Scientific Computing Using Matlab
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Lecture 36
Continued

Hello viewers, welcome back to the course on Scientific Computing using Matlab. So, now we will continue with the previous topics, we are starting with the Interpolation. So, today we will discuss how we can make the finite difference table to approximate the value of the function at any x required or for any x in the given interval. So, today is lecture number 36.

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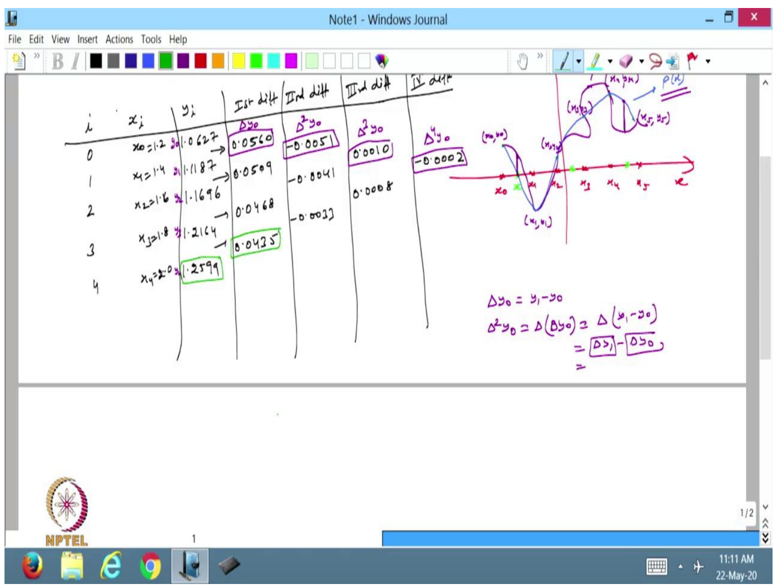

Now, today we will discuss the Finite Difference table. So, this is my x axis and this is y axis and I have some data that is distributed equally spaced for the given interval. So, suppose this is my x_0, x_1, x_2, x_3, x_4, x_5. So, let us take that they are the total number of six mesh points or the nodal points. And these nodal points, we have the value the function that is given to us is this one then this may be this value this value then this value and then this value.

So, this value is given to me. So, I will call this.as so this point is x_0, y_0. So, this is the y_0 .then this is my x_1, y_1, x_2, y_2 then $x_3, y_3, x_4, y_4, x_5, y_5$. So, all these points are given to us. Now, somebody asked me if I want to approximate the value at this point. So, that is suppose I take the x or somebody asked me I want to find the approximate value of the given function for some x that is lying here or some x in between.

So, everything depends upon that where you want to approximate the value of the function for a given x will make the finite difference table. Now, and we also know that if I want to interpolate this function with a polynomial then my polynomial will be like this one it is passing. So, starting when I am here, it will go like this because it should be passed from all the given data points. So, it should be like this then this and then this value.

So, that is my approximated polynomial and that we represent by P(x). And what is the value what is the function here. So, function we do not know but let these points are satisfied by some function. So, maybe let I take that this function as a my function is like this one like this one then going this this way and then going this. So, let this be my function.

So, that function we do not know but we are taking that there is some function and the points are the few values of the function at the given value of these nodal points. So, that data is given to me. Now, based on these data points. I want to approximate a polynomial. So, this P(x) is a polynomial that is giving me the interpolating polynomial passing through all these data points.

Now, suppose I want to find the value of the function at this x, so at this x if you see the actual value maybe different from the approximate value. So, maybe I can call it that this is the error or maybe I want to find the value of the, for this x so at this x this may be an approximated value and this may be the exact value. So, that is the error. So, this error everywhere you will see that if I want to approximate the value at any x in between there is some error involved in this case. So, that we also discuss today.

So, let us make the finite difference table. So, in the finite difference table. Suppose so this is given to me. So, I start with the table. So, this in my indexing, so that is 0, 1, 2, 3, 4, 5, 6. So, we have the value up to 5. So I will take only up to 5. Now, at this I have the x coordinates that is given to me. So, this is x_0, x_1, x_2, x_3, x_4, x_5, So I just for the time being I just take

$x_0 = 1.2, x_1 = 1.4, x_2 = 1.6, x_3 = 1.8, x_4 = 2.0$ and $x_5 = 2.2$.

So, in this case, maybe I can because I am dealing with the six values. So, this table will be very large. So, I will just remove one value also, so you can remove the value just based on the large number of values the bigger this table will be. So, I will take only five values and this value I am taking. And suppose the value given to this is given like this 1.0627 so this is value given to me 1.1187, 1.1696, 1.2164 and 1.2599. So, this value is given to me.

Now, based on this one. I will try to find out the differences. So, I will first find out the first difference. So, first difference, you know that I will take this value minus this value. So, this minus this if I find then it is giving me 0.0560. So, 87-27= 60 and 11-6 = 5 so this value is. So, this value I am writing here in between this so that shows that this value is the difference of this minus this.

Now, I will find the next difference. So, the next difference will be 0.0509. Then the next value I will write here. So, 0.0468 and the next value is 0.0435. So, that is the first difference I have taken this minus this, this minus this, this minus this and this minus this. So, this is the first difference, then I will write the second difference.

So, you know that from the we started with the 5 values the first difference will be the only 4 values and the second difference will be the difference of these values. So, this minus this will be the second difference and if I take this value, this will be -0.0051. So, it is smaller than this one. So that is why negative signs are coming and 51. So, it is coming. Then next will be -0.0041 and the next value will be -0.0033.

So, that is a second difference and will now left with only three values then I will take the third difference. Now, we also know that we have five values. So, in the previous lecture we have discussed that if I go there, with these five values and so the fourth difference will be the constant value.

So, that we already know from the previous lecture. Now, the third difference will be this minus this so if you take this one, it will be 0.0010 and this will be 0.0008 and the fourth difference I have taken so this will be the difference of this, so it will be -0.0002. So, now from here you can see that they are the five values and the fourth difference is the constant value. So, if I take the fifth difference, then it will be 0 that we also have discussed in the previous lecture.

Now, from here you can see that. So, this is my so this value is basically if you see this is y_0, this is y_1, y_2, y_3, and y_4. Now, what about the forward operator of y_0, so this will be $y_1 - y_0$. So, this is $y_1 - y_0$, so from here I can say that this is forward operator of y_0, what about the second forward operator of y_0. So, this will be again, I am taking the two times of this, so y_0 so this I can write forward operator of $y_1 - y_0$ and from here I can write $\Delta y_1 - \Delta y_2$. So, from here, I can say that I have taken this value minus this value. And what is this value minus this value.

So, from here, you can see that this is the second order finite difference forward finite difference operator and similar way this value is $\Delta^3 y_0$ and this is $\Delta^4 y_0$. So, this the value on the top they are giving me the first forward difference, second forward difference, third forward difference and the fourth forward difference. Now, from here I can see now what about this values. So this is my y_4. Now, what about this value? So, this value is giving me this minus this.

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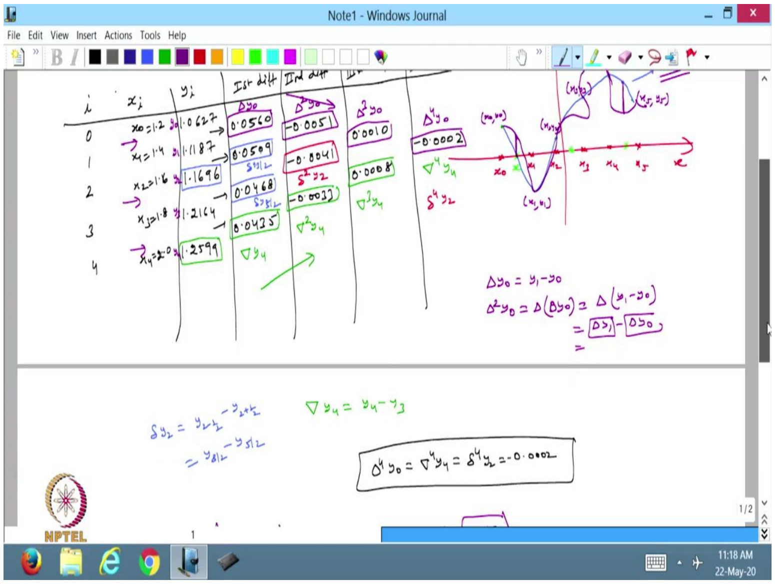

So, I want to check what will happen. Now, I want to play the backward so backward y4. So, it will be y4 minus y3. Because it is going just one backward. So, I want to find this value. So, this will be this minus this so this is the value so I can say that this is the first order backward difference. What this one the same way this minus this so this is the second order backward difference.

This is the third order backward difference and this is the fourth order backward difference. So, now if you see from here, it is the downward going, this is the forward difference and this is going up so that is the, so this is going up. So, this is the backward differencing and this is going downward. So, this is the forward differencing.

The same way I can define the central differencing, so from here you just see that centre differencing means I just take the value that is as in the centre. So, let us choose y_2 now if you see from this this is my y_2. Now based on this one, I will choose this value and this value. So, this is my y_2. Now, from here, I can say that this value is now I want to find δy_2. So, what is the δy_2? It will be $y_{2-\frac{1}{2}} - y_{2+\frac{1}{2}}$. So, it will be $y_{\frac{3}{2}} - y_{\frac{5}{2}}$.

So, the next will be δ^2. So, this value if I choose this one so this will be this minus this so I can this is $\delta^2 y_2$ and so on. So, now I can choose these two values based on this and this value will be this minus this. So, this is δ^2 and you will see that this becomes $\delta^3 y_2$. So, now the same value is either presented by the forward fourth order difference, or backward fourth order difference or the central y_2 third order difference. So, from here you can see this will be four fourth order.

So, I can write this is $\delta^4 y_2$. So, first order difference second order third order and fourth order. So, from here, I can say that based on this one this $\Delta^4 y_0 = \nabla^4 y_4 = \delta^4 y_0$ so all values are equal to -0.0002. So, that is also the verification that we have the data five data points. Then the fourth difference will be a constant value and the fifth difference will be 0. So, that is also one of the verification we can do.

So, this is the basically we can make the difference table whenever some data values are given to us. And based on these values we want to find an approximate value at any x. So, in this case what will do, now suppose I want to find based on these values I want to approximate so we would like to find the approximate value of y at x is equal to say I just want to choose at x = 1.3.

So, I want to find the value in between that if I put the x = 1.3. What will the value y? So, this one I want to approximate or in other sense I can say that I want to find the value of y at x = 1.65 or I want to find what is the value of y at x = 1.97.

So, I want to approximate the values at these different different values of x and if you see from this value of x lies so 1.3 is lying here 1.65 is lying here and 1.97 is lying here. So, it is up to me. It depends upon where the value of the x is lying. So, from here I can say that x is lying in the upper top of the table, another value the x lying in the central or another one is lying in the bottom. So, based on that where the value of this x lies we can, we can find the different different methods to approximate or to find the interpolating polynomial. So, that is the way we can make the finite difference table.

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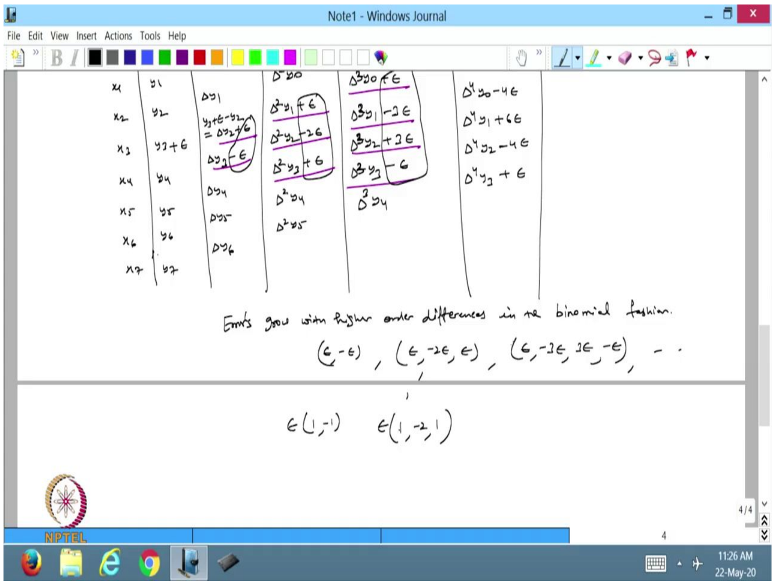

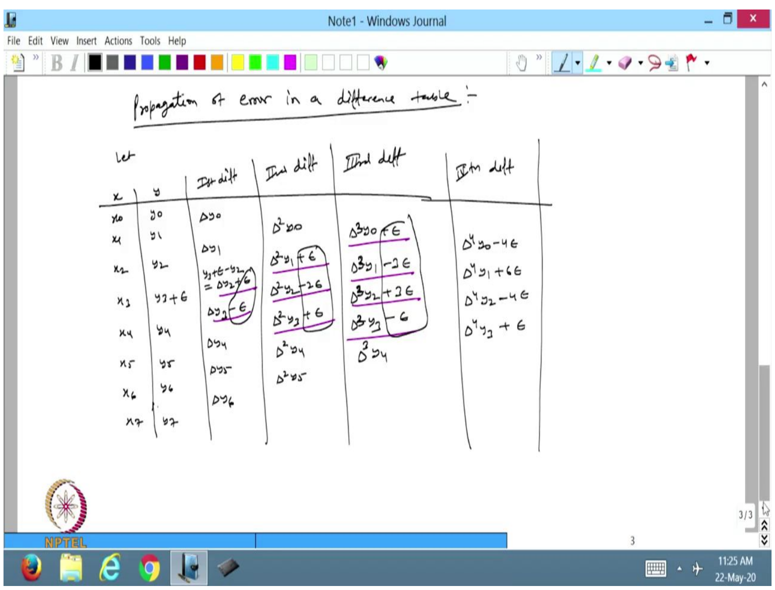

So, next thing after this we want to find out what will happen if there is some error introduced in the values of the table. So, propagation of error in a difference table. So, now I want to find out how the errors propagate in the difference table. So, let I have a difference table like this one.

Suppose I have this is the x and this is y. So, let us take this is $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7$. So, let us take the eight values. So, this is $y_0, y_1, y_2, y_3, y_4, y_5, y_6, y_7$. So, let us take that in between there is some error introduced. So, $+\epsilon$ is the error. Now, I take so let us take the first difference. So, first difference if you see, maybe I can now write in the forward differencing.

So, I can write it here. This will be equal to Δy_0 this would be equal to Δy_1 this will be equal to, this minus this, so I will get $y_3 + \epsilon - y_2$. So, that is equal to $y_3 - y_2$. So, $\Delta y_2 + \epsilon$. Next value will be y_4 minus this. So, from here I can write that y_4 minus this so $-\epsilon$ will be there.

So, here I can write $\Delta y_3 - \epsilon$ then $\Delta y_4, \Delta y_5$ and Δy_6. So, here I am using the forward differencing operator. So, this is the value. Now, this is the first difference this error propagates and it appears at the two places $+\epsilon$ and $-\epsilon$. Now, let us take the second difference. So, in the second difference if you see this minus this will be $\Delta^2 y_0$.

So, this minus this will be $\Delta^2 y_1 + \epsilon$ this minus this this minus this will be $\Delta^2 y_2$, this minus this so it will -2ϵ and then it will be $\Delta^2 y_3 + \epsilon$. So, from here $\Delta^2 y_4$ and this will be $\Delta^2 y_5$. Because we are taking it forward. So, in the forward if you move with the higher operator high difference value, then this value keeps reducing. So y_7 has been eliminated. Now after that Δy_6 has been eliminated. So, in this way we can go. I take the third difference.

So, in the third difference this minus this so it will be $\Delta^3 y_0 + \epsilon$ this minus this will be $\Delta^3 y_1 - 3\epsilon$, $\Delta^3 y_2 + 3\epsilon$, this minus this it will be $\Delta^3 y_3 - \epsilon$ and then $\Delta^3 y_4$ so that value will.

So if you see from here, the next will be so here you can see that the error has been distributed or error is spreading in the two places here. So, in the two places it is coming. The error is coming here in the second difference error is spreading at three places. Then error is spreading at the fourth place.

So, from here you can see that if I take the next one. The fourth difference. So, this will be in this case this minus this so that will be $\Delta^4 y_0 - 4\epsilon$, this minus this will be $\Delta^4 y_1 + 6\epsilon$, this minus this, so it will be $\Delta^4 y_2 - 4\epsilon$, so this minus this it will be $\Delta^4 y_3 + \epsilon$.

So, from here, you can see that the error propagates as we go for the higher difference and the pattern if you see the pattern of the error is like this one. So, here is a pattern like this one here is the pattern is like this one and so on. It means that I can see from here that the errors grow with higher order differences in the binomial fashion.

So, in the binomial fashion means, if you see from here, the error is coming ϵ and $-\epsilon$ then it is coming $\epsilon, -2\epsilon, \epsilon$. Then the third one is $\epsilon, -3\epsilon, 3\epsilon$ and then $-\epsilon$. So, like this one it is coming. So, from here, you can see that this one I can write as ϵ and this is 1 and -1 this one I can write as taking ϵ out so it will be 1, -2, 1.

So, if you see from here, it is $a^2 + b^2 - 2ab$ so $(a - b)^2$ from here it is $\epsilon(1, -3, 3, -1)$. So, it is $a^3 + b^3 - 3a^2b + 3ab^2$. So, this is also if I take this one I can write as $(1 - 1)^3$. This I can write as a $(1 - 1)^2$ this I can write as a 1-1 like this one. So, it is a binomial fashion the error is growing and it grows with the higher order differences.

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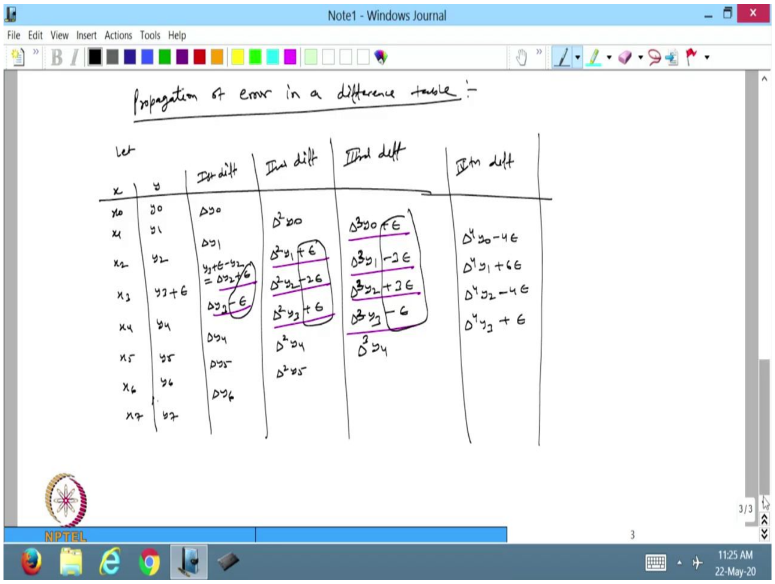

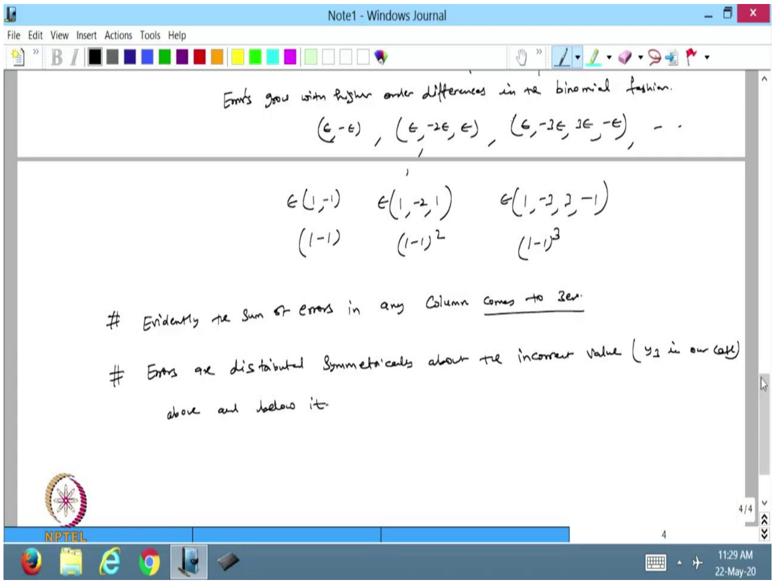

So, from here also that I can write that evidently the sum of errors in any column comes to 0. So, if I choose any column and add that error so $+\epsilon - \epsilon$ so it will be 0, $+\epsilon - 2\epsilon + \epsilon$ so that will be 0 so this will be 0. So, if I take I choose any column and add all the errors so that comes to 0. So that is one of the other factors of other properties of the final difference table.

And the second one is errors are distributed symmetrically about the incorrect value. So, an incorrect value means like this one I have taken y_3, y_3 in our case. So, errors are distributed symmetrically about the incorrect value above and below. So, from here also, we can see that the errors are distributed one above one below then one more above one more below. So, it is distributed symmetrically in the above way in the above and then the below.

So, that is also one of the characteristics of the errors in the final difference table. So, I should stop here. So, today we have discussed how for the given data we can make the finite difference table and in the next lecture, we will discuss how the finite difference table is used to approximate a value. So, thanks for watching. Thanks very much.