**Scientific Computing Using Matlab**
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**Lecture 34**
**Interpolation**

Hello students, now welcome back to the course on Scientific Computing using Matlab, so now will go further with this course and will start the next unit and that is the Interpolation and Curve fitting.

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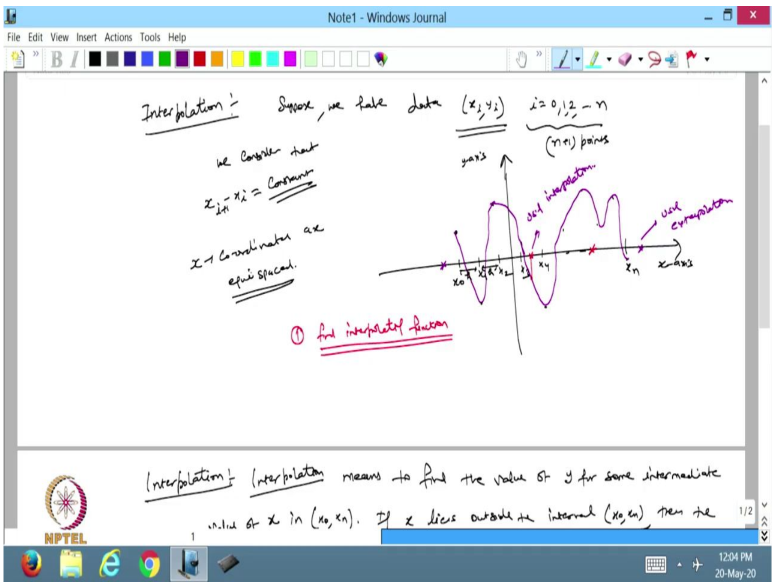


So now, will start with the concept of Interpolation, so now in the interpolation, suppose we have data and that is given to me like in the x coordinate and corresponding y coordinate and i can start with 0, 1, 2 up to n, so in this case total I have n+1 points and this data is given to me, so if I want to plot this data, so lets we plot this one, so this is x axis and this is y axis, so suppose I start with plotting the point so lets, it is my x0 so this is my x1, this is x2, this is x3 and in the last I have xn.

Now, in this case we consider that the difference between $x_{i+1} - x_i$ that is constant, so I can say that in this case the coordinates, the x coordinates are equispaced that means that this difference is h, this difference is h, so all the points are given to me that are equispaced.

Now, for the corresponding x node, suppose this is the point that is given to me corresponding to x1, suppose this is the point corresponding to x2, suppose this is the point then x3, then x4 so similarly I can take different, different point and in the and I have suppose this point.

So now, I have this my data that is given to me so in this case, so first thing is that I want to approximate that which function is represented by all these data points, so that is the first question and based on this one, now suppose I want to find the value of the y at some value in between x3 and x4 or some value in between x0 and xn, so in this case I want to interpolate the value of a function based on the given values that is provided to me so, in this case, so what I want to do? The first thing is that I want to find an interpolating function, so this is my first criteria so how can I find the interpreting function?

Interpreting function means that that I want that my function should pass through all this point so, this is my point then it should go like this then, it should go like this, then it will be like this one, then this one, this one, this one, this one, this one, so the condition is that the function I am going to approximate should pass through all these points and now once I know this function then based on this function I can find the value of x, so this is the given corresponding x will be given to me, so based on this x I am able to find the value of y.

So, this is the way we can find or we can approximate the value at a given, at any x that is lying in between x0 to xn. So, this is the way we are going to introduce the concept of interpolation.

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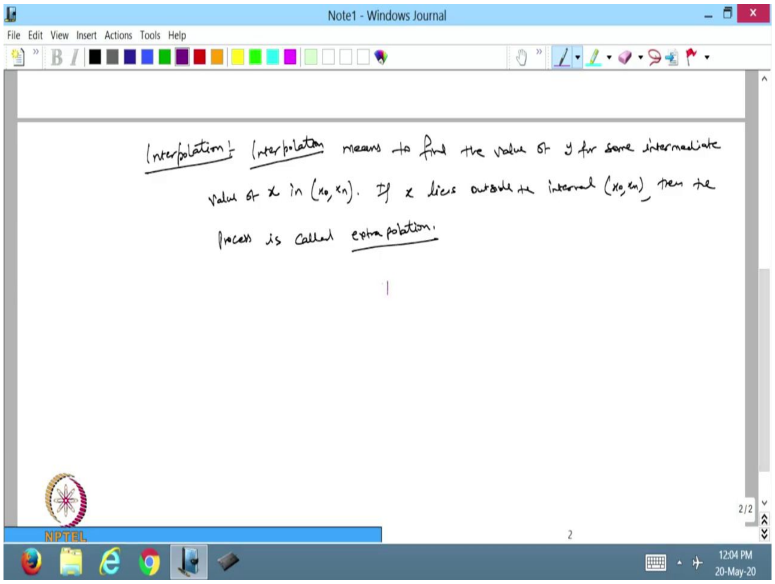


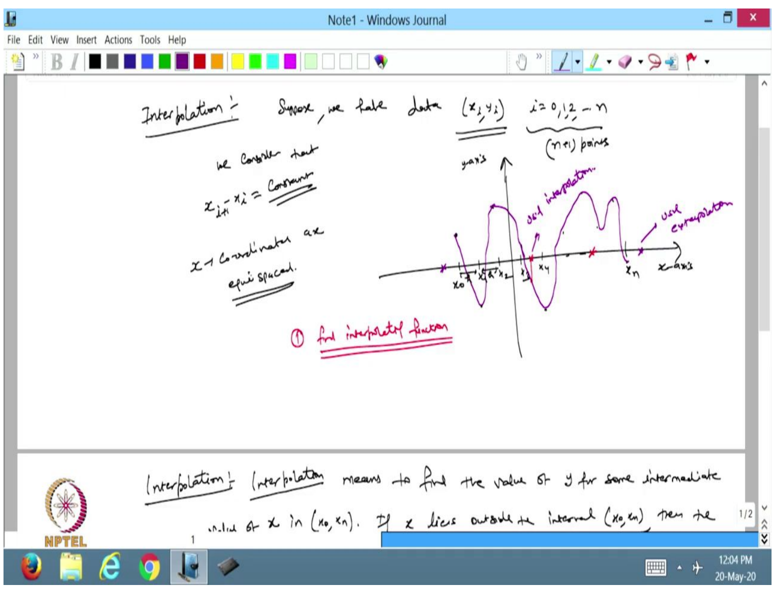


So, I can write the definition so interpolation, so interpolation means to find the value of y for some intermediate value of x, so that is given to me in x0 to xn, if x lies outside the interval that is x0 and xn then the process is called extrapolation. So, in that case we will call it extrapolation otherwise this is the interpolation, so in the extrapolation basically this is the function given to me and somebody asks me what will be the value of the function for this x?

So, this x is outside this one or somewhere here, so in that case if I want to approximate the value so this will be done by using extrapolation and in between this is it is done by using interpolation.

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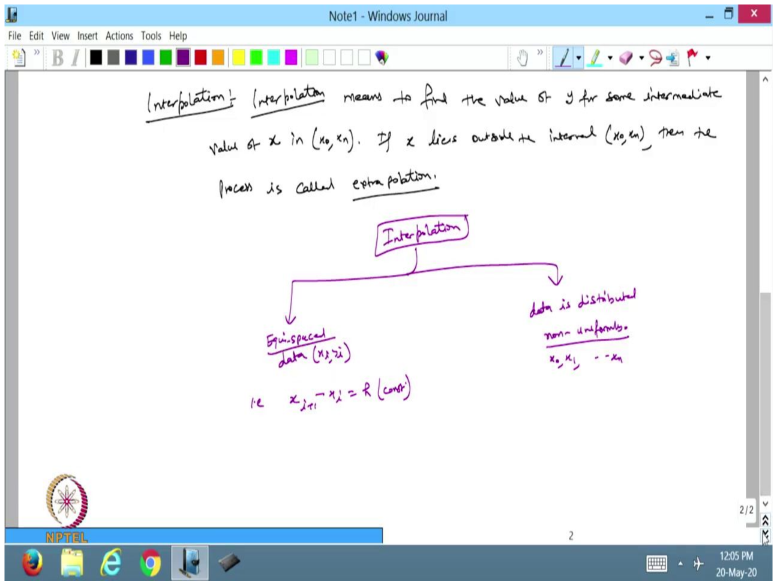

So, in this case, so now our topic is the interpolation, so interpolation is done for two type of data is given to me based on this one so this is the equispaced data that is xi and yi, so this is given to me but this is equispaced so that is that if I take any two xi's the difference of this one, so this will be h and that will be constant.

So, this is the equispaced data and another data is non equispaced or the data is distributed non uniformly. So, in this case we have the data that is given to me only in the form of x0, x1, xn and the difference between these two values is not a constant, it is varying. So, in that case we have data that is non uniform distributed and this is the data which is given to me that is equispaced.

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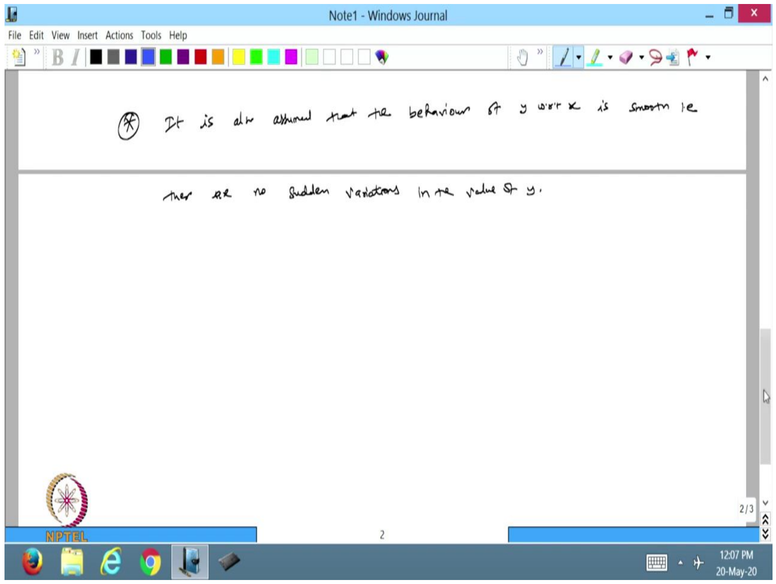


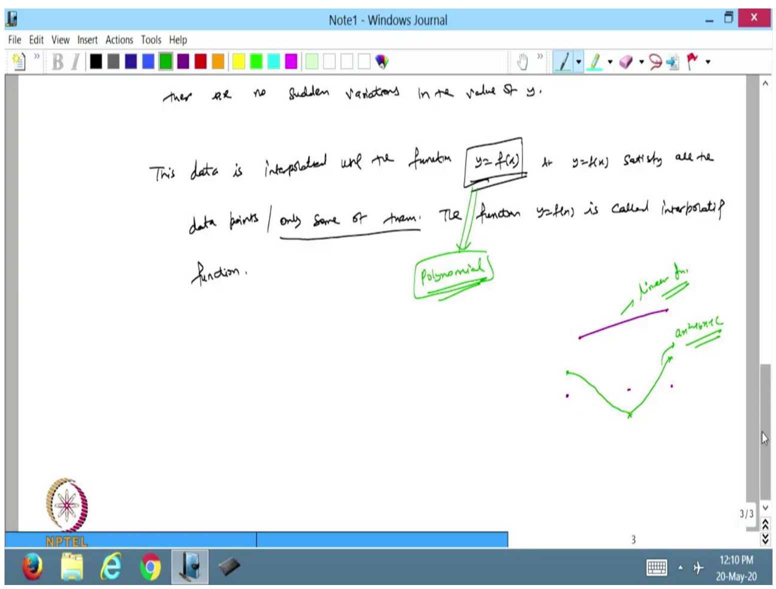

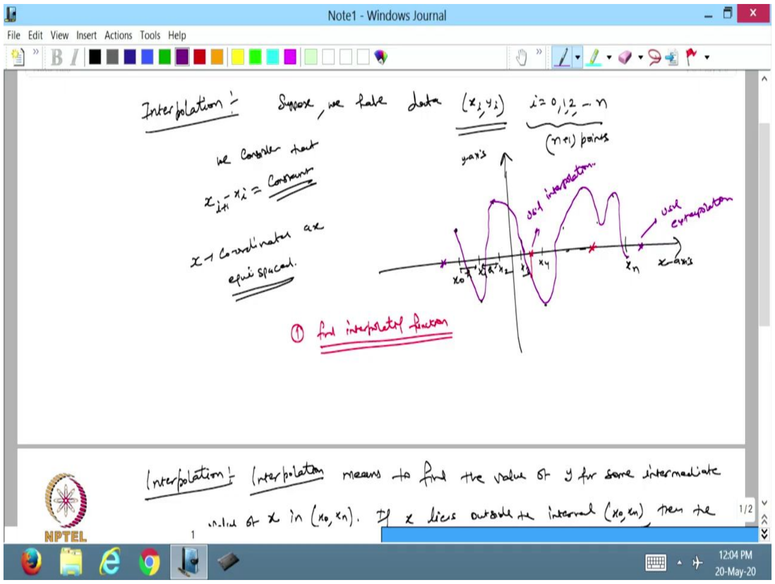

So, for the, so in this case one is, an option is also made that it is also assumed that the behavior of y with respect to x is smooth that is there are no sudden variations in the value of y, so that is the condition given to me that like in this case if you see that I say that the, the given data is smooth it means that function is changing very smoothly from one point to another.

And in this case I am not getting a point like this one, that this type of data is not there like this, so sudden change in the data so this is not there in this case, so that is not there. So, we are taking the assumption that the given data is given to me, the given data is smooth enough so that there is no sudden change in the data anywhere in the given domain. So, that is the condition for this one.

Now so, from here based on this one, now the condition is that I want to approximate so this data is interpolated using the function $y = f(x)$, so this is the, I will going to introduce the interpolation using the function $y = f(x)$, such that that $y = f(x)$ satisfy all the data points or I can say only some of them.

In the interpolation it is passing through all the points but I am also going to introduce the other method, that is curve fitting, so in that case only few of them will be satisfied by the given function. So, this function $y = f(x)$ is called interpolating function, so I want to find this interpolating function, so like suppose I have two points and I want to find a functions which is passing through these two points so, I know that based on two points I can find a line, that is line passing through these two points.

So, in this case I have two points then I can interpolate by a linear function having two points, this point and this passing through these points suppose I have three points, one point is given to me here, one is here, one is here and suppose I have three points the value is here, here and here so based on this one I can find a quadratic function.

So, this is a function of type $ax^2 + bx + c$, this is a linear function it means that based on the given function I want to approximate or I want to find a function $y = f(x)$ so this f(x) is a polynomial. So, the simplest one is the polynomial in this unit or in this course we will find, try to find out first how a polynomial can be approximated for a given number of mesh points, for a given number of data points. So, that is we are going to have this one.

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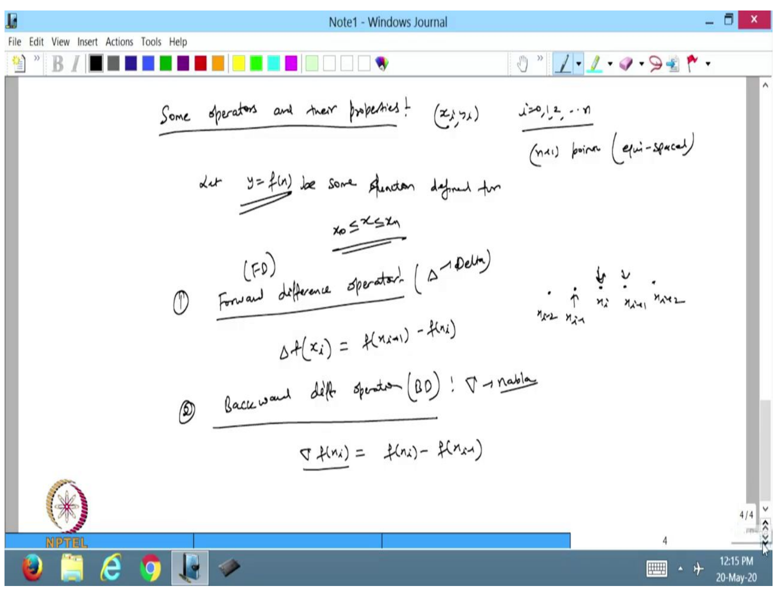


Now, so based on this one I will going to introduce some operators before starting the interpolation, so let us discuss some operators, some operators and their properties, so the only thing we have to keep in mind that we have a data, so this data is given to me xi and yi and i is moving from 0, 1, 2 up to n, so this is n+1 points and I am also considering that this is equispaced it means now I want to introduce, so let $y = f(x)$ be some function defined for all $x_0 \leq x \leq x_n$.

So, suppose, the given points are given to me and I find approximate a function $y = f(x)$ which is passing through all this point and then it is also true for all $x_0 \leq x \leq x_n$. So, that is given to me, now with this one first I will find out the forward difference operator.

So, forward difference operator means that suppose I have a point here, another point is here, another is here and I apply the operator here, then I will get the, I will use the information, this information or this information so ahead of this information I need to use.

So, this is called the forward difference operator and this is represented by delta so what is this? If I apply the forward difference operator or the function $f(x_i)$, so this will be $\Delta f(x_i) = f(x_{i+1}) - f(x_i)$, so in this case I want to apply this finite difference operator on any xi, so suppose this is my xi this is $x_{i+1}$, this is $x_{i-1}$, next is $x_{i+2}$, next is $x_{i-2}$ and I apply this operator at this point.

So, in this case I am using the value of the function here minus this, so it is giving me the difference of these values. So, that is called the forward difference operator, it is giving you the difference and forward means it is using the value forward of that one, so after that one. So, that is called the forward difference operator, the second one we are going to use is the backward difference operator.

So, the short form is FD so it is BD and it is represented by the inverted triangle and this is called nebla, what is this one? Suppose I want to apply this at function value at any xi so it is, what is doing? It is going to use the value before that one, so backward that is why, so in this case I apply this one so this will be equal to $f(x_i) - f(x_{i-1})$.

So, in this case if I want to apply this operator on any function at xi, so it will use the value before that one and it gives the difference between $f(x_i) - f(x_{i-1})$. So, it will use the value just before that one, so that is called the backward difference operator.

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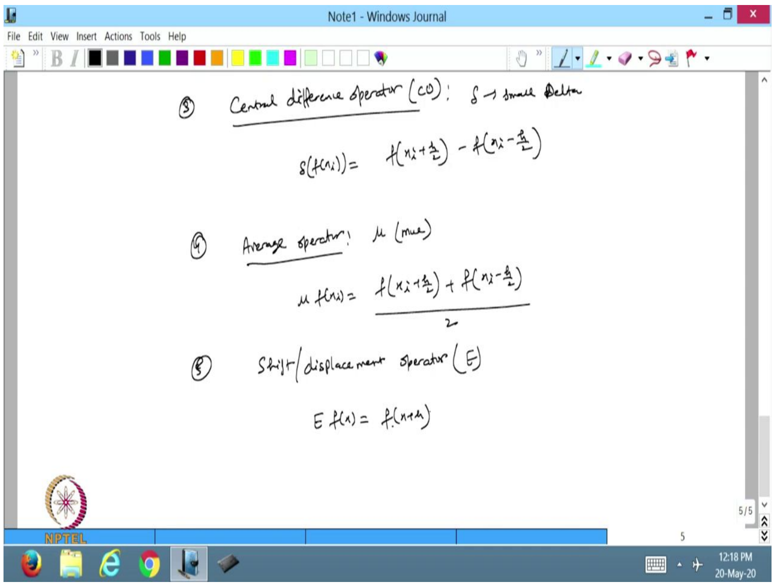

Third one we are going to use is the central difference operator, that is CD so this is represented by a small delta, so what is the small data? If I apply this one on any $f(x_i)$ so it will give you $\delta f(x_i) = f(x_i + \frac{h}{2}) - f(x_i - \frac{h}{2})$. So, in this case the value is given to me at this value and if I suppose apply here then it will use the value somewhere in between this here and somewhere here. So, it will go one forward and one backward so that is why it is called the central, so this is the half it is going forward and half it is going backward and giving you the difference so that is why it is called the central difference.

Next is the fourth one that is average operator and this is sometimes represented by $\mu$, so if I apply here on the function at xi so it will just give the average $\mu f(x_i) = \dfrac{f(x_i + \frac{h}{2}) + f(x_i - \frac{h}{2})}{2}$. So, it will give you the forward by half, backward by half adding and dividing by 2. So, that is why it is called the average operator.

So, the next one is the fifth one, so fifth one is we call it shift or displacement operator and we call it, represented by E, so in this case if I apply E on fx then it will give $Ef(x) = f(x+h)$, so it has shifted the function by its next value so that is x+h so that is why it is called the shifted operator.

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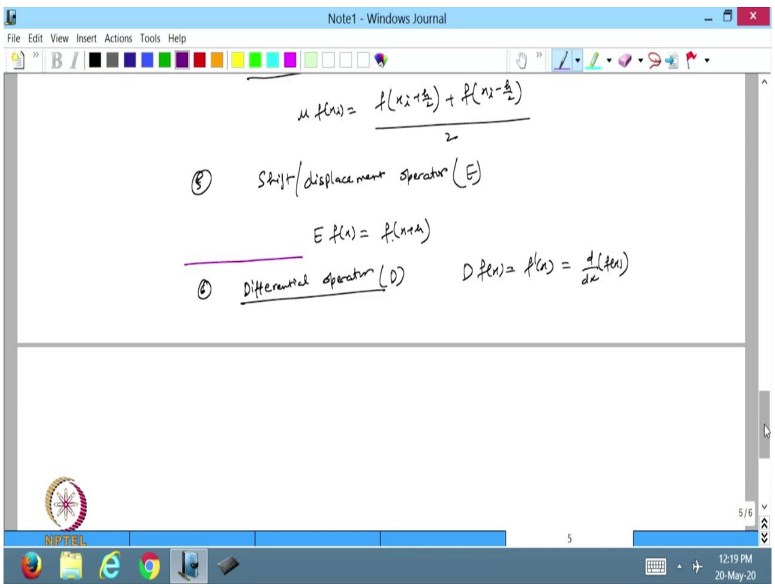


And the next one is the sixth one, so the sixth one is that we already know that is called the differential operator. So, the differential operator we have to apply on the function and that is printed by capital D, so $Df(x) = f'(x) = \frac{d}{dx}f(x)$. So, the differential operator is applied only for the function that is a differentiable function but before that whatever we have used that we are applying on the given data so that is why this all is called.

So, before that one up to the fifth they are all called the difference operator because they are applying on the data and giving you the difference, the last one is the differential operator that is only given for the function that is a differentiable function. So, that is the few operators we have introduced here. Now, and we will use this operator at the different, different levels to find out the interpolating polynomials.

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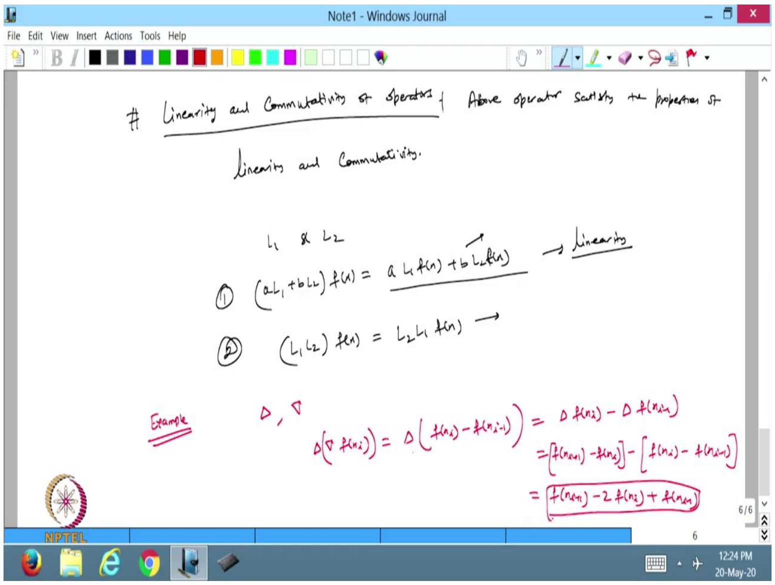


Now we will introduce another concept of Linearity and Commutativity of Operators, it means that whatever the operator we have defined so if they are the linear operator it means that so above operators so satisfy the properties of linearity and commodity, so what is the meaning of linearity? Linearity means that I have two operators supposed to be represented by L1 and L2.

So, it may be a forward operator, central operator, shift operator and suppose I had two operator and then I apply this multiplied by some constant L1 plus multiplied by some constant L2 and if I apply on this function $f(x)$ then I will get

$(aL_1 + bL_2)f(x) = aL_1 f(x) + bL_2 f(x)$. So if this is satisfied then we call that operator the linear operator. So, that is the concept of linearity.

And the second one is the Commutative, so commute means that

$(L_1 L_2)f(x) = L_2 L_1 f(x)$ so it does not matter you apply L2 first and then L1 or L1 first L2. So, that is also if this is also satisfied then we say that this operator L1 and L2 are commutative, they can commute and this here the L1 and L2 are called linear.

So above all the operators I have defined they are also satisfying all these properties, so suppose I apply this forward operator and the backward, so it does not matter that you applied forward first and the backward or backward first and the forward. So, this operator is the linear operator, for example, I take forward operator and backward operator, so what I do is that I apply, I take a function fx so I have a function now I want to apply this then this on any fxi.

So, this one I am doing. So, let us find out what will happen? So, first I am applying the backward and then forward, so backward means I will get $f(x_i) - f(x_{i-1})$, so this is my backward difference operator applying on this one, so it gives the value of fx minus the previous value, so that is why they are backward.

Now, I apply the forward so this will be forward applying on fxi minus forward applying on fx minus 1 because this satisfies the linear property, these operators are linear so I can take this one in the separate form. Now, from the forward operator it will be

$f(x_{i+1}) - f(x_i)$. So, this is forward I applied here minus and this will become $f(x_{i-1})$ I am applying, so I will get $f(x_i) - f(x_{i-1})$. So, from here I will get the value $f(x_{i+1}) - 2f(x_i) + f(x_{i-1})$. So, I will get this value based on this operator.

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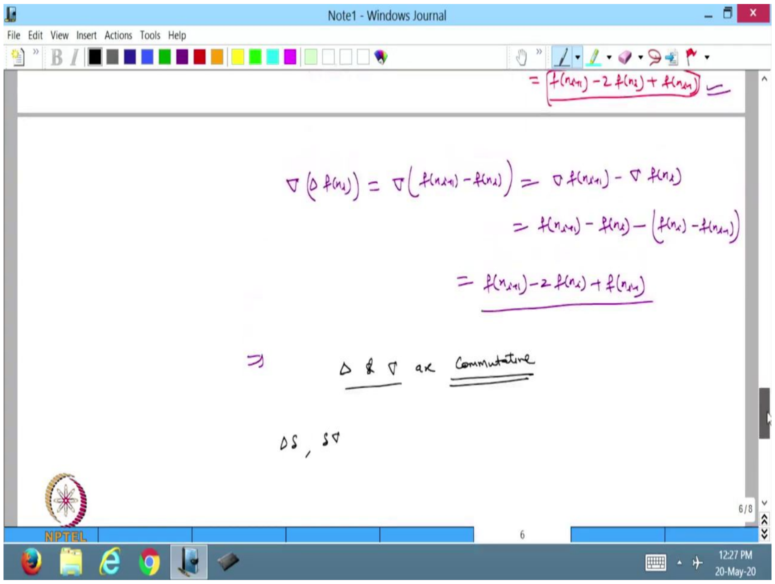

Now, if I change the order of this operator and suppose I apply backward and the forward fxi, let us see what will happen. So, in this case this is backward and the forward so forward will be $f(x_{i+1}) - f(x_i)$, so this is the forward operator and this operator I can take inside so this will be $f(x_{i+1}) - f(x_i)$. Now, I can apply the backward operator for $f(x_{i+1})$ it means I will get the value $f(x_{i+1}) - f(x_i)$, so I am going backward, one step backward then for this one it will be $f(x_i) - f(x_{i-1})$.

So now from here using this one if you see it becomes

$f(x_{i+1}) - 2f(x_i) + f(x_{i-1})$. So, in this case I will get the same value I am getting from here. So, it does not matter that I apply the forward operator first and the backward or the backward operator first forward because I am getting this value. So, from here I can say that forward operator and backward operator are commutative, so they are commutative and linear. We have used this place that the function is linear so I can take inside and that will be applied so from here I can say that these two operators are commutative.

Similarly, I can discuss with the other operators may be forward with the central or central with backward and so on, so that we can do in the examples or in the assignments that show whether these are commutative or not, whoever is satisfying this condition that is called the commutative. So, from here I can say that the forward operator and the backward operator are commutative in nature. So, this is our starting point with the operator, so I will stop here.

So, today we start with the concept of interpolation for the given data that how we can, so in this case we want to interpolate a function that is $y = f(x)$ for the given data to approximate the value of the function at any point in between the data. So that is called the interpolation and we have started with some other basics about how we can define the operators, that is finite difference operators and then we have also discussed the differential operator. So, in the next class, in the next lecture we will continue with this one, so thanks for watching thanks very much.