Scientific Computing Using Matlab
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Lecture 33
Matlab Code for Power Method/ Shifted Inverse Power Method

Hello viewers welcome back to the course on Scientific Computing using Matlab. So, today we will try to make some Matlab code based on the Power method and shifted power method to find out the Eigenvalues of the given matrix. So, in the previous lecture, we have discussed the Gershgorin theorem and the bounds for the Eigenvalue.

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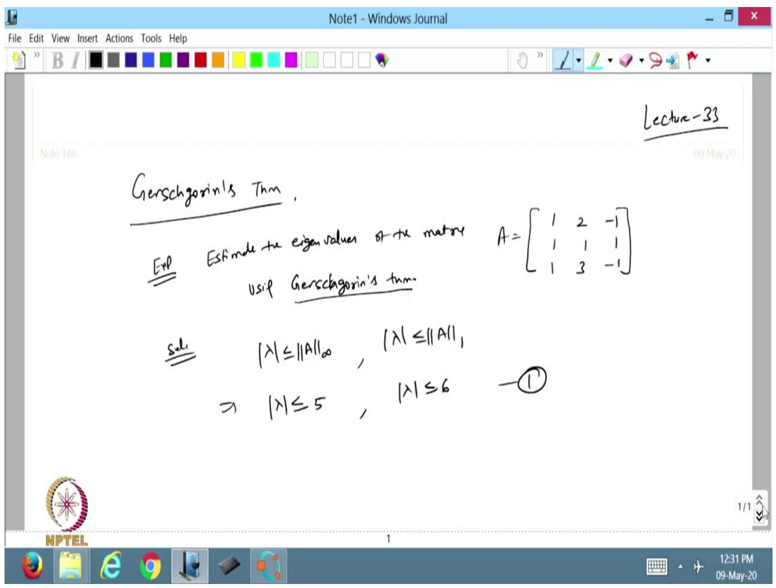

So, in the previous lecture we have discussed that. So, now we used based on this one we will try to find the, so let us do one more example based on this one that estimates Eigenvalues of the matrix 1, 2, -1 and 1, 1, 1 and this is 1, 3, -1. So, in this case we have taken this matrix. Now from here I want to find all the estimated Eigenvalues of the matrix, so I can do this one using the Gershgorin theorem so that is my solution.

Now, if I take lambda less than infinity and lambda norm 1, so from here I can write my lambda will be less than equal to infinity means the row sum, the maximum row sum, so 2 plus 3 plus 4 and this is 3 and this is 5, so from here it is coming 5 and this is my column sum, maximum columns sum it is 3, 6, 3, so it will be 6 so this is my first bounds on the Eigenvalues. So, it is case one.

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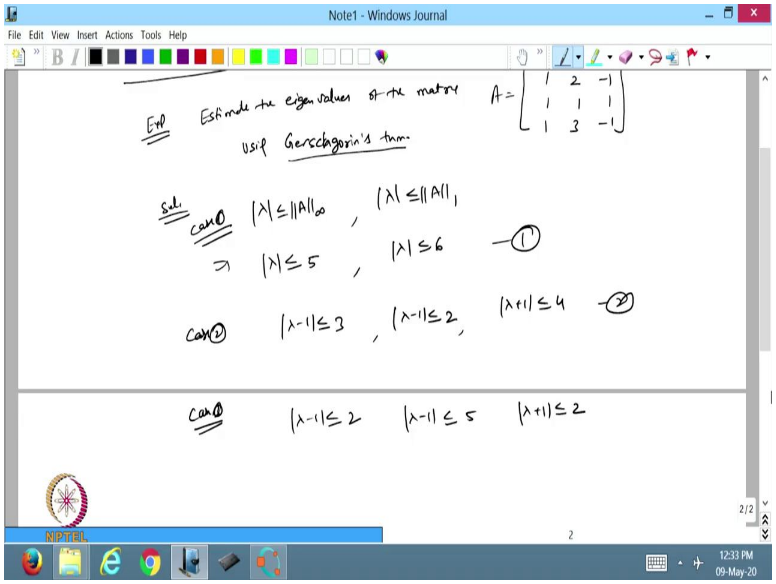

Now if I take case two, so that is it will be $\lambda - 1$less than equal to the sum of the remaining elements. So, 2+1, 3 $|\lambda - 1| \leq 2$ and $|\lambda + 1| \leq 4$. So, that is the bounds on the Eigenvalues, so this is coming from the Gershgorin theorem this is also coming from the Gershgorin theorem and this is also we have seen that this is equal to the norm of the matrix.

And this is a case number three, I am taking the column sum, so from here I can write that my $|\lambda - 1| \leq 2$, $|\lambda - 1| \leq 5$ and $|\lambda + 1| \leq 2$from the last column. So, this is my third bound.

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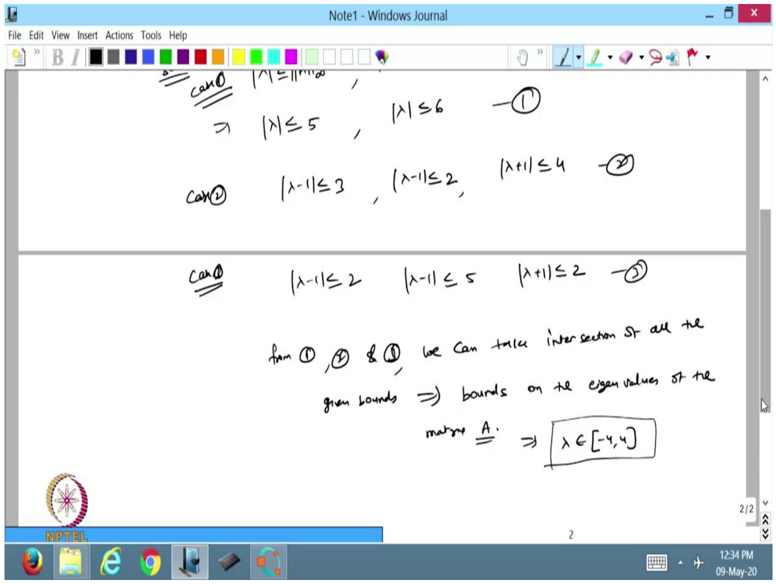

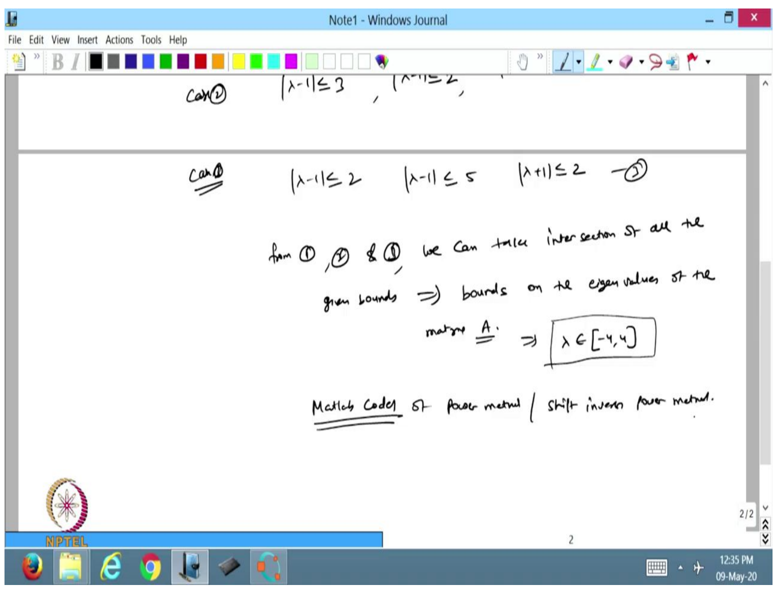

Now, from here, so from 1, 2 and 3 we can take the intersection of all the bounds, of all the given bounds and that will apply the bounds on the Eigenvalues of the matrix. So, based on this one we are able to find the bounds on the Eigenvalues of the matrix A. So, if I take the intersection of A so from here we will get that our $\lambda \in [-4, 4]$ so that is my intersection of that given all these bounds.

Because first it will give me lambda from -5 to 5, then -6 to 6, so from here I will get all the bounds on this one and based on this one I will take the intersection on the left and the right so that gives me this. So, that is the bounds of the Eigenvalues. So, all Eigenvalues of this matrix lie from -4 to 4 and we will verify this one with the codes of the power method and other methods.

So, now we will move to, so we will go for the Matlab codes now of power method and shifted power method, shifted inverse power method. So, let us do the, go to the Matlab.

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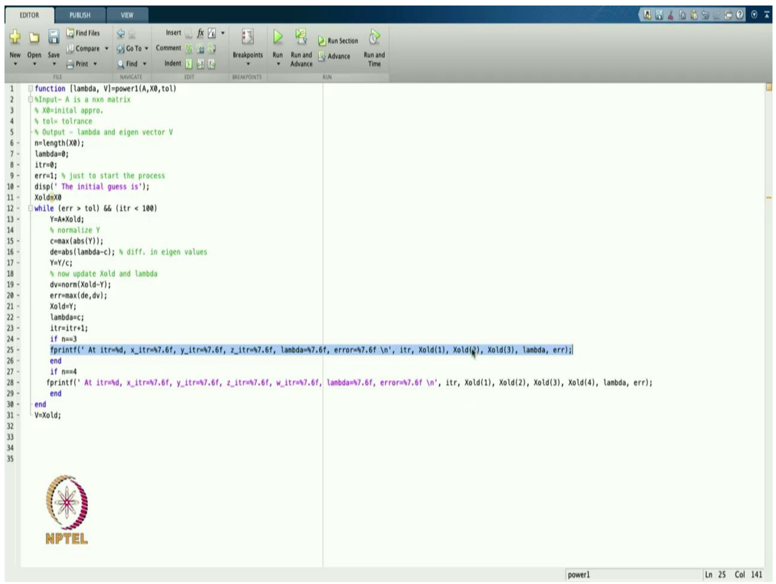

So, let us start with the Matlab code, so if you see this, this is the Matlab code, I have made then this is, I call it the Power 1, so this is the name of the function, so it is the power1 and if I see the arguments of this one so A matrix I will pass to this x_0 is the initial solution or initial Eigenvectors I had pass and this is the tolerance I had to pass and from this routine, subroutine from this function I will get the value of lambda so that is the Eigenvalue, the dominant Eigenvalue and V is that corresponding dominant Eigenvector.

So, what I will do, so let us do this and so I have written here that what is the meaning of these input values and what did output value. Now, let us take $\lambda = 0$ here and then I will start with the iteration, so I am starting with 0 and the error1 I had taken just to start the process.

Now, I will display the initial guess is this X, $xold = x_0$ if you will see that x_0 I am passing to this function, so this will be saved in the Xold and here I am finding the $n = length(x_0)$, so this one I can maybe I can write at the top, so that will give me the what is the dimension of the length of this vector, so that is n because this is the n cross n matrix, so that n, this A will be n cross n.

Now, if I go to the while loop so it will give me while error is greater than tolerance, so I to show you that what is tolerance I will pass from here and the iteration is less than 100, I do not want to do the iteration more than 100 in this case so I will go, if this both are true then it will go inside and I will find $A * xold$, so $A * xold = Y$.

So, that is the Y naught I will get so I call it Y and now what I do is, I want to normalize the vector Y so from here I can, what I do? I take the absolute Y and then I take the maximum values because I have to choose them, the maximum value in magnitude, so that is why first I applied the abs that is the absolute value of Y and then I am taking the maximum and calling it C.

Now, what I do is that, I find the difference between the lambda, that the Eigenvalue I am going to find minus the C, so lambda I have already started from here, so I am finding the lambda and whatever the C I am getting and I am finding the difference, absolute difference between this two, so that I call it de, the difference in Eigen value.

Now, what do I do? I normalized this vectors, so $Y = \frac{Y}{c}$ so I will take the c common from this Y so this is Y by C, so now my Y is normalized and in the lecture we have shown this one with the X1, so this X1 is basically Y over C1, so this is C1.

Now, from here I will take the norm of Xold so whatever the Eigenvector we are getting from the previous iteration and this is Eigenvector from this iteration, I will find the norm, I will take norm from there, so that will call it dv. Now, I showed you that I will find the maximum of these two errors, so de and dv and that is my error.

So, I want that this error should be always whenever it is greater than tolerance. This program will continue otherwise it will stop and it will give me the results. So, from here so after doing this one I will find, I will give the Y to Xold, lambda will be C iteration is incriminated by one, now in the same as in the previous I will show that how this happens, how the accuracy or happens when we move with the iterations or in this case, if I have taken for only two Eigen, two matrices for $n = 3$ and $n = 4$.

So, when $n = 3$ I am printing on the screen at the iterations, what are the iterations number X component, Y component, Z component and this is the Lambda, that Eigen value and this is the error. So, I am writing this one in this order and similarly for $n = 4$ I am finding this order.

So, both the orders we are giving for n is equal to 3 and 4, so once it is keep doing here, then what I will do after once my error becomes less than tolerance or in other case our iteration becomes more than 100. So, this will be false, so I will not move into the while loop and that will be the result, final result $V = Xold$.

So, X old will move to the V, this will go as the output and lambda, whatever the lambda I am getting that will be our output. So, in this case I will input A, X_0, tolerance and I will get lambda and V, so this is we save as the name power1 because I cannot use the power because if you see in the Matlab there is an inbuilt function that is power. So, we cannot use, give the same name as the inbuilt function, so I have given this power1, you can write it power method M, power2 or whatever you want to give.

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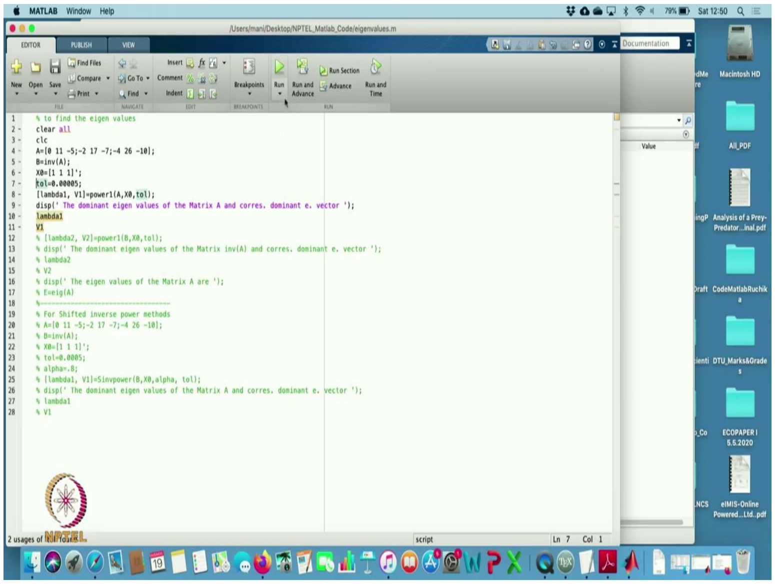

So now, I want run this code, so for this one so input file I have to make, so I have written input file here. So, let us make it this comment and make this uncomment. Let us do this one. Now, this is the input file, so I will use this file name as Eigenvalue dot m. Now, this is clear all, CLC now defines the matrix, what are the matrices we have done in the lectures that are 0,11, -5, -2, 17, -7, -4, 26, -10. So, that the same Eigenvalues we have taken, now what I do? I will apply this matrix, this is my initial approximation.

So, I am finding the initial approximation of the Eigenvectors, I started with the 1,1,1, so in this case you can see that the largest value is 1. So, it is a normalized form, no problem, the tolerance I am giving you 10^{-5}, so that is my tolerance $\frac{1}{2} \times 10^{-5}$, this is my tolerance, now I will call the method power 1 by applying this and then I will display the dominant Eigenvalues of the matrix and the corresponding Eigenvector.

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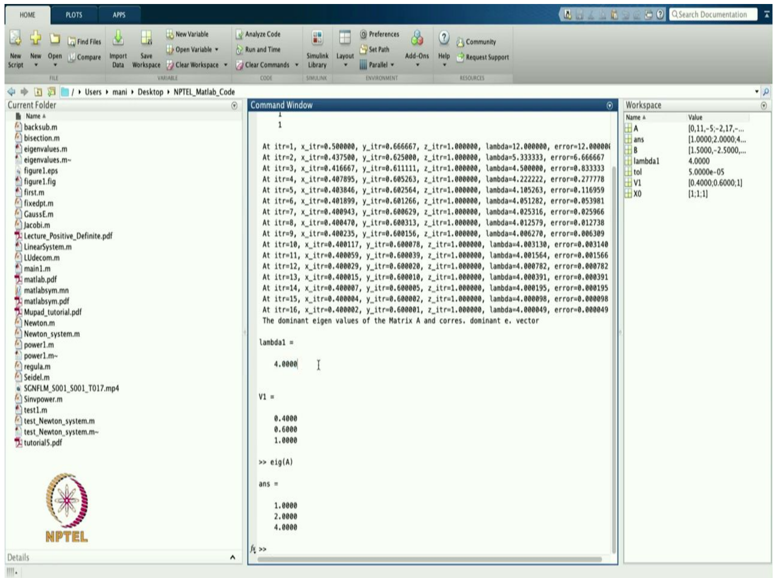

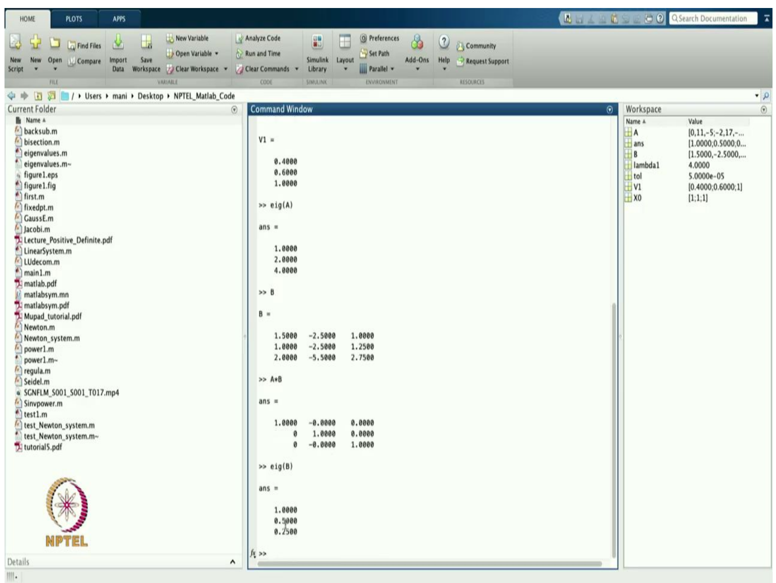

So, let us run this one, so this is the solution I am getting, so I started with the initial guess.1,1, now at the iteration 1 my X component was 0.5, 0.6, 0.1 or 1 and the lambda was 12 that we have already done with pen and paper. So, the error was 12 so next iteration it goes to 0.4, 0.6 and 1 and I will take the lambda outside, so another lambda is coming 5.33, so this is what we also have taken in the previous lecture. So, from here my error goes this and after that if we keep running this code, then after 16 iteration you will see that my X component is 0.4, my Y component is 0.6, Z component is 1 and the lambda is 4.0049 and this error is giving me this value.

So, you can see that even after you can see from here after the 10th iteration you are getting, you are going convergent to your Eigenvalue and that is going to be 4 and that is the corresponding Eigenvector. So, in this case it will get the Eigenvalue that is lambda so that is 4 and this is the corresponding Eigenvector that is 0.4.6 and 1. So, this is my now dominant Eigenvalue.

Now, I want to see what is the Eigenvalue of the given matrix? So, this Eigenvalue I want to find, so from here that you can see that I have the Eigenvalue 1, 2 and 4 so from the power method I am able to find the Eigen value that is 4. So, in this case we are able to do this one. Now what do I want to do?

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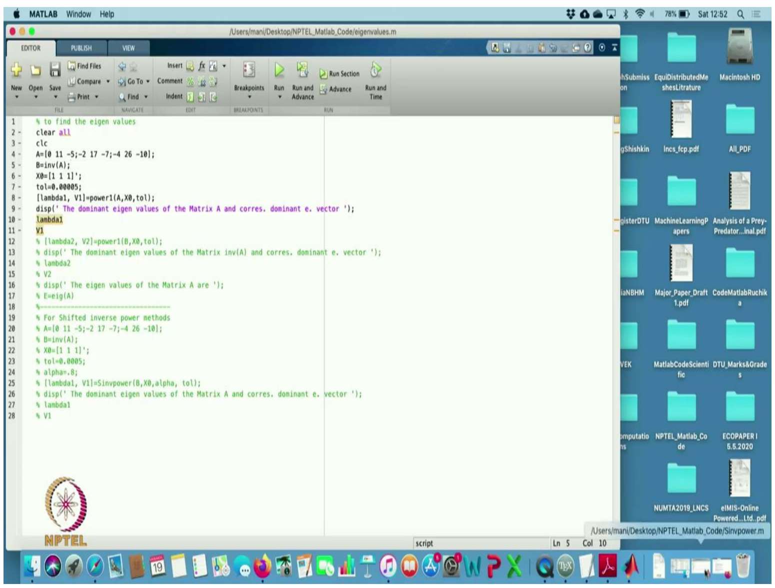

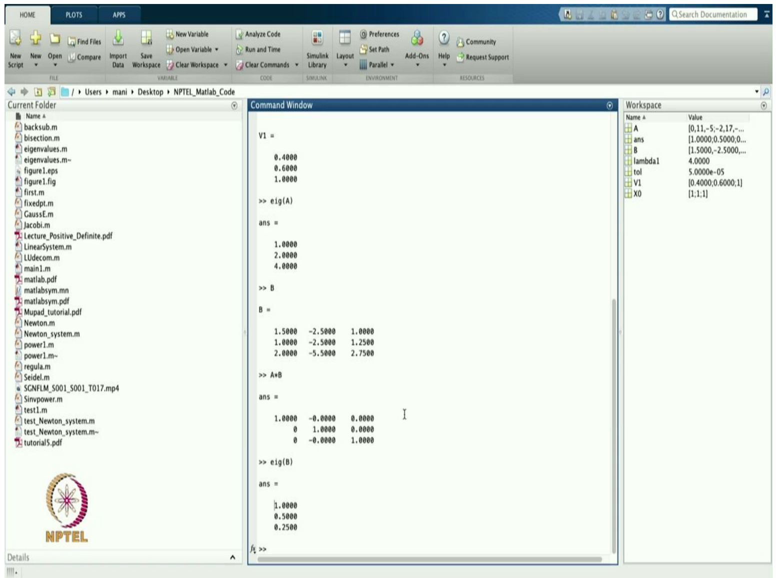

Now, I am able to find the dominant Eigenvalue, so what I want to do? Now, I want to take the inverse of the matrix, so let us take the inverse of the matrix B, so inverse of the matrix A that we call it B. Now what will be the Eigenvalue of B? So, Eigenvalue of B will be, just let us check this one.

So, be it this one, my B is this one, so that is my inverse of A and this one I can check from by multiplying A and B so this is giving the identity matrix. Now if I want to find what is the Eigenvalue of B so that you can see one, earlier Eigenvalues were 1, 2 and 4. So its value is 1, 1 by 2 that is 0.5 and 1 by 4 is 0.25. So now, if I want to find, if I apply the power method to this matrix B, then I should get the value 1. So, let us check whether we are getting the value one or not in this case.

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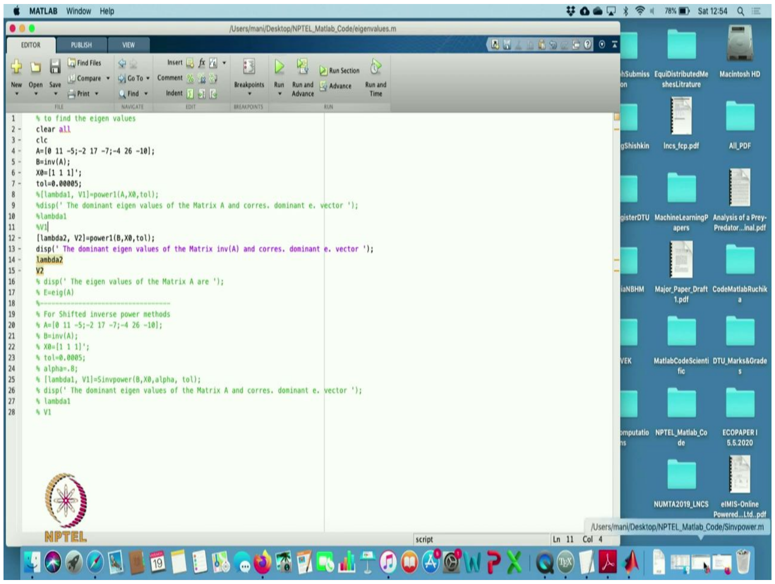

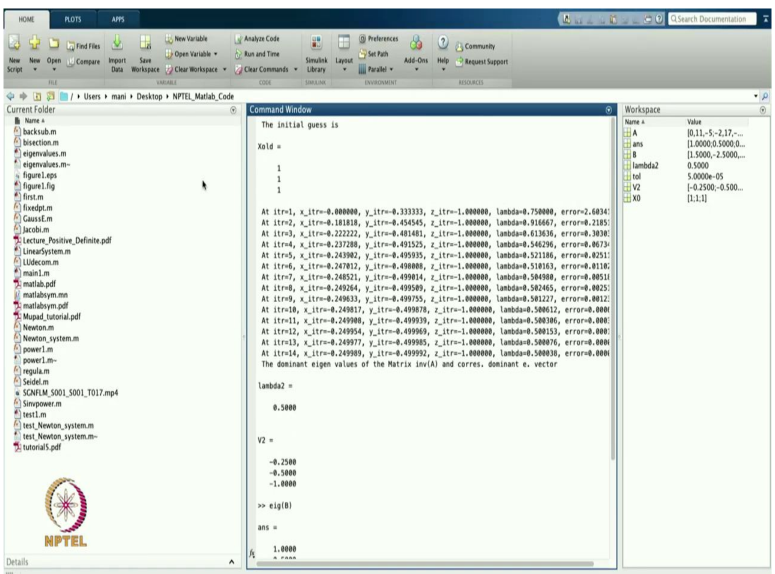

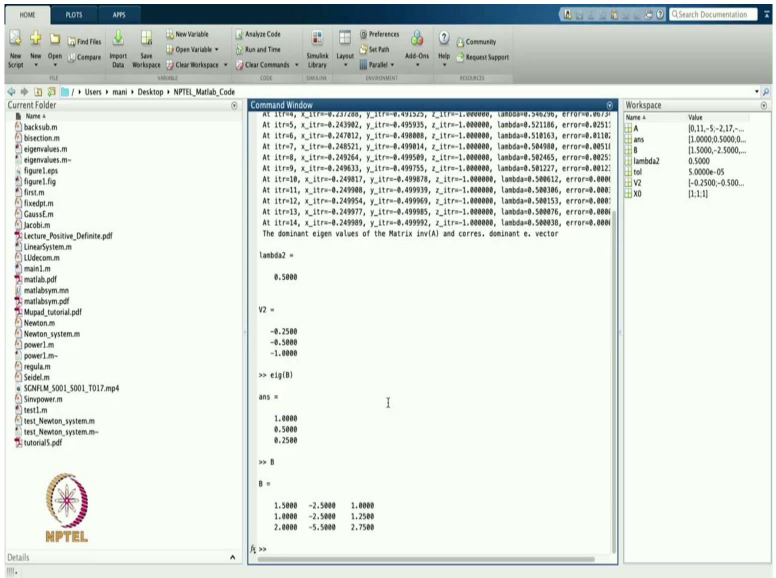

So, let us take this one. Now what I do is, I will just ignore this one and I will try to apply this one. Now, I will call again but now I want to find the Eigenvalue of B so let us try to find out. So, after doing this one now from here you can see that initial guess I have taken the same one, so for B I am getting the value so that is 0.5 I am getting instead of getting.

Instead of getting the value 1 I am getting 0.5 and the corresponding Eigenvector we are getting is this one. So now in this case, what is happening? That it is taking a dominant Eigenvalue is 0.5 not 1 instead of, but why is this happening? So, this is happening because the difference between the Eigenvalues is very small here, you can see that the difference is 0.5, 1 to 0.5 and it is 0.25 so in this case the error may happen that it started with the given matrix, so if I take the matrix B so this in my matrix. So, in this case I am getting the Eigenvalue close to 0.5, so this is the wrong thing we are getting from this method, in the case of inverse.

So, let us try to resolve this one using the help of a shifted inverse power method. So, let us see what will happen in that case, the shifted inverse power method. So let us, now I want to also discuss letting us do the same thing for the other equation, another matrix, so I want to define this for the other matrix.

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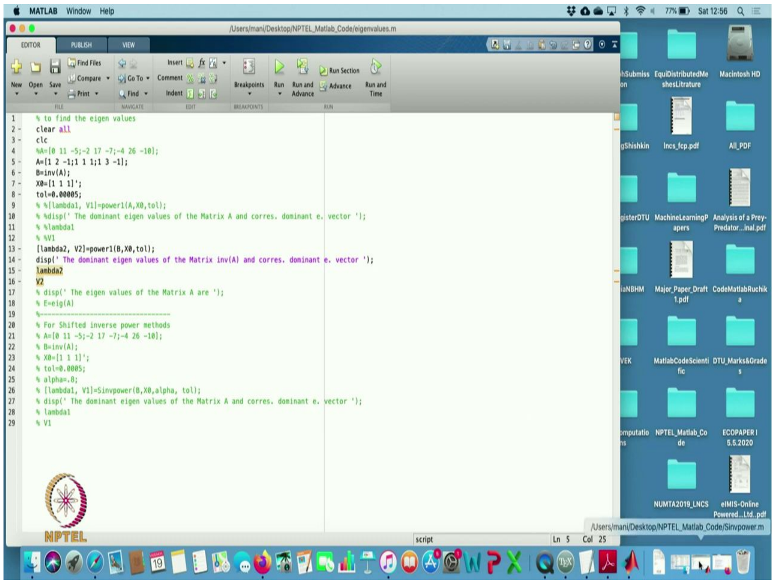

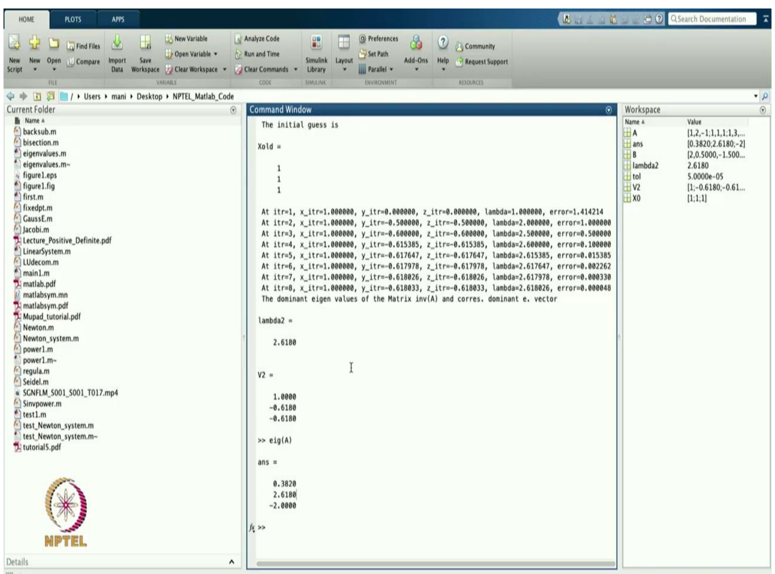

So, let us take the another matrix now, I m taking the same matrix which I have discussed in the example, as example in the lecture, so let us take A is equal to 1, 2, -1, 1, 1, 1, 1, 3, -1, so that is the matrix we are getting. 1, 2, -1, 1, 1, 1 and 1, 3, -1 so let us run this one, keeping the same everything other than that so let us see that what is, so in this case initial guess is 1 and after doing this one you can see from here, the Eigen value in this case is 2.6180 so this is Eigenvalue we are getting and this is a corresponding

Eigenvector 1, 0 and I want to find the Eigenvalue of A so that is 0.3, 2.6 -2 so we are getting the largest one, that is this one.

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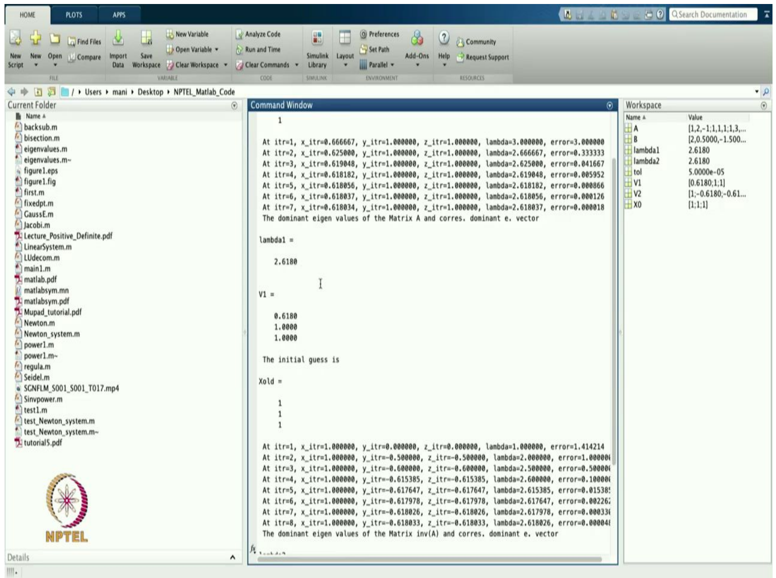

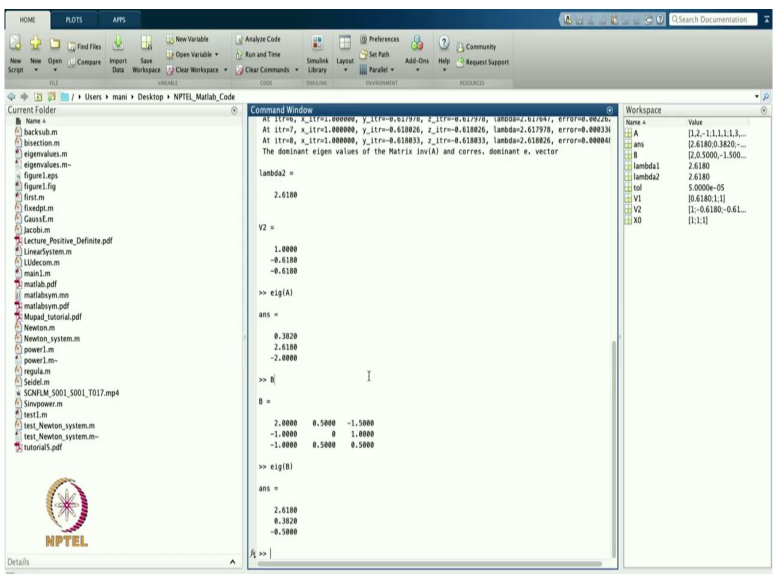

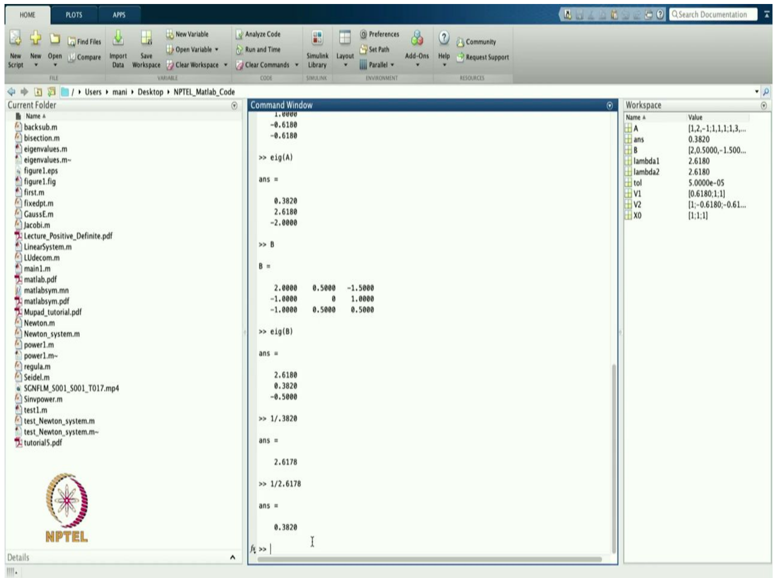

Now, it will come, so let us check, so initial guess is this, the given matrix is this and I am getting 2.66 so that was my corresponding Eigenvector. Now, if I want to find the inverse of the given matrix so finding the inverse I am again getting the value 2.6180 and this is my corresponding Eigenvector, if I want to see Eigenvalues of A, so this is corresponding 2.6 and then if I want to find what is my B, B is this one and Eigen value of capital B, this one.

So, can see from here that the Eigenvalue of B is same, the largest is same so it is 2.61, it is 2.61, 0.3820 3820 and the last two Eigenvalues are the same but the last one is in this case is 2 and in this case is 0.5, 1 by 2 so 0.5. So, in this case it is giving me the exact value, this is the value, Eigenvalue we are getting maximum so that is 2.6 in A also and 2.6 in B also, so it is giving me a good result.

So, from here you can see that why it is giving good result in this case maybe it happened that the difference between the Eigenvalue is quite large, so it is easily to determined that which is the dominant Eigenvalue so here in this case, we are getting the dominant Eigenvalue for A and A inverse.

But the problem is coming in this case is that it is okay with the A matrix, so the A matrix if you see, the dominant Eigenvalue is this and I am getting and in the B the dominant Eigenvalue is again 2.6180 and we are getting so that means if I take the least

Eigenvalue, so least Eigenvalue is this one 0.3820 so from here you will see that if I do 1 by 0.3820 so it is 2.617 so from here you can see that it is 2.6180 and this is 216.6178 so I am getting the value here, it is 2.6180 so that is why in this case whatever the least Eigenvalue was there for matrix A that become the dominant Eigenvalue for A inverse and were able to find that vale also.

So, from here you can say that if I take 1 over 2.6178 so it should be this one 0.328. So, in this case we are giving good results for the matrix A and A inverse. So, from the help of power method we are able to find Eigenvalues that this 0.3820 and 2.6180 so this two Eigenvalue we are able to find, the third Eigenvalue is lying between so this two, this Eigenvalue we are unable to find using the power method.

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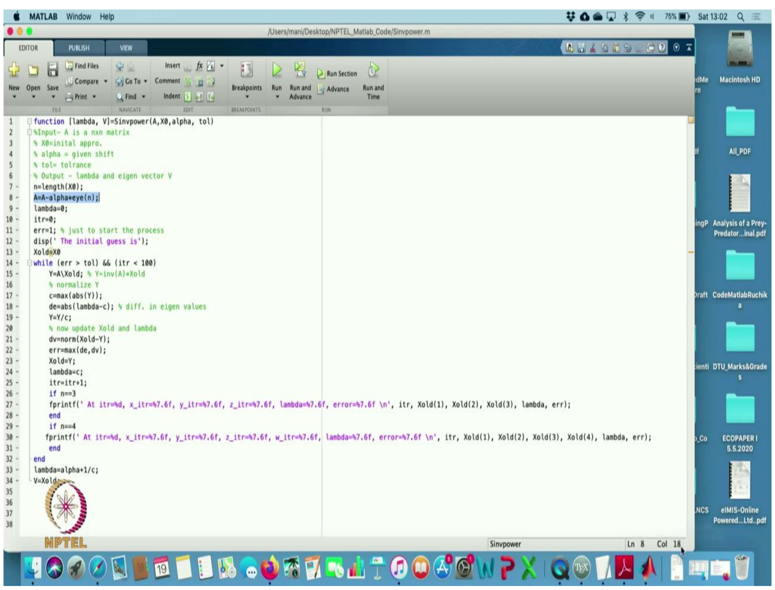

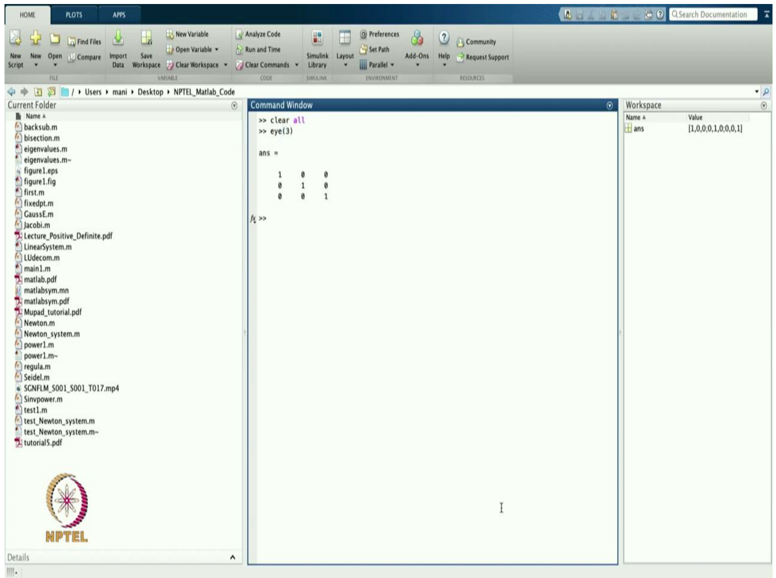

So, let us try to find this one using the in shifted inverse method. So, that is the code I have made for using the, so this is the code for S in power, so that is the shifted inverse power and in this case, also I am giving the input A x_0 and alpha so this alpha we have to give that is the shift in A and that is the given tolerance, so while other things are the same in this case, I will get the output as the lambda and V.

So, let us start with this one n is equal to length, now I defined the matrix

$A = A - alpha * eye(n)$ so we can write in this case as eye(n) so let us write this method CLC clear all. Now, if I write eye(3) so I will get an identity matrix, 3 by 3 identity matrix so this is what we have written there.

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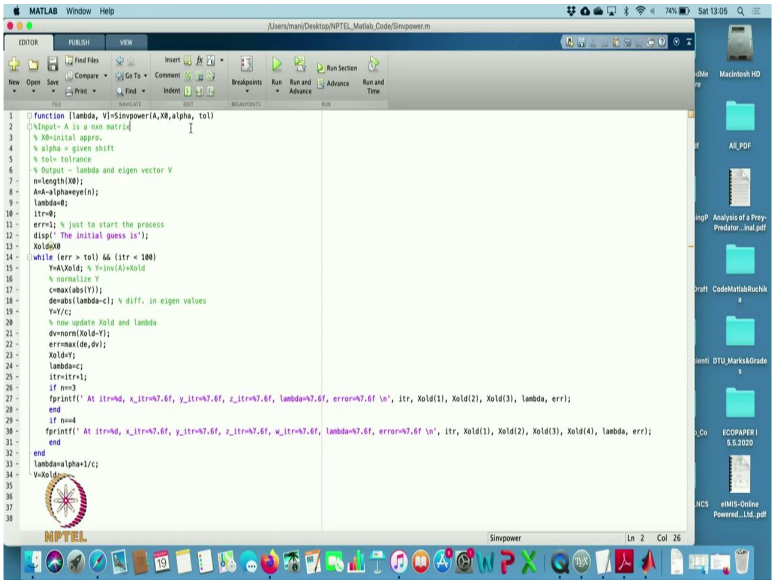

So, we got A minus alpha I, so that is my matrix and I call it again the A and lambda started with 0 iteration 0 error 1 display and then Xold. So, this is the same as a different one. Then we go inside while the error is greater than tolerance and iteration less than 100. So now, whatever doing this we are finding the solution using this. So, A back slash X old so, what is the meaning of this A back slash X old?

So whenever we write A backslash it means this is equals to $A^{-1} * Xold$ because I wanted to find the inverse of this A minus alpha i into Xold so this I have taken as put, so this all values we have taken as A now I am putting the A backslash Xold that is same as $A^{-1} * Xold$, so from here I am finding the value of Y_0.

Now, everything is same, I will normalize the Y with the finding the value c, then I am finding the difference of this c and this is $Y = \frac{Y}{c}$ so here I am dividing by c so I have normalized, then I am finding the error in the Eigenvector so then I find the maximum error, maximum error in both the cases. So, Y, $Xold = Y$, lambda=c and this iteration will be increased and the same thing we are doing.

So, after the while loop is over then I will apply the lambda and you know that the lambda was equal to alpha plus 1 by c so that we have already seen from the previous lecture that my $\lambda = \frac{1}{c} + \alpha$ because c will go to the lambda here. So, c and lambda are the same, so I can write there the lambda, updated value of lambda that is alpha plus 1 by c. So that is the value we are getting and then my X old will go to V and this will be giving you output. So, I am able to find this value. The only thing is that I have to choose what is in my alpha.

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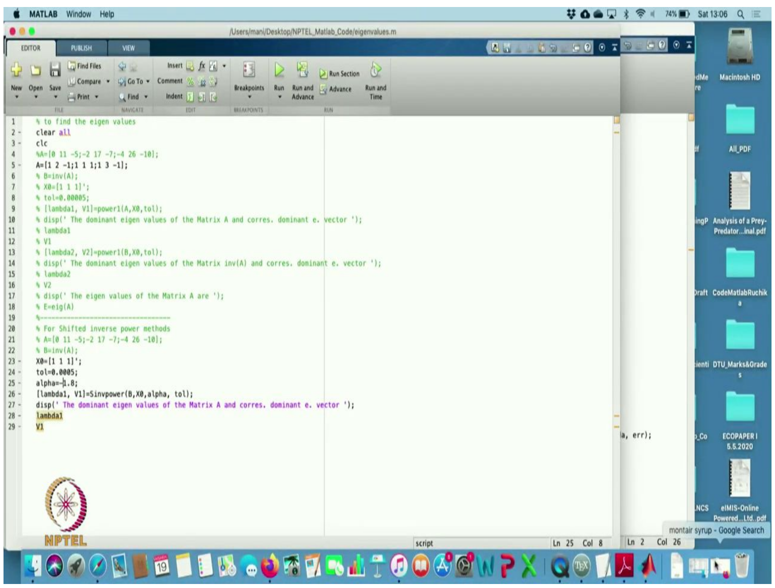

So, let us run this code. So, let us start this one. Now, I know that I will take the first this matrix, so this matrix, if you remember, then I was able to get the maximum value 2.6 and the minimum value 0.3 only in between there was -2.0 value so that I want to find, so let us, I want to find that one using the shifted.

So, this is the value I am getting and this is the Eigenvalue I am getting, matrix A and all this I will comment. Now, I have this matrix X_0 is there, tolerance is there, alpha now I want to find close to 2, so in this case I can take this as 1.8, so let us see what will happen and with the negative sign -1.8.

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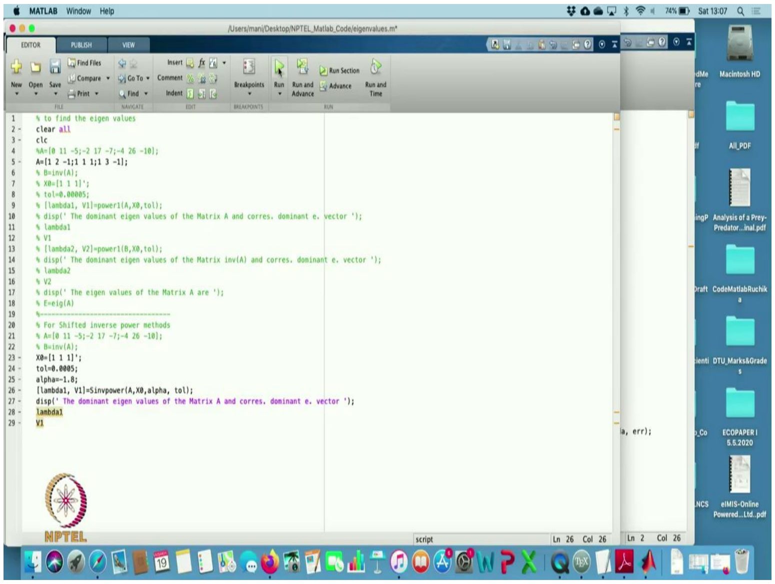

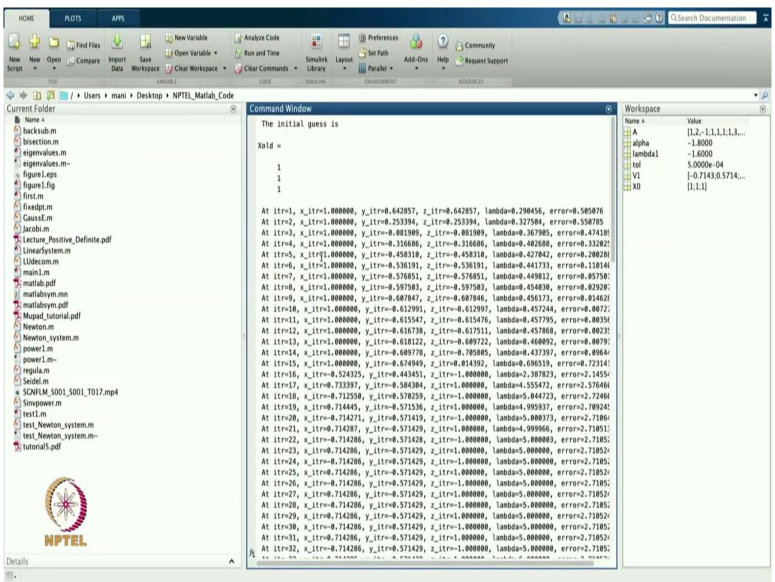

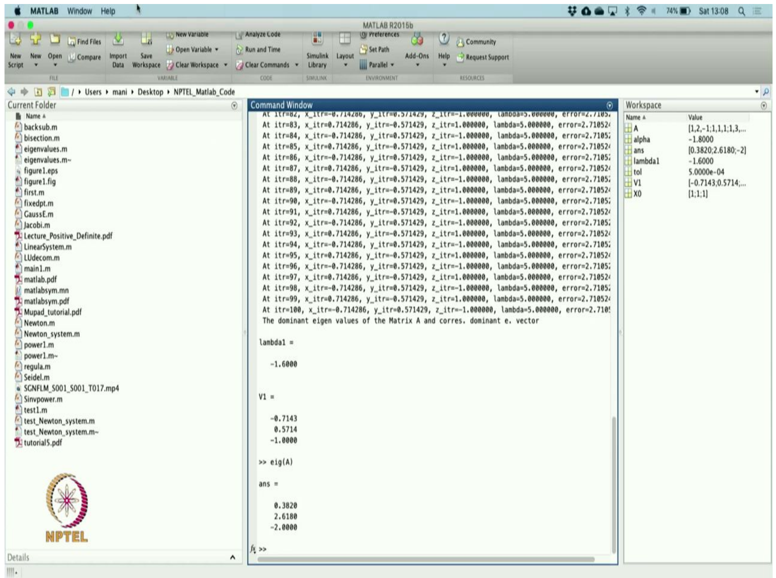

So, let us see what will happen in this case, undefined function variable B, so I have to, where is B? This is a B, this A, not B I mean, now let us see from here, I started with 1,1 and then after 100 iteration we are able to get our lambda is equal to minus 1.6 and this is the corresponding Eigenvalue and I want to find Eigenvalue of A so that is this value 0.3820 to 2.6 and so from here because I have stopped here after 100 iteration so let us try to increase the number of iterations in this case.

So let us make it maybe 200. So, after 200 iterations I am getting this, so I have to go much closer, it means that I have to go much closer to this one, maybe I will find 1.9, 2 let us take this one. Try to run this one so in this case, it is not giving the convergence after even 200 iterations so maybe it will happen that it will give me the solution after more than 200 iterations.

So, let us change the matrix maybe we can take another matrix and let us try to try to find this so I will take the matrix this one and I know that in this case Eigen value as 1, 2 and 4 so I will be able to find the value 4 so let us see what will happen in this case if I want to find Eigen value closed to 0.8 so let us see.

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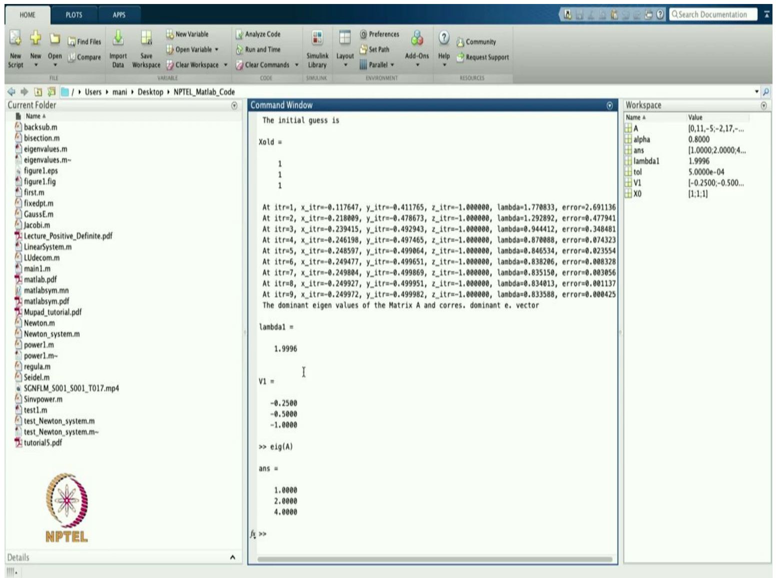

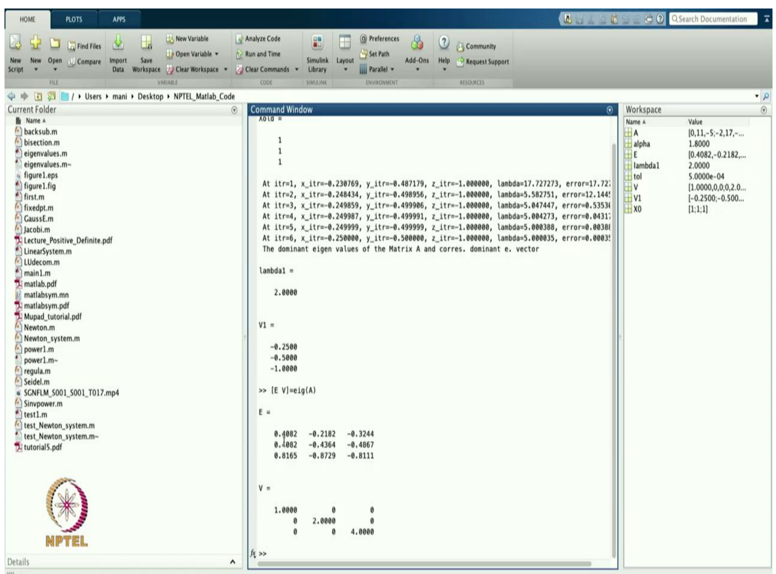

Now, can see from here that in this case after the nine iteration I am getting the value 1, 0.99, so 2. So in this case I'm getting the Eigenvalue closed to 2. So, we have a 1, 2 and 4 so 4 was I was able to find Eigenvalue of, so that is 1, 2 and 4. Now, with the help of this one I am able to get the value that is close to 2.

So, I have taken the alpha=0.8. Let us see what will happen if I take close to 1.8, so in this case also I am getting the value 2 even after the six iteration and that is the corresponding Eigenvector, this one I can find as, from here you can see that this is the corresponding, I have written the, maybe I should take this value and I should make it E, this is V and let us see them, so this is the corresponding Eigenvectors and this is the corresponding Eigenvalue.

So Eigenvalue is 1, 2, 4, so corresponding to 1 the Eigenvector is this, corresponding to 2 Eigenvalue is this so if we take the inbuilt function, use the inbuilt function this is my Eigenvector and we are using the shifted inverse power method then this is the Eigenvector we are getting corresponding to. So, it is -0.21 it is -2.5, it is -0.43, it is 0.5 and this is -0.87, this is closed to -1. So, this is just giving the overview of how we can find the Eigenvalue and the Eigenvector using the shifted inverse power method.

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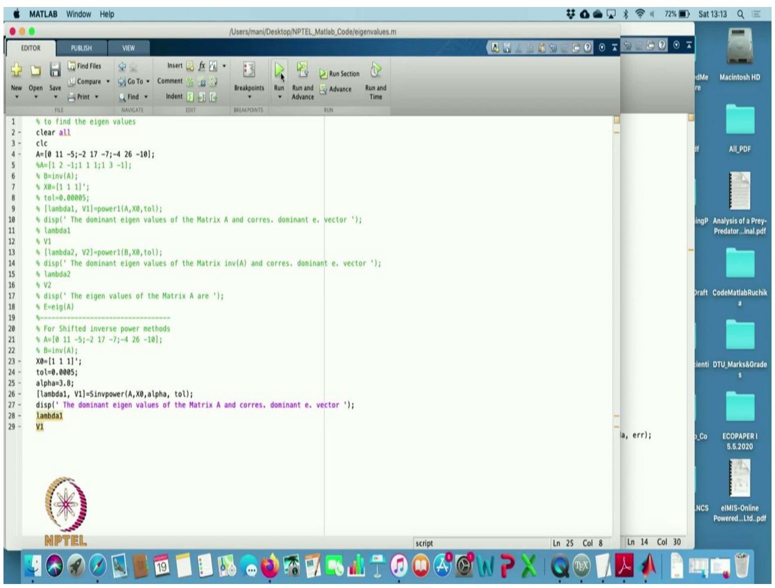

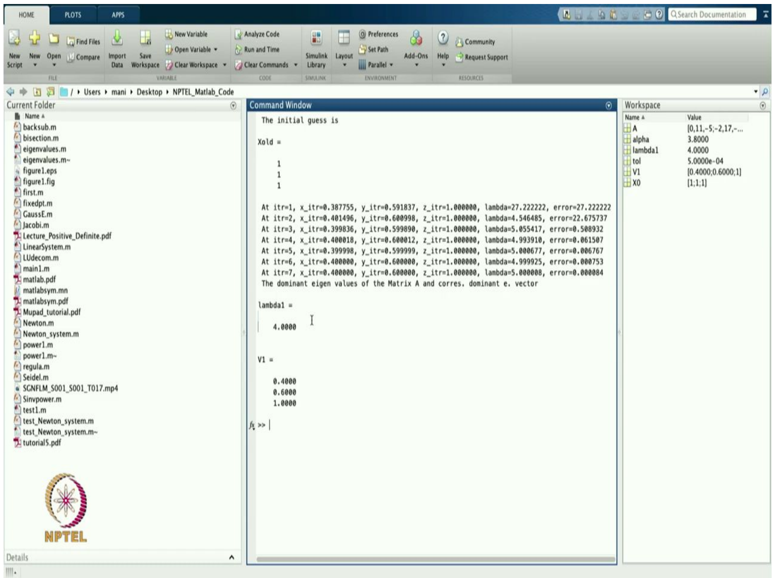

Now, so with the help of this one I may change the tolerance also so in this case I am giving the value of alpha A X_0 alpha and tolerance and I am able to find the values so let us see what will happen if I want to take close to maybe 3.8, let us see, now I am getting the value close to 4 and that is, so this is the dominant Eigenvalue we are able to find.

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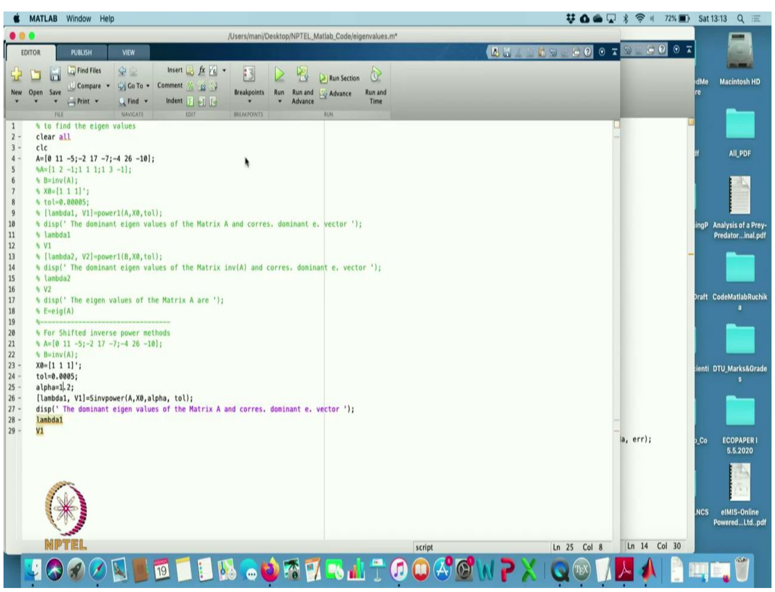

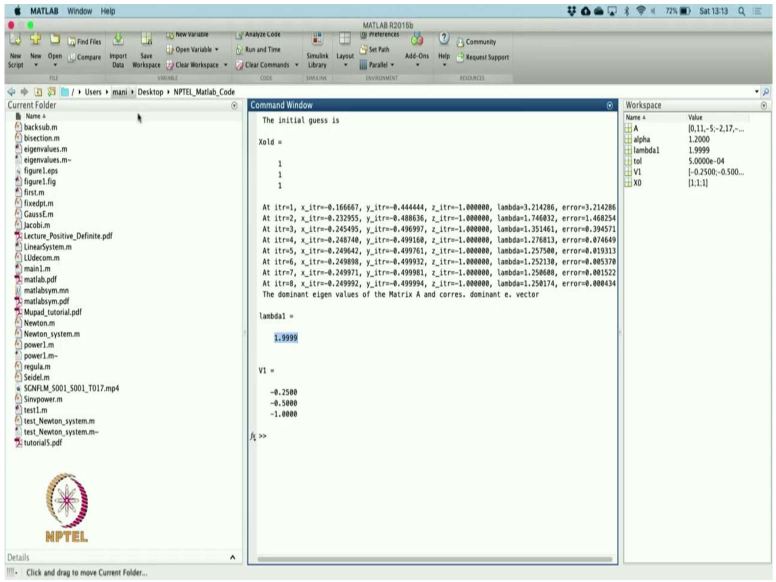

Now, I want to change it a little bit and let us take it to 1.2. Let us see what will happen. Now, I am getting two Eigen values.

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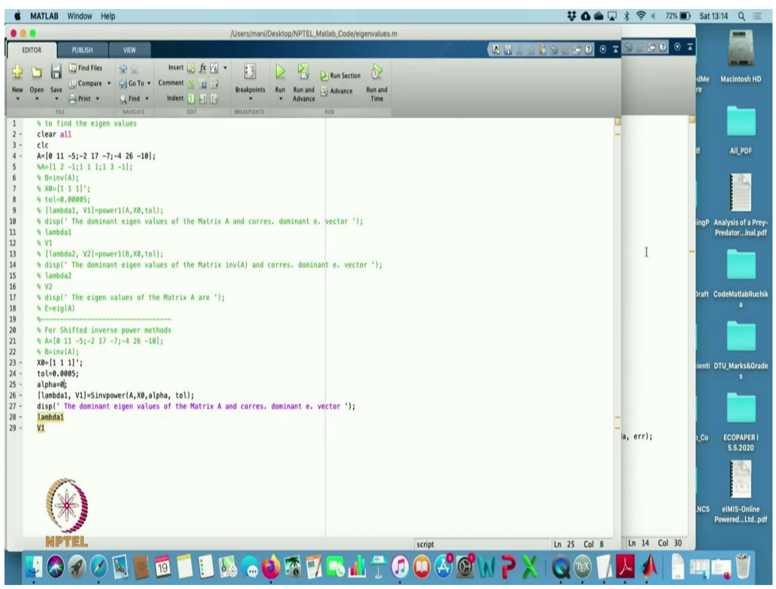

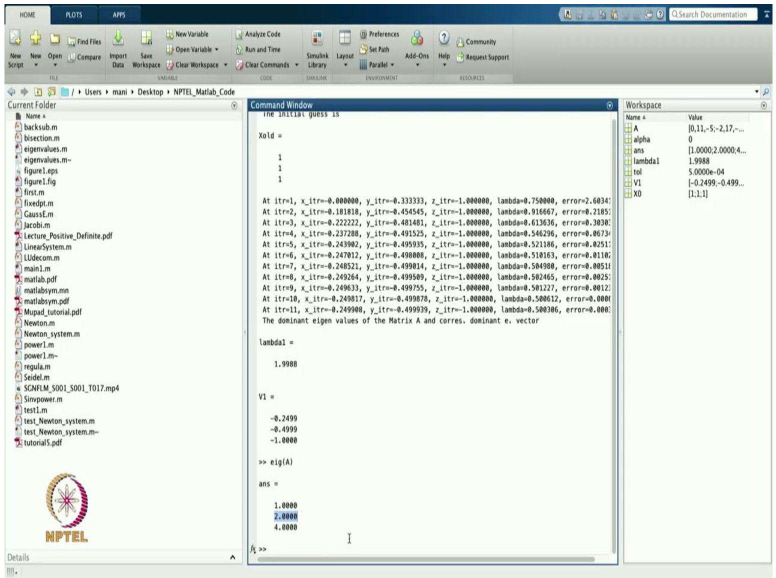

Now, if I choose alpha is equal to 0 so if I take alpha is equal to 0 then it is just a power method. So, from here you can see that which is the inverse power method and the solution is coming 2 because from here I am able to find the value that is this 2. So, with the help of this we are able to find Eigenvalues corresponding to whatever the alpha value we are taking, so now I hope that we should stop here and with this code.

So, today we have started with the example based on the Gershgorin theorem and then we have discussed the Matlab codes for power method and the shifted inverse power method. So, I hope that you have learned how to make the code for power1, power and the shifted power methods. So thanks for watching, thanks very much.