Scientific Computing Using Matlab.
Professor. Vivek Aggarwal & Professor. Mani Mehra
Department of Mathematics
Indian Institute of Technology, Delhi
Lecture No. 39
Interpolating Polynomial Using Newton's Backward Difference Formula

Hello viewers, welcome back to the course on Scientific Computing Using Matlab.

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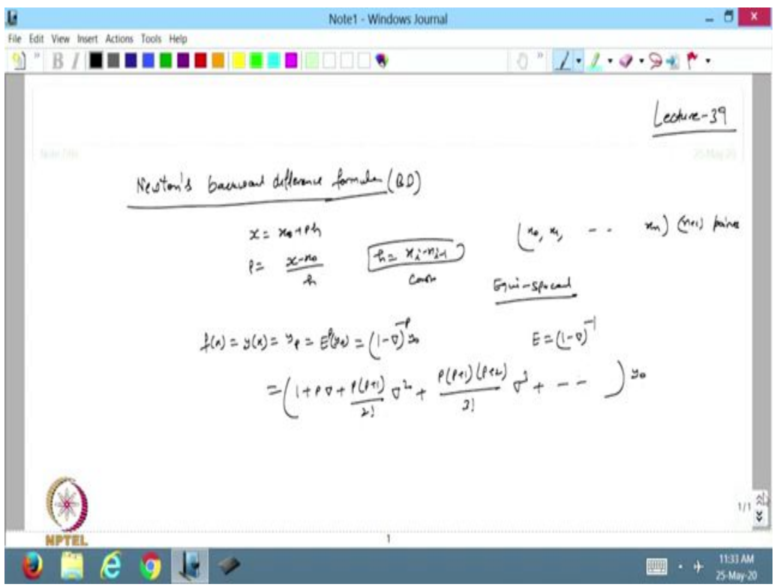

So, in the previous lecture we have discussed the error estimate involved in the interpolating polynomial. So, today we will continue with this one with that, and today we will start with Newton's backward difference method. So, today I will discuss Newton's backward formula, backward difference formula or I should call it that is BD backward difference. So, in this case the same as we have done the Newton forward, so I have my x that is $x_0 + ph$.

I have the points this is $x_0, x_1, \cdots, x_n$. So, these n+1 points are given to me and I want to choose any x that is lying in between somewhere. So, I choose $x = x_0 + ph$, where my p can be written as $\dfrac{x - x_0}{h}$ and where h I know that is $x_i - x_{i-1}$ and this is constant. So, this is all equi-spaced. So, in this case I want to find the value of the function at x or I can want to find what is the value of y(x). So, this one I can represent as y_p, because x is involved with the parameter p.

So, this can be written as $y_p = E^p(y_0) = (1 - \nabla)^{-p} y_0$, because we know that the relation between the shift operator E and the backward operator which is $E = (1 - \nabla)^{-1}$, so I use this one. Now, I apply the binomial expansion, so,

$$y_p = (1 + p\nabla + \frac{p(p+1)}{2!}\nabla^2 + \frac{p(p+1)(p+2)}{3!}\nabla^3 + \cdots)y_0$$

So, I know that this infinite series will terminate whenever we reach the higher order operator.

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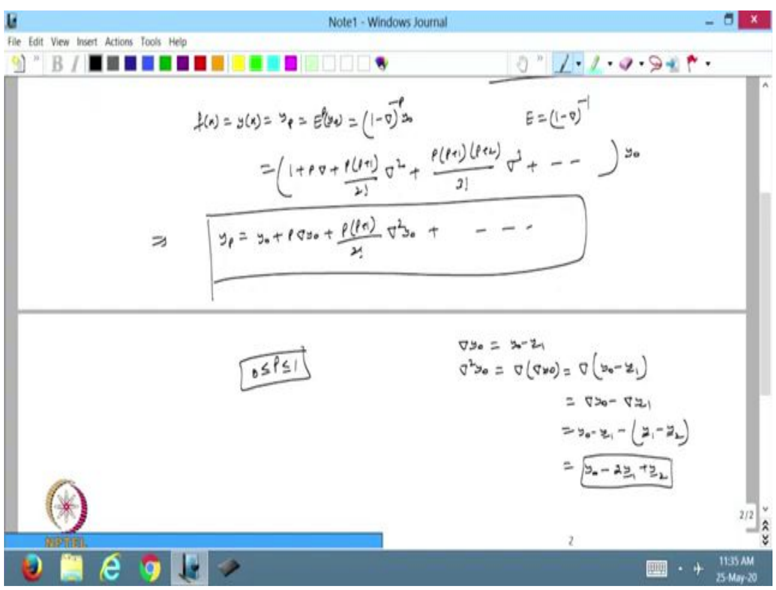

So, from here I can write, now I can write that my yp will be

$$y_p = y_0 + p\nabla y_0 + \frac{p(p+1)}{2!}\nabla^2 y_0 + \cdots$$

Now, for here I know that the $\nabla y_0 = y_0 - y_{-1}$. And, $\nabla^2 y_0 = \nabla(\nabla y_0) = \nabla(y_0 - y_{-1}) = \nabla y_0 - \nabla y_{-1} = y_0 - 2y_{-1} + y_{-2}$ So, the 3 points are involved in the second backward difference operator. So, from here I can say that in this case I am going backward using the Newton backward difference method.

So let us do examples, then it will be more clear to me. And also in this case the value of the p should be between 0 and 1. Only then I can apply the Newton backward formula.

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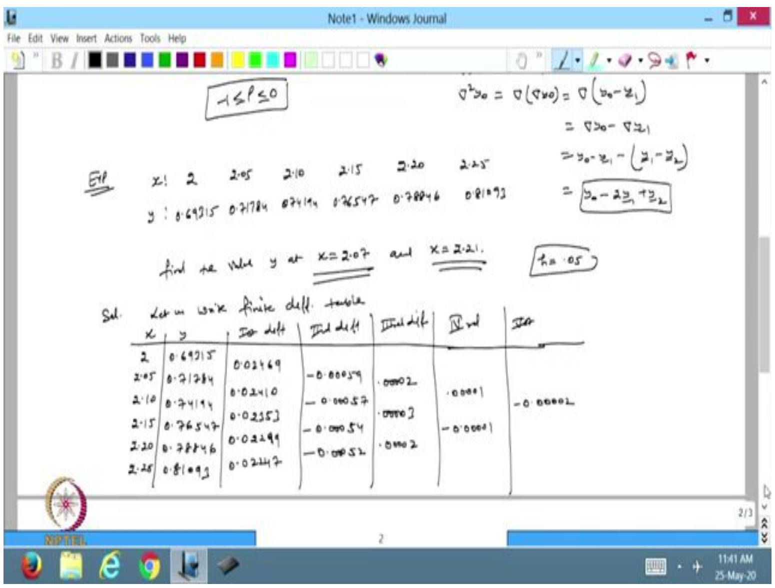

So let us do one example based on this. Or I can say from here that this value p between -1 and 0, this value should be there. So, let us take one example. I have the value, so this x is given to me 2, 2.05, 2.10, 2.15, 2.20, and 2.25. So, at this value of x, my y is given to me. So, y coordinates are given to me, it is 0.69315, 0.71784, 0.74194, 0.76547, 0.78846 and 0.81093. So, these 6 value is given to me. Now, the question is find or approximate the value, find the value y at x = 2.07 and at x = 2.21?

So, these two values I want to approximate the value y, at this x. So, let us do this one solution. So, now in this type of question, first of all I want to write the finite difference table. So, let us let us write a finite difference table. So this table, first we have to make this my x, so x is 2, 2.05, 2.10, 2.15, 2.20, 2.25. So in this case, you can see that my h will be 0.05, because this minus this and these are equal space, so no problem.

Now from here my y is given to me, because there are a large number of values involved in there, so it is more prone to write wrong values. So, we have to be very careful writing these values. Now I will apply the first difference. So, the first difference will be, so this will be 0.02469, 0.02410, 0.02353, 0.02299, and 0.02247. So, this value is the first difference, then we will go for the second difference.

So, the second difference will be -0.00059, -0.00057, -0.00054, -0.00052, then we will go for the third difference. So, this will be 0.00002, 0.00003, 0.00002 then I can go for a fourth difference. So, this will be 0.00001 and -0.00001, the fifth difference will be -0.00002 and I know that the fifth difference will be a final, will be a constant and the sixth difference will be 0. So, that is my finite difference table.

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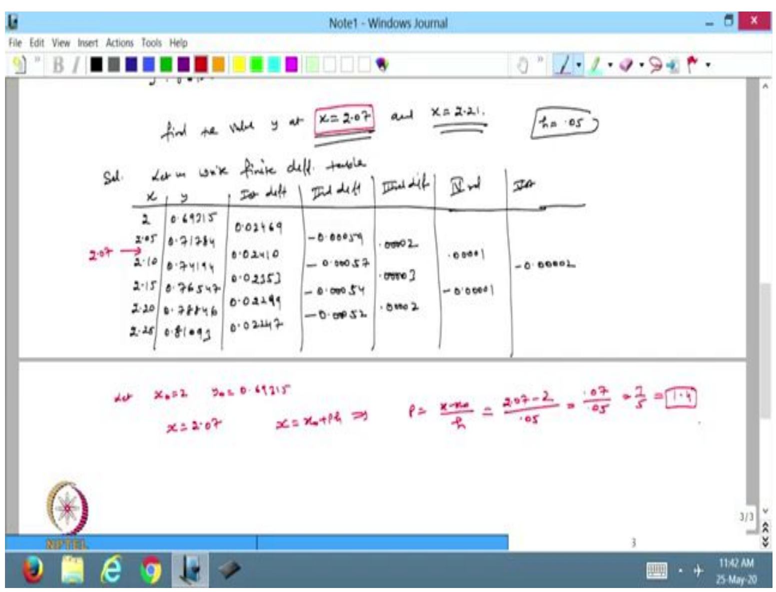

Now, based on this one, first I want to find the value of this. So, now x = 2.07, so it is here lying, so 2.07 lies here. So, in this case, let we choose let $x_0 = 2$, $y_0 = 0.69315$ and my x = 2.05. Now, from here I want to find the value of p, so, I know that this $x = x_0 + ph$. So, from here my $p = \dfrac{x - x_0}{h}$. Now x = 2.07 so, p = 1.4. So, the value of the p is coming greater than 1.

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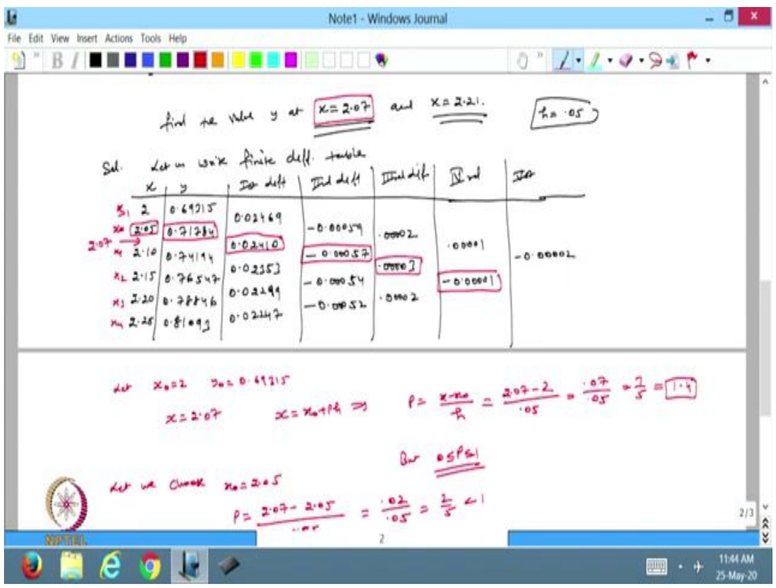

But p should be in between 0 and 1. So, in that case what we will do, so let we choose $x_0 = 2.05$. So let us choose this one. So, for once I choose this one, the value of p will be in this case, it will be $p = \dfrac{2.07 - 2.05}{0.05}$ So, that value is less than 1. So in this case, what we can do, I can write this as x_0, x_1, x_2, x_3, x_4, and this one as x_{-1}. So, now I want to apply the, in this case, I want to use the maximum value, so we will apply Newton forward difference operator formula.

So for that, this is the value I am going to use a y_0, this is x_0 and this is the y_0 and then I will use this value then this value, this value and this value. So in this case, I will be able to use only 1, 2, 3, 4, 5 values. If x_0, if I choose the $x_0 = 2$, then I was going to use 6 values, but now I have to use only 5 values. So, let us do this one.

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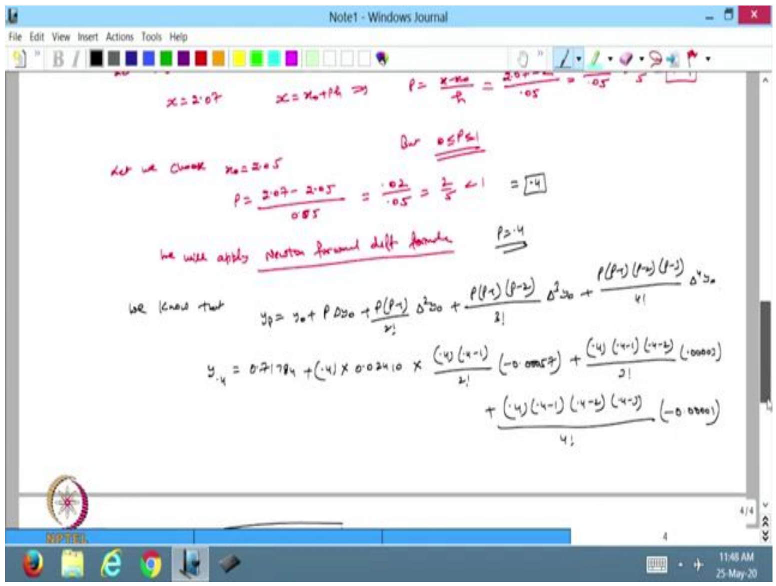

We know that my y_p the formula is

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

So, $y_0 = 0.71784$ and p = 0.4 and using a difference table, So in this type of question we have to take care about the calculation.

So, from here if I calculate all this value using the computer or calculator, the value is coming, so the value of $y_p = 0.72755$. So, this is the value, I am able to approximate using the Newton forward difference formula. And you can see that its value is 0.72755. So, it is in between 0.71784 and 0.74195, so this value is lying in between here somewhere. So, from here I can see that this function is an increasing function. So, that is my approximated value. So, that is my answer.

Now, the next case at x = 2.21. So in this case, what I am going to take, 2.21 is lying here. Now, if I choose the 2.21 here and apply the Newton forward, so, in that case it might happen that I will choose this as my x_0. And if I choose x_0, this will be my y_0 and this will be the first derivative, first finite difference y_0 and that is it. So, if I apply the same Newton forward to approximate the value at 2.21, in this case, I will use only 2 values, this and this.

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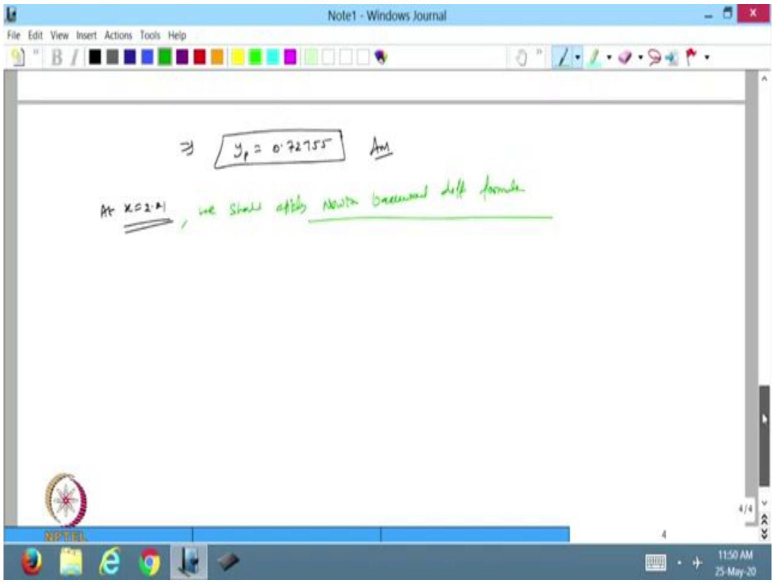

So, in that case the approximation will be not well defined, well the approximation will be not good in this, we can say, because I am using only 2 values.

So, in that case, to find out the value at x = 2.21, we should apply Newton backward difference formula.

So in the new Newton backward difference formula, so in that case what will I do, let us take this one as green color, this one, so if I apply backward, so I will use this value and I will call it x_n, in the formula also I have used x_0 . So, this y_0 is there maybe I can call it x_n, y_n also because I can take it as the last value or some in some books, this is also chosen as y_0. So, if I choose is y_0, then the previous value will be in the terms of x_{-1}, x_{-2}, x_{-3}.

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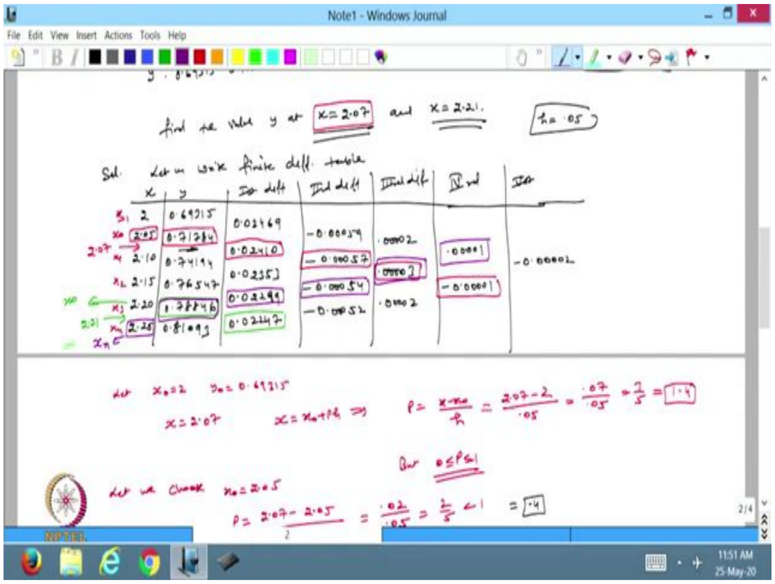

Or if I choose, let us take this value, then the corresponding value, then I will choose this value, this value, again this value and this value. So, I am using these values because I know that in the forward I will go this way downward and the backward I will go in this. So, I will go by this way.

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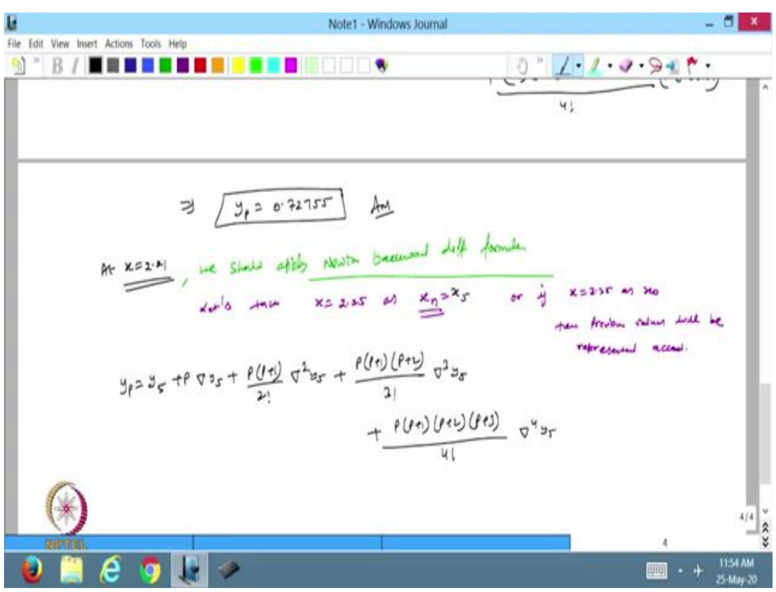

So, let us take x = 2.25 as $x_n = x_5$. I can also choose this, so if I took, choose as the x_5 then the previous value will be x_4, x_3, x_2 or if I choose, if we choose x = 2.25 as x_0 then the previous value will be represented accordingly. So, I can call them a x_0, y_0 or y_n does not matter. So, let us apply this formula. So, in this case I know that Newton backward difference formula is y_p is equal to,

$$y_p = y_5 + p\nabla y_5 + \frac{p(p+1)}{2!}\nabla^2 y_5 + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_5 + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_5$$

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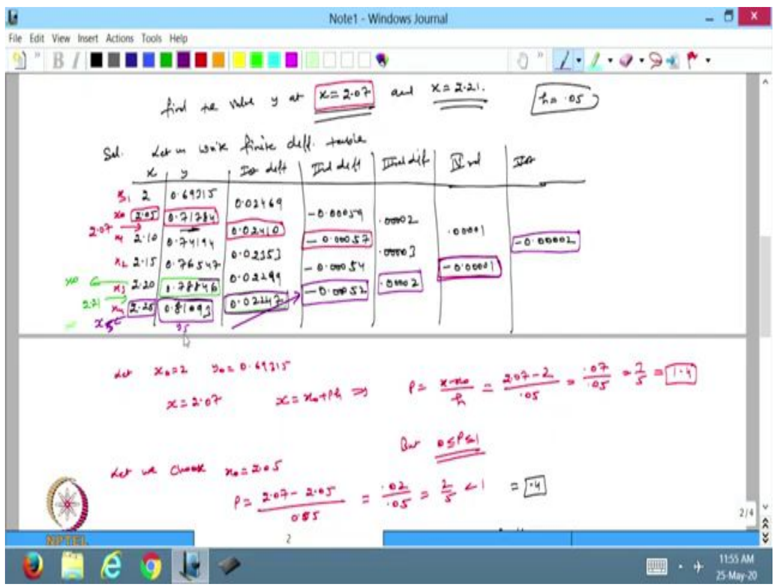

This value, no this value is not this value. I have to make little changes here. Because I am choosing this value, so the value will be this one now. So, this is my so this is my x_0. So, this is my x_5, and this would be my y_5. So, this is the y_5, then this value, then this value, then this value, then this value, and then this value. So, in this case I am using, in fact, 6 values. So, this I am using the maximum value I can use 1, 2, 3, 4, 5 6, because this is the y_5.

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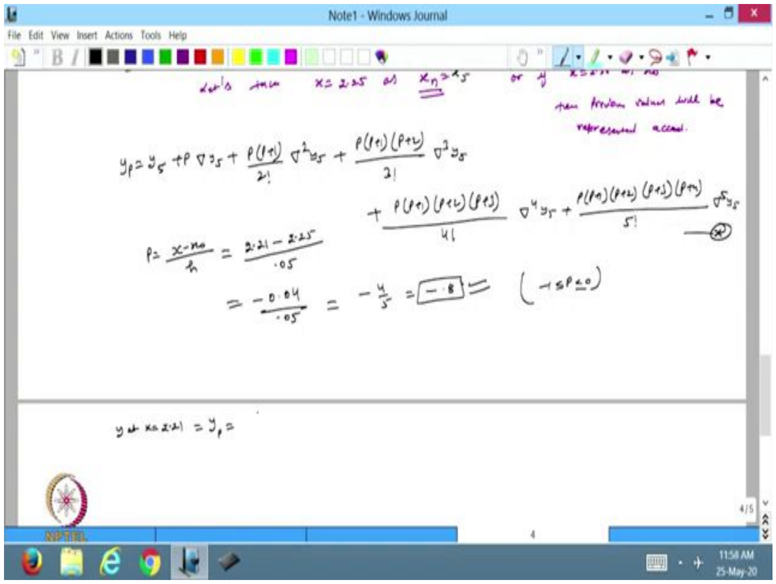

So in this case, I have to go one term more. So, that will be $\frac{p(p+1)(p+2)(p+3)(p+4)}{5!}\nabla^5 y_5$. Now, if I substitute the corresponding value, so, first off I want to find what is my p? So, in this case my p will be $\frac{x-x_0}{h}$. Now, my x = 2.21 and x_0 is basically the last value I have chosen because we have represented this one, so I told you that this can be x_0 also okay. So, x_0 is same as x_n, the last value, so this will be 2.25. So, that should be clear that what is the meaning of x_0 here, the last value and h = 0.05.

So, in that case, $p = \frac{2.21-2.25}{0.05} = \frac{-0.04}{0.05} = -0.8$ so this value is coming and I know that our value p should be between -1 and 0. So, that should be there. So, it is qualifying for this one. Now, from here I can write my y at x = 2.21.

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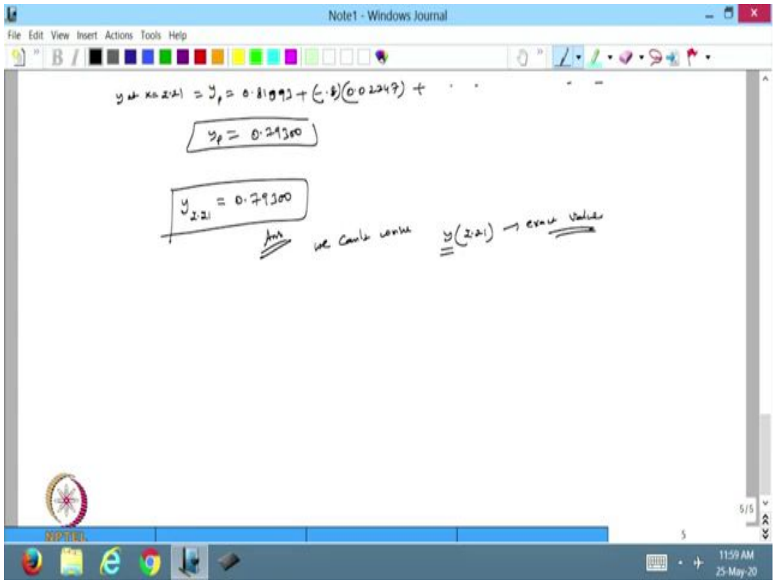

So, this is equal to y at this point p, so, this p is whatever the P is there, I will substitute all these values in the given formula. So, this is my formula, I will substitute all these values.

So, I can substitute all this value and then using the calculator we can find out this value and this value is coming 0.79300. So, that is my approximate value of this one. So, I call it $y_{2.21} = 0.79300$. So, that is my approximate value. I cannot write this as, we cannot write this as y(2.21) because this means that I know the value of the function and substitute the value there. So, this is the exact value.

So, an exact value I can find only if I know the value of y, so, that is my approximated value and this is my answer. So, from here I can say that, whenever the value is to be found out that lies in the upper part of the table, I will go for the Newton forward difference and when this value lies in the bottom of the table, then I will apply that Newton backward formula. So in the next lecture, we will see that if the value lies in between some values in the middle of the table, then we can apply the central difference formulas.

So, that is all about this Newton backward and Newton forward. So, I should stop here. So, today we have discussed the Newton backward difference formula and we have also tried to solve one example and to approximate the values that lie in the upper part of the finite difference table and the lower part of the finite difference table. So, that is all about today. In the next lecture, we will go further. So, thanks for watching. Thanks very much.