there. So, it should be a quadratic, then I can write my $P_2(x)$ taking the help of $P_1(x)$, so from here I can write this as $P_2(x) = f(x_0) + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2]$ it will be my quadratic polynomial.

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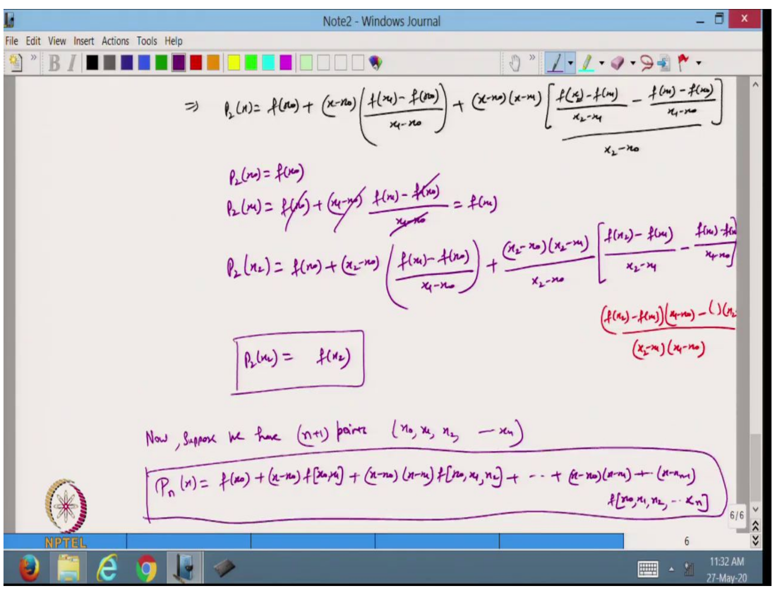

So from here, you can verify that my $P_2(x)$ will be

$$P_2(x) = f(x_0) + (x - x_1)\left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right) + (x - x_0)(x - x_1)\frac{\left[\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}\right]}{x_2 - x_0}$$

Now from here, I can write. Now, if I put $x = x_0$, I will get $P_2(x_0) = f(x_0)$ Now,

$P_2(x_1)$, so this will be $f(x_1)$.

Similarly, I can verify $P_2(x_2) = f(x_2)$.

So, based on this one, now suppose, so now I want to do the generalization. So, suppose, we have n+1 points that is $x_0, x_1, \cdots, x_n$.

So now, from here I can write directly my nth degree Newton's interpolating polynomial, so that will be equal to

$$P_n(x) = f(x_0) + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2] + \cdots + (x - x_0)(x - x_1)\cdots(x - x_{n-1})f[x_0, x_1, \cdots, x_n]$$

So, that is the nth degree interpolating polynomial using the divided difference formula.

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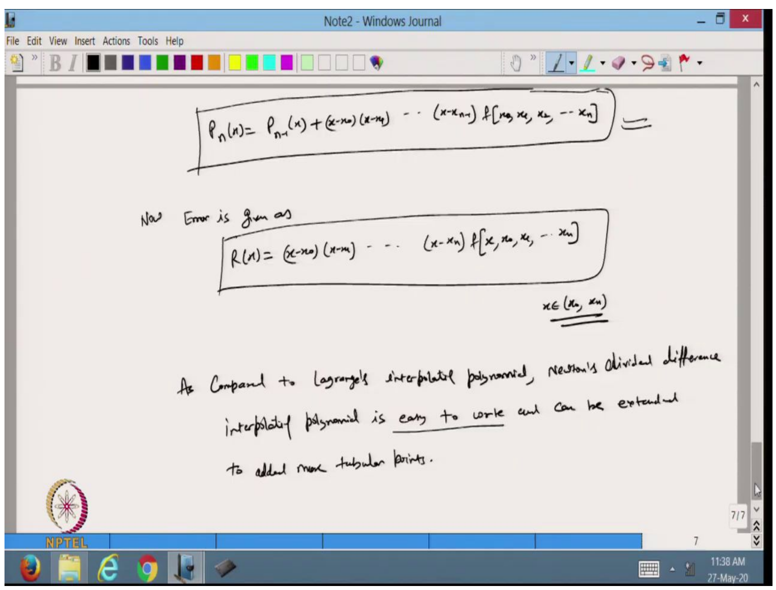

So from here, also we can see that my $P_n(x)$ can be written as

$$P_n(x) = P_{n-1}(x) + (x - x_0)(x - x_1)\cdots(x - x_{n-1})f[x_0, x_1, \cdots, x_n].$$

So, now from here, I can say that in the Newton's divided difference formula, whatever the, the interpolating polynomial is exist at the given number of nodal points, if we add one more nodal points, then they no need to rewrite the whole expression again, just add one more terms in the previous written interpolating polynomial. So, this is the way we can define the nth degree interpolating polynomial using Newton's divided difference formula.

Now, so in this case the error is given as, as we are discussed in the previous lecture also. So, R(x) is given as $R(x) = (x - x_0)(x - x_1) \cdots (x - x_n) f[x, x_0, x_1, \cdots, x_n]$, $x \in (x_0, x_1)$.

So, that is my error. So, in this case, I choose any x in between and then I write the divided difference multiplied by this factor, so that will be the error, it is given, error we can find when we apply Newton's divided difference formula. So, in this case, I can write as compared to Lagrange's interpolating polynomial, Newton's divided difference interpolating polynomial is easy to write and can be extended to and can be extended to add more tabular points.

So, as compared to the Lagrange's interpolating polynomial, Newton's divided difference interpolating polynomial is easy to work, easy to write because we can see that we can write very easily and it can be extended to the added more tabular points, because in this case we are using the the interpolating polynomial which is given to us for the less number of tabular point as we have already discussed. So, this is all about how we can define the Newton's divided difference formula, so let me stop today.

So, today we have discussed the drawback of the Lagrange's interpolating polynomial, and then we started with the divided difference formula given by Newton and then we have discussed how we can write Newton's divided difference interpolating polynomial. So, in the next class we will continue with this one. So, thanks for watching this. Thanks very much.