Scientific Computing Using Matlab
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Lecture 35
Continued

Hello viewers, welcome back to the course on scientific computing using MATLAB. So we will continue with the concept that is called interpolation. So, let us continue with this one.

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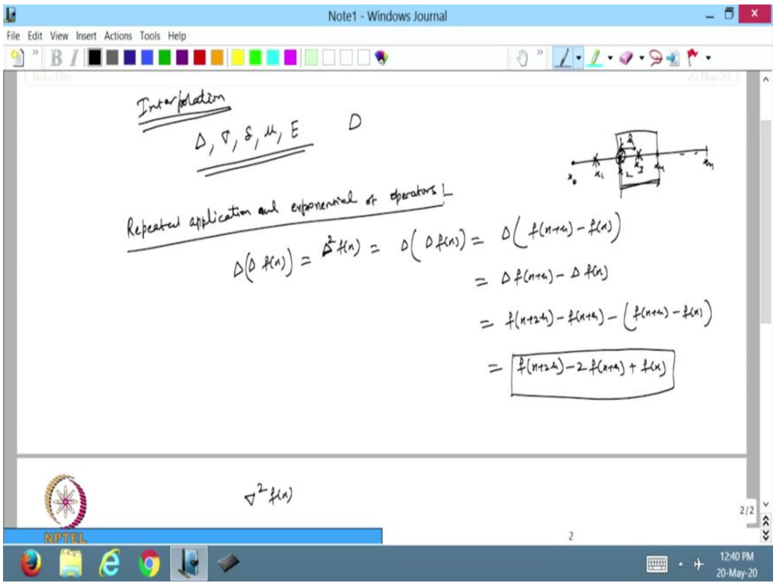

So, in the previous lecture we discussed we started with the interpolation and then we have discussed the various operator that is forward operator the backward operator central difference operator, the mute that is the average operator and this is a shift operator. So, these are the difference operator and then we have also discussed the differential operator that is D

Now, we can use this so, I want to Discuss the repeated use of application and exponential of operators. So, this is a. So, what is the meaning of repeated applications? It means that suppose I want to apply a forward operator two times on the given function fx. So, this one I will applying the two times So, I can write this function as operator $\Delta^2 f(x)$.

So, in this case we are repeating this process two times. So, let us see what will happen. Now, suppose I want to apply this, so $\Delta(\Delta f(x)) = \Delta(f(x+h) - f(x))$ So, I am applying at an x. So, it will move further by is a step h. So, I have the data that is given to me. So, this is my x0 suppose x1, x2, xn.

So, I am applying this operator on any x. So, it will go further by the distance h and this will be x3 and minus fx. So, this is the difference operator and now again applying this one. So, this will be $\Delta f(x+h) - \Delta f(x)$. So, from here this is equal to

$f(x+2h) - f(x+h) - (f(x+h) - f(x))$. Now, from here I can see that

$f(x+2h) - 2f(x+h) + f(x)$. So, in this case, you can see that when we apply this forward operator two times, then the value of the function is involved at three points. So, in this case, if I know the value of the function at x3 x2 x1 only then I can take the second forward operator add the value x2, because in this case I have to go further not only x3 even x4 also.

Because in this case wherever the value I am considering applying the operator, suppose, I am applying here, so, I have to go two steps further. So, I will go from here to here, so two steps further and then one step And that value the function. So, in this case, I am applying the forward operator two times. So, I need the value of the function at two places after that. So, this is the forward operator two times. I am applying for the forward operator. Similarly, I can apply the two times backward operator like this one.

So, in this case you will see that in that case I need the value of the function to value the function before the value of the given function value before the value fx. So, in this case I need to go backward at this place and this place to find the value of the functions using the two times backward operator.

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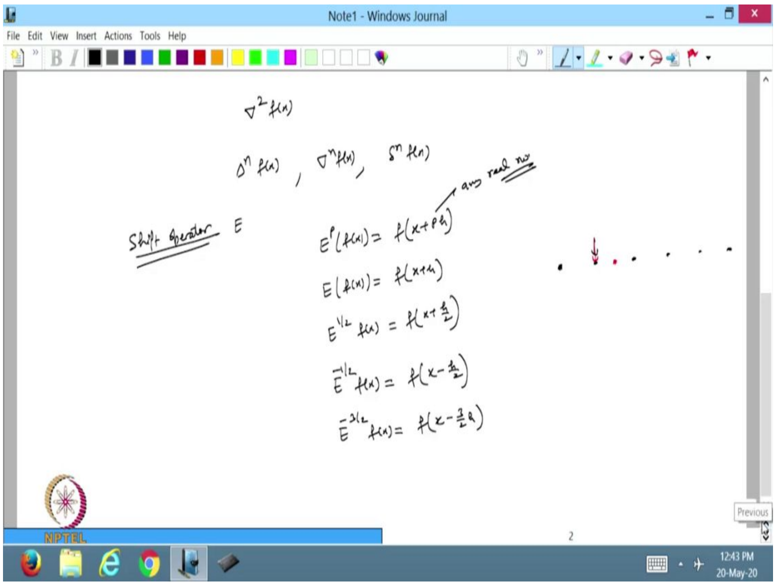

So, based on this one I can apply n times forward operator and n times backward operators or I can apply n times center difference operator. So, this way we can apply that any number of times the same operator we can apply on the given function. Now, I want to apply and I want to use the shift operator. So, shift operator I know that this is represented by E. Now what is the meaning of E, so suppose I take the $E^{p} f(x)$. So, this will be equal to $f(x+ph)$.

So, I am doing this by shifting the function by ph, because it is the E raised about P. So, P is any real number. So, it can be any real number. For example, if I put p=1 and this is fx. So, in that case my $E(f(x)) = f(x+h)$, it means that I have the value of the given data and I am applying the shift operator suppose at here, then the value of the shift operator will be this value one forward. Now, suppose I want to apply $E^{1/2}(f(x))$, so, this will be valued $f(x+\frac{h}{2})$. So, now, here the h by 2 is here this value. So, now, if I apply the operator E raised to power half or at this value then I will get the value or the function at this point. Similarly, I can define my value $E^{-1/2}(f(x))$. So, that will be $f(x-\frac{h}{2})$. So, it is Going to be one half step backward to that value. So, this way we can define

$E^{-3/2}(f(x)) = f(x - \frac{3h}{2})$. So, this way we can move with the shift operator for either forward direction or in the backward direction.

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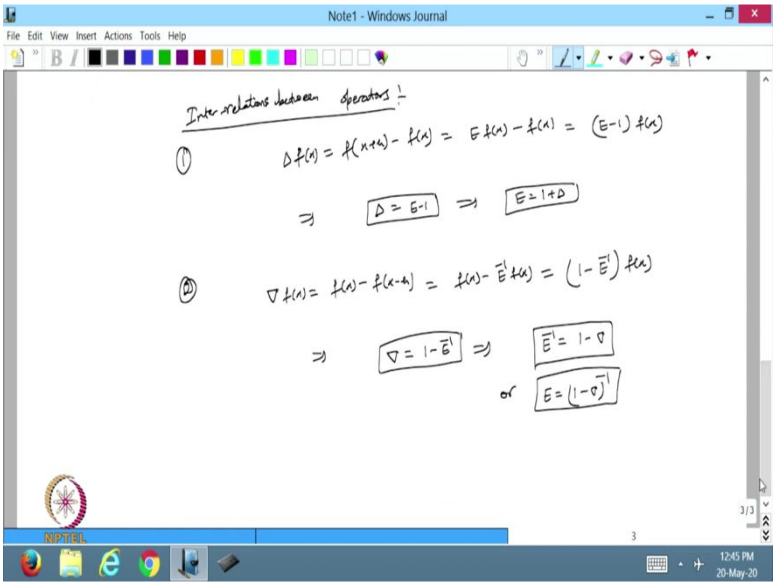

Now, we want to find the relations. So, I can define the other one: the interrelation between operators. So, what is the relation between the forward operator or the backward operator or the shift operator So, that we want to see. So, this is the interrelations between operators. So, suppose I want to display the first one I know that if I have the forward operator to the function at x, then it will be $f(x+h) - f(x)$ and this one I can write as.

So, this is the f(x) so, I can write as a $Ef(x) - f(x)$ Because E of x I know that if I apply this one I will get the value at the next step that is x+h from here I can take fx common and its value will be this one. So, based on this one I can say that the forward operator is always equal to E-1 and from here I can apply this here also.

So $E = 1 + \Delta$. So, that is the relation between the shift operator and the forward operator. The second one is that I know the nebula the backward operator on the fx, so this one I can write as $f(x) - f(x-h)$. And this one I can write as $f(x) - E^{-1} f(x)$.

So, in this case I am taking the E^{-1} because I am going backward one step backward. And from here I can write $(1 - E^{-1})f(x)$. So, from here, I can say that $\nabla = 1 - E^{-1}$ and based on this one, I can write that $E^{-1} = 1 - \nabla$ or from here I can write that the $E = (1 - \nabla)^{-1}$. So. that is the relation between the shift operator and the backward this see the shift operator and the forward now I can find the other relation.

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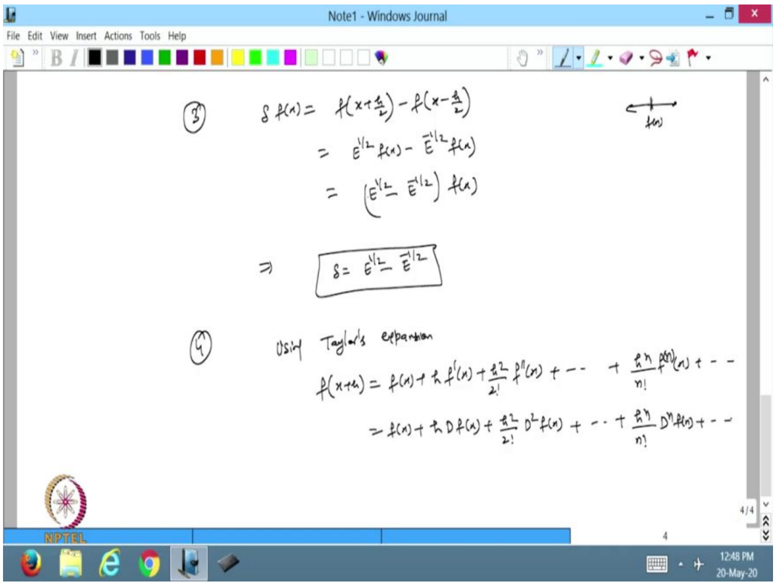

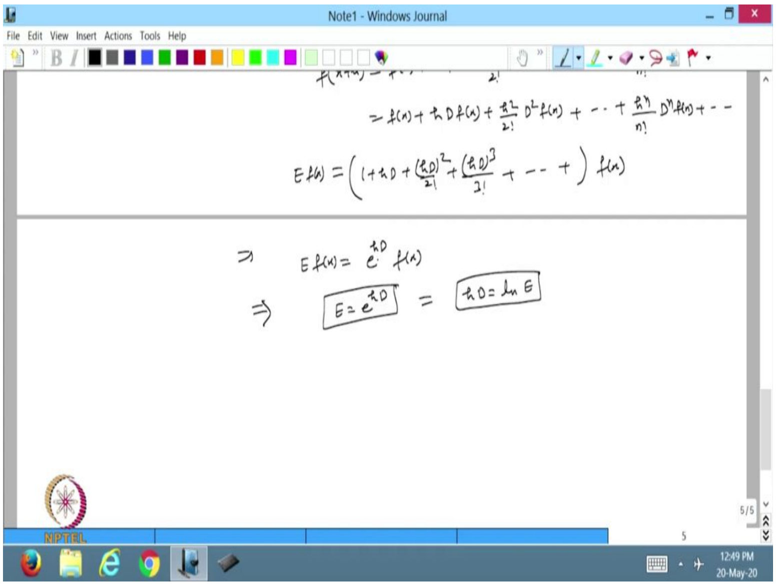

So, the third relation I want to define for this one, I know that the central difference

$\delta f(x) = f(x + \frac{h}{2}) - f(x - \frac{h}{2})$ so, it is going half forward and half backward and this is the value of the function f and this one I can write as

$E^{1/2}f(x) - E^{-1/2}f(x) = (E^{1/2} - E^{-1/2})f(x)$ and from here I can say that this gives me the $\delta = E^{1/2} - E^{-1/2}$. And that is the relation between the central operator and the shift operator.

Similarly, I can define the next one as the fourth one. So, the fourth one is I want to define the relation, I know that using Taylor expansion I can define I want to find what is the $f(x+h)$ and I apply the Taylor expansion. So,

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \ldots + \frac{h^n}{n!}f^{(n)}(x) + \ldots$$

. So, this is my Taylor expansion about x. So, from here I can say that I can write this as a f(x)+h. And this one can be written as $Df(x)$. Because in this case the function I am considering is differentiable, well defined and differentiable it has an infinite number of time differentiable functions, because I am able to get the differentiation.

So, I can apply the differential operator on this one, then I can write

$$f(x+h) = f(x) + hDf(x) + \frac{h^2}{2!}D^2f(x) + \ldots + \frac{h^n}{n!}D^nf(x) + \ldots$$

.

So, from here, I can write as, so I can take my function fx. So, from here I can write this as

$$Ef(x) = (1 + hD + \frac{(hD)^2}{2!} + \frac{(hD)^3}{3!} + \ldots)f(x)$$

.

So, this one I can apply and fx. So, this is applying this operator on the function of x and on the left-hand side this is equal to $Ef(x)$. Now, from here I can write that

$Ef(x) = e^{hD}f(x)$. So, that is given to me. Give me that $E = e^{hD}$. So, shift operator is equal to exponential hD, or I can say that $hD = log(E)$ So, that gives me Relation between the E and hD. So, that is another relation we have now. So, that is a very important relation we should always keep in mind. So, this way we can define many relations. So, but here we are doing only a few relations.

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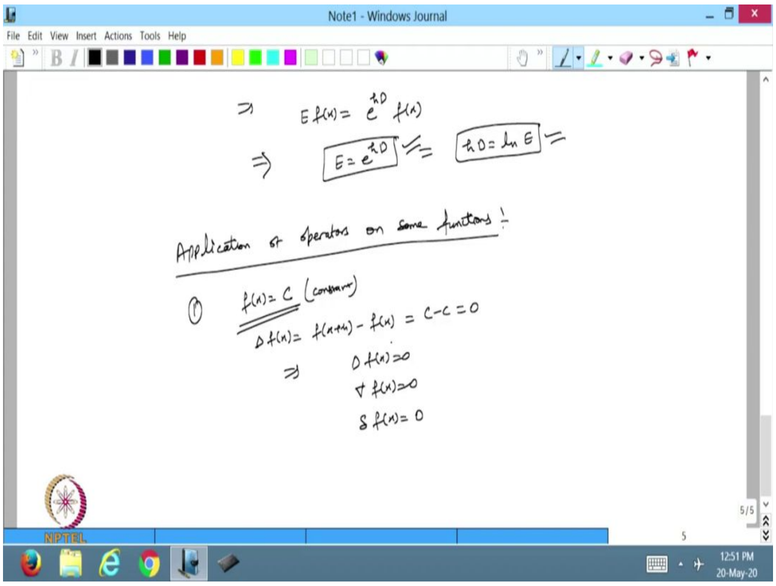

Now, we will discuss another thing, the application of operators on some functions, so let us see how we can apply the operators on some functions. So, I will start with the very simple function, select my function $f(x) = c(\text{constant})$. So, I want to apply the constant just take the constant function and I want to apply this operator. So, let us apply my operator forward operator what do you the fx I know that the forward apply on the f(x) will be equal to f(x+h)-f(x). Now the value of the function is constant.

So, it will be c-c=0. So, from here I can say that $\Delta f(x) = 0$. Similarly, if I apply the backward that will be also 0. Similarly, if I apply the central then that will be also 0. So, from here I can say that if I take a constant function and apply my difference operator on that one forward backward or this one, so, then the value will be 0. So, the same as the derivatives if you take the derivative of a constant function that is valued 0. So, let us say this is a constant function. So, let us take another one.

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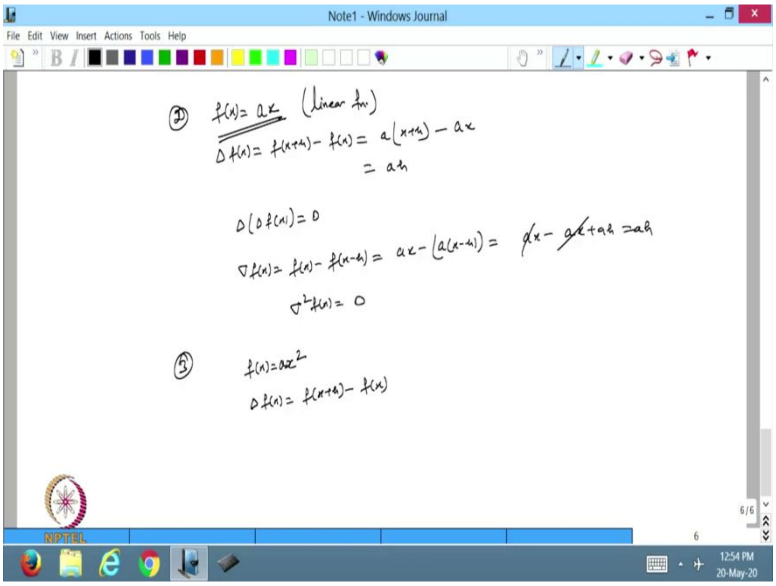

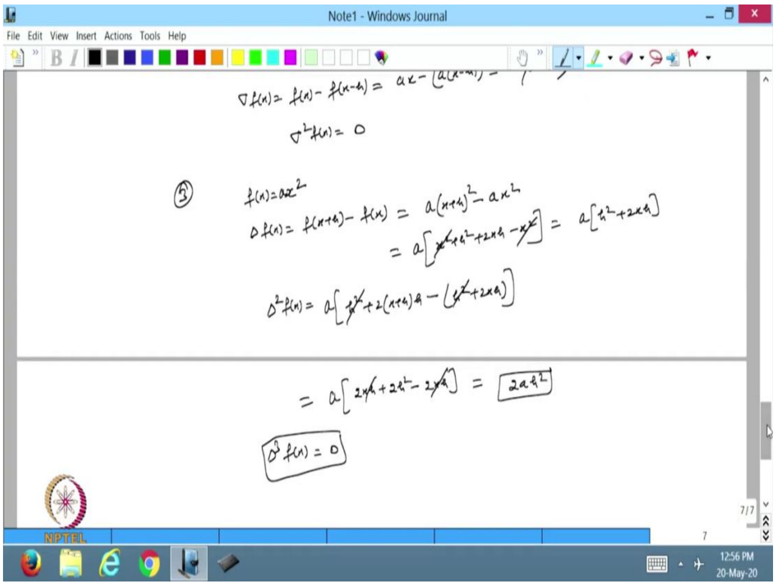

The next function, if I take $f(x) = ax$, is the linear function so, let us see what will happen. Now I suppose I apply the forward operator. So, it will be f(x+h)-f(x)=a(x+h)-ax=ah, it is a constant value because h is constant and a is constant.

So, if I take the one differential operator on this, I will get ah now I apply. I want to apply one more time to this forward operator. So, this is the constant function. So, the value will be 0 we have done in the previous one. So, from here I can say that if I have a linear function And if I apply the two times forward operator its value will be 0 if I apply one time forward operator valued the constant and that will be ah.

Similarly, I can find the backward operator $\nabla f(x)$. So, that will be again this will be f(x)-f(x-h)=ax-a(x-h)=ah. So, this will again the same value ah, if I apply the two times the backward operators its value will be 0. So, from here one is able to understand that you my function is a linear function and I can apply any operation in one time. So, if I apply the operator one time it's value will be the constant and two times its value will be 0.

Third one suppose, I take the function fx is equal to some quadratic. So, that's I take $f(x) = x^2$. So, it is a quadratic function or I can take $f(x) = ax^2$. So, in this case, I just want to apply my forward operator. So, the forward operator in this case will be

$a(x + h)^2 - ax^2$. So, this will be again. I can take the common So, this will be

$a(x^2 + h^2 + 2xh - x^2)$. So, this will cancel out and I will get $a(h^2 + 2xh)$. So, h is constant it means this is a linear function. Now, if I apply one more time on this one, so what I will get, I will get $a(h^2 + 2(x + h)h - (h^2 + 2xh))$, I am just applying directly from here.

And from here you can see that this will cancel out with this and I will get it and now I am applying my operator here. So, it will be this one. Now, from here, I will get this. So, yeah, so I will get $a(2xh + 2h^2 - 2xh)$. So, this will cancel out and it would be $2ah^2$ and that is a constant value. Now, if we apply the third one third times this operator its value will be 0 So, from here I can say that, that if my function is a second order, then I can apply my operator up to second or only and that will be the constant value. If I apply the higher order operator more than two, then all the values will be 0.

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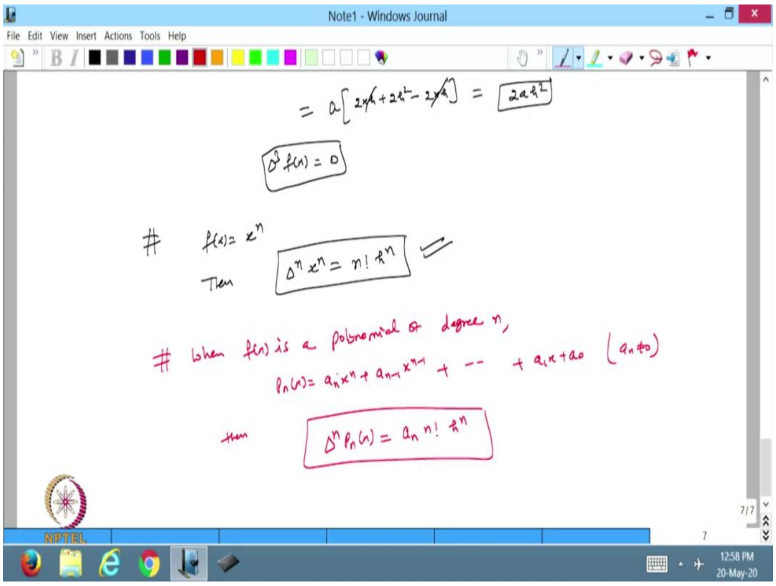

So, based on this one, I can say that from here I just make a conclusion that if I have a function $f(x) = x^n$, then if I apply a forward operator on the function, that is n times, then $\Delta^n x^n = n!h^n$. That we can also verify from here that whenever I have taken a square, just a you take one, so, it is x square. So, it will be two times it will be 2 x square.

So, it is two factorial x square. From here I can say that this is equal to if I take a is equal to one, then it will be one factorial into h. So, based on this one, I can very easily show that this is equal to this and this is a very important relation we are going to use for any polynomial of degree n.

So, now from here I can say that So, using this one when my function fx is a polynomial of degree polynomial of degree n that is

$p_n(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$. So, this is equal to a0 such that $a_n \neq 0$ then now if I take the nth forward operator on this one, so, in this case it will be $\Delta^n p_n(x) = a_n n! h^n$.

So, that is the same way we are doing only thing is that, here we are getting the coefficient of x is to power n as a n here also. So, that is the concept of the nth derivative of any polynomial is going to give us. Now, we will use this concept later on to find out the interpolating polynomial.

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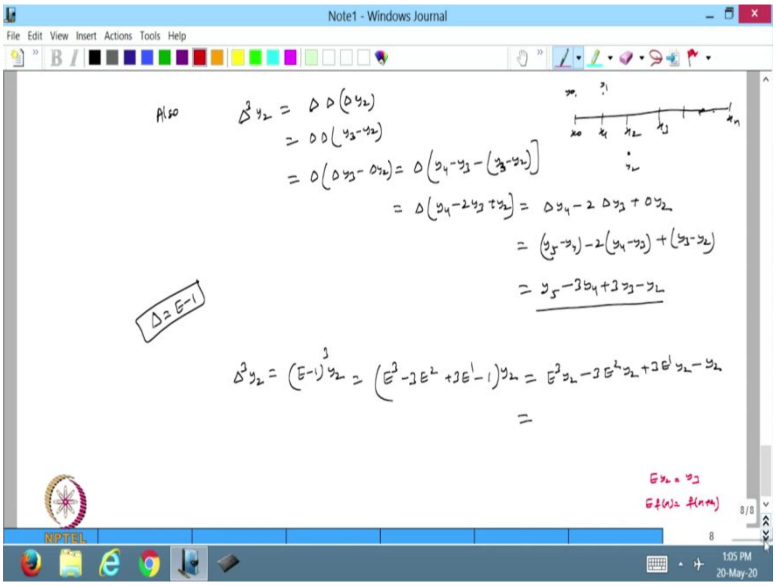

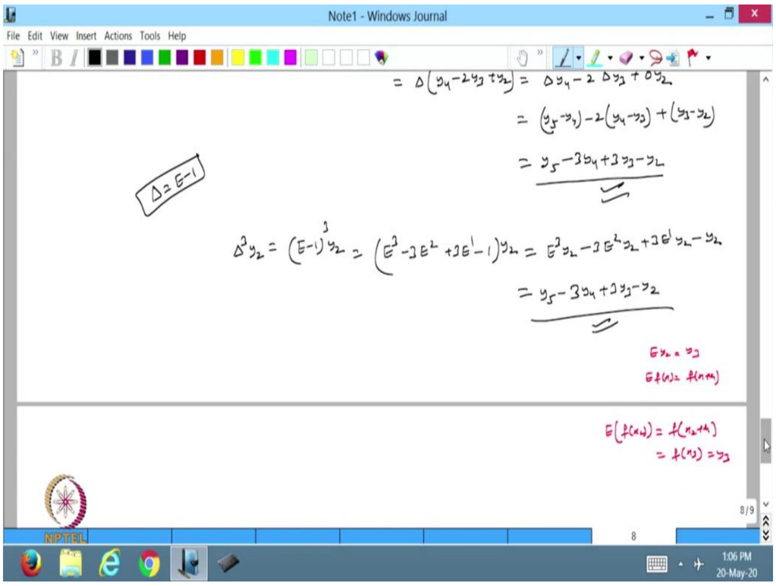

Now, we also know that I also have a dell cube and I want to apply I suppose, I take the value of the function at x1 x2 x3 up to xn. So, all this value is given to me and suppose, I apply this one on any function, so value is given to us.

So, suppose I apply this value on some value, so, I have a value x0 suppose this is y0, this is y1 some value is given here it is y2 like this one. So, suppose I apply this one on y2 now, in this case you can see that the value that third forward operator is applied on a value of the function at the point x2 it is not applying to a function that is like a polynomial we have discussed that a polynomial of nth degree like this one or this one I am applying this one only at the discrete value.

So, in this case, let us see what will happen. So, this is I want to apply, so, you can apply directly also or maybe you can apply like this one, forward y2. So, from here, I will get forward y2, so, y2 I am applying here, this is my y2. So, it will be forward so, it will be y3 minus y2 because I am applying this on a given mesh point that is given nodel value x2 and has the value y2. So, I am applying this one, because we apply this one on a function, and in this case I have a difference value of the given value of the function in the form of discrete values, I have x not y not x1 y1 x2 y2 like this one.

So, this is clear now, again I apply the forward so it will apply forward y3 minus forward y2. So, this will be y4-y3 minus y4-y3 yeah and y3-y2 and this one can be written as y4-2y3+ y2 and this one again I apply here so it will be forward operators apply on y4, y3, y2.

So, this is y5-y4-2(y4-y3)+y3-y2. And then from here I will get y5-3y4+3y3-y2. So, now you can see from here that it is giving you the symmetric value like a binomial coefficient.

So, 11 1 -1, and -3 3. So that is the value if I apply a third operator forward operator on y2. So, from here you can see If I apply the third operator, on the y2, I am getting the value I am the value, y5 is involved. Then 5 y4 is also involved. So, if I want to apply the third operator on y2 my forward operator, we will go three steps away from y2 that is up to y5.

So, that is we have to keep in mind which order we are able to apply, or the final, the forward different operator on y2. Now if I want to apply the same thing, I can do it directly also with the help of the relations, so I wanted to find $\Delta^3 y_2$. So, this one I want to apply to $\Delta^3 y_2$. So now I know that $\Delta^3 y_2 = (E-1)^3 y_2$. So, we already know that forward operator is equal to E-1. And now, from here I can assume that it will be $(E^3 - 3E^2 + 3E - 1)y_2$.

So, this one I have just opened the cubic. So, this cube minus this minus this time plus this, so, that will be the formula for this one. Now, if I can take this inside y2, so, it will be

$E^3 y_2 - 3E^2 y_2 + 3Ey_2 - y_2$. Now, this is the shift operator three times So, it will be what is the meaning of this E y2, that will be y3, because I know that Ef(x)=f(x+h).

It means and what is the y2, y2 means I am applying E on f(x2) and f(x2)=y2. And from here I will get this will be equal to x2+h and x2+h is what? f at x3 and that is y3. So, I am shifting to one step. So, I am shifting here in three steps. So, from here I will get y5-3y4+3y3-y2 . So, if you see from here, I am getting this value and this value the same. So, here I am applying this forward operator again and again 3 times, but here we are getting the values by the shift operator by the relations. So, this is the way we can apply either the given operator directly or we can use the relations between them. So, similarly, I can apply various methods like I want to find out dell cube or dell 4 on the given function, when the data is given to me.

So, maybe we can discuss this in the next lecture. So, we will stop here today. So, today we have discussed some relationship between the operators. And then we showed that how the operators can be applied to a given function, it may be a constant function, a linear function or a nth degree polynomial. And from there, we will take some help from these relations and we will define the interpolating polynomial in the coming lectures. So, thanks for watching this. Thanks very much.