Scientific Computing Using Matlab
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Lecture No. 30
Continued…

So, welcome back to the course on Scientific Computing Using Matlab. So, we will continue from the previous lecture. In the previous lecture, we have started with the power series method. So, we will now continue with that method.

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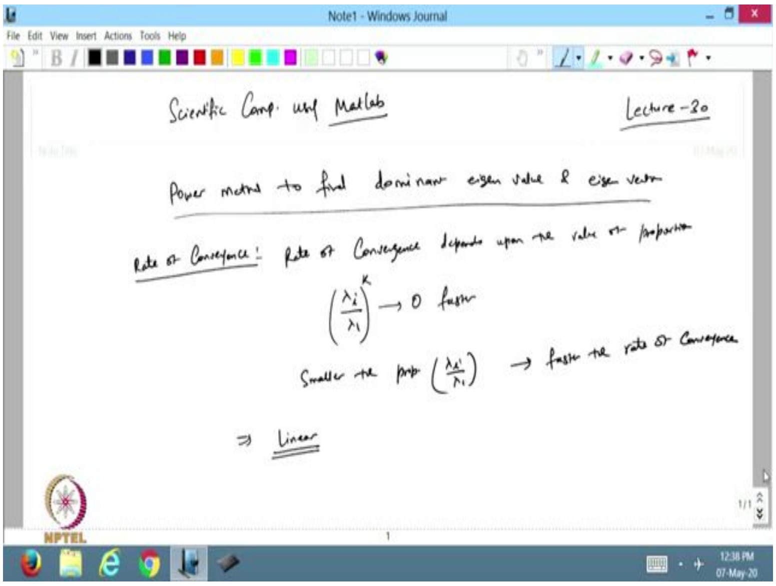

So, in the previous one, we have discussed the power method to find the dominant Eigenvalue and then Eigenvector. Now, it was the iterative method so, from the conversions rate so, the rate of convergence basically depends upon the proportion. So, there is $\frac{\lambda_i}{\lambda_1}$ because, λ_1 be the dominant value and this λ_i is all the other values. So, depends on what is the $\frac{\lambda_i}{\lambda_1}$.

Because, if this value is smaller then factor, the power k will be much smaller so, then it will converge to 0 faster. So, smaller the smaller the proportion $\frac{\lambda_i}{\lambda_1}$, implies faster the rate of convergence. Otherwise it is a rate of convergence that is linear, so the power method has a rate of convergence that is linier. So, this is the way we are able to find that the dominant Eigenvalue. Now, I will find out the inverse power method.

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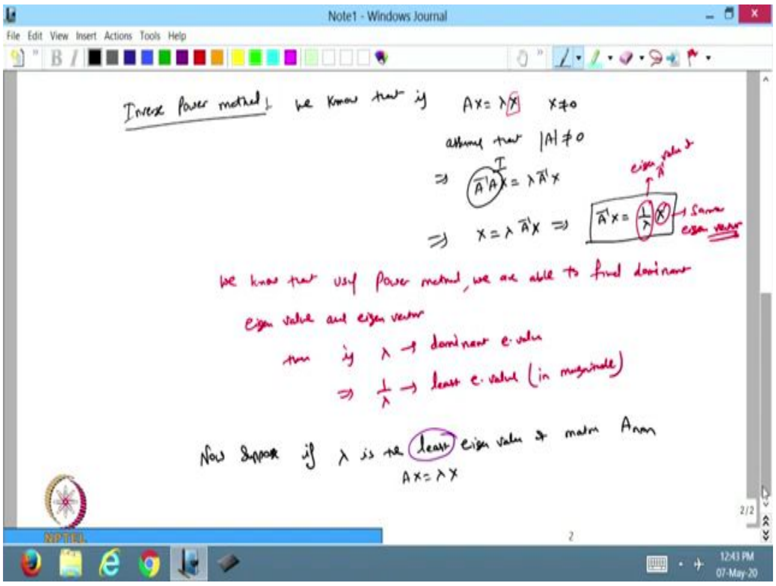

So, what is the inverse power method? Now we know that if $Ax = \lambda x$ where $x \neq 0$, so lambda is the Eigenvalue of this. Then, from here assuming that A is non singular which implies, I can pre multiply by A inverse so, I will get this value, $\lambda A^{-1} x = x$ from here I can write, $A^{-1} x = \frac{1}{\lambda} x$. this one.

So, now from here, I can say that if λ is Eigenvalue matrix A then, $\frac{1}{\lambda}$ is the Eigenvalue of the matrix A^{-1} with the same. So, Eigenvalue of A inverse and this is the same, same Eigenvector. So, the same Eigenvector whatever the, I have started with this one. $\frac{1}{\lambda}$ will be the Eigen value corresponding to A^{-1}. So, this is our same Eigenvector we are choosing.

Now you know that from the power method so, we know that using power method we are able to find dominant Eigenvalue and Eigenvector then if λ is the dominant Eigenvalue then which implies $\frac{1}{\lambda}$ will be so, lambda is the dominant Eigenvalue so, this is the minimum

Eigenvalue so, I can say that this is the least Eigenvalue in magnitude. So, I am talking about the magnitude.

Now, suppose if λ is the smallest or the least, is the least Eigenvalue of matrix A and that is n cross m that is I write $Ax = \lambda x$ so, this is now I am considering that this is the least Eigenvalue. Least or the minimum Eigenvalue in magnitude of course, everything is in terms of magnitude.

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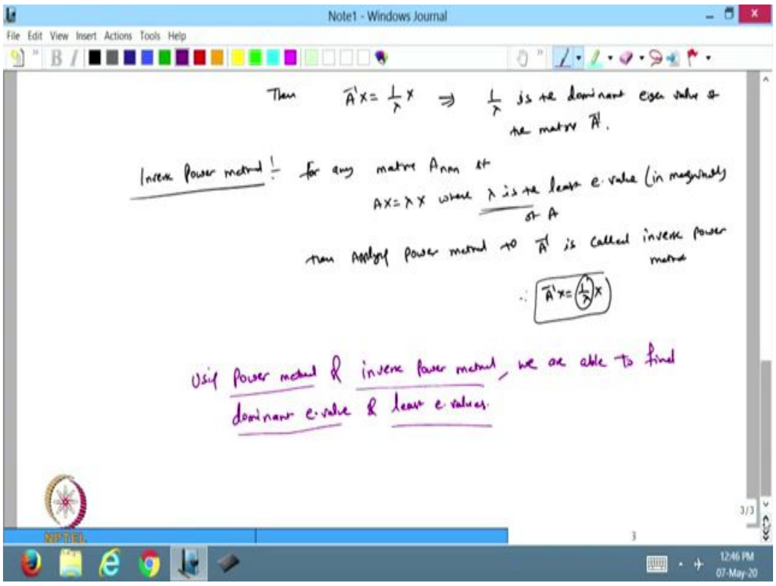

Then, $A^{-1}x = \dfrac{1}{\lambda}x$ which implies that $\dfrac{1}{\lambda}$ is the dominant Eigenvalue of the matrix A^{-1}. So, now from here I can say that now, so, what is the inverse power method? So, inverse power method is that that for any matrix so, this is for any matrix A that is n cross m such that $Ax = \lambda x$ where, λ is the least Eigenvalue in magnitude, the least Eigenvalue of matrix A then, applying power method to A inverse is called inverse power method.

Because, we know that $A^{-1}x = \dfrac{1}{\lambda}x$ so, then it will be the dominant Eigenvalue and I can apply the power method to inverse to find this value. So, from we are able to find 1 over lambda then, using this I can find this lambda that has the least Eigenvalue. So, now from here I can say that using power method and inverse power method we are able to find dominant Eigenvalue and least Eigenvalues and least Eigenvalues.

So, now we are able to find at least two Eigenvalues that are the dominant Eigenvalue and least Eigenvalues. And the corresponding x will be the same in both the cases.

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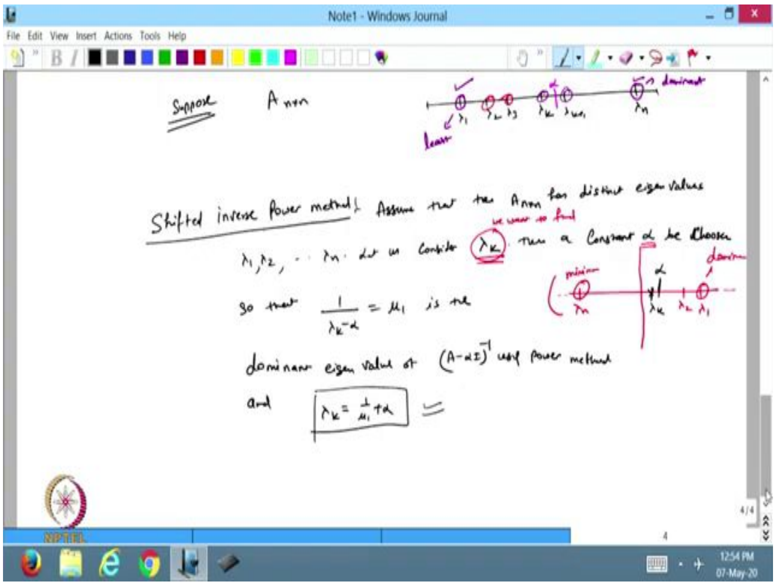

Now, suppose I have matrix A n cross m and suppose, its Eigenvalue is given like this one, this is the range of the Eigenvalues suppose, I have so, let say that we have a Eigenvalues so, this is suppose λ_1 this is $\lambda_2, \lambda_3, \lambda_4$, maybe I can choose this one λ_{k+1} to lambda n. So, suppose I have these Eigenvalues.

Now, I am able to find using the help of power method I may be able to find the least values and the highest values so, this is my least and this is the dominant. So, I may be able to find this and this now what will happen if I want to find in between? Suppose I want to find this value, this value. So, I cannot use my power method, the inverse power method to find the values, Eigen values in between the maximum or the minimum.

So, no what we can do with this one? So, for this one there is another very important method and that is called the shifted inverse power method. Shifted inverse power method, so, in this case what we will do? Suppose I take a α so, let me choose one α here. This value and that is α. Now, what will I do? I try to find the Eigen value close to α so, let us write this one, assume that that the matrix A n cross m has distinct Eigenvalues so, that is I called it $\lambda_1, \lambda_2, \ldots, \lambda_n$ so, let us consider, let us consider my λ_k.

So, this one I want to find then a constant, then the constant α can be chosen so that so let us assume that now I can assume that this is my λ_1, this is my λ_2, and up to here λ_n so, it is supposed I am taking the axis now, so it is supposed to be the x axis and y axis, so this quantity is the largest one, so this is the largest one dominant and this is s the minimum.

Now, from here then we consider, therefore we chose so that now we will have chosen alpha close enough to λ_k which is we want to find, so this one we want to find. Then a constant alpha is chosen such that, I take $\frac{1}{\lambda_k^{-\alpha}}$ and that I call it μ_1. So, let us called it μ_1 so, let us take this as a μ_1. Then what will happen? That then so that this is the dominant Eigenvalue of $(A - \alpha I)^{-1}$ using power method and then λ_k can be written as so, what we will get the λ_k so, I can take this one so, it will be $\lambda_k = \frac{1}{\mu_1} + \alpha$, this should my λ_k so, I can find the λ_k from here.

So, that is the, we are able to find my λ_k. so, that is a shifted inverse power method, so what are we doing? Basically, we are choosing to suppose this is the place and my λ_k is falling at this place now, what do I do? I chose a number alpha closed to λ_k and the I will finding the value $(A - \alpha I)^{-1}$ and using this one I will finding the power method so, power method will be the dominant eigenvalue of this, with the help of this I am able to find the λ_k so, that is called the shifted inverse power method.

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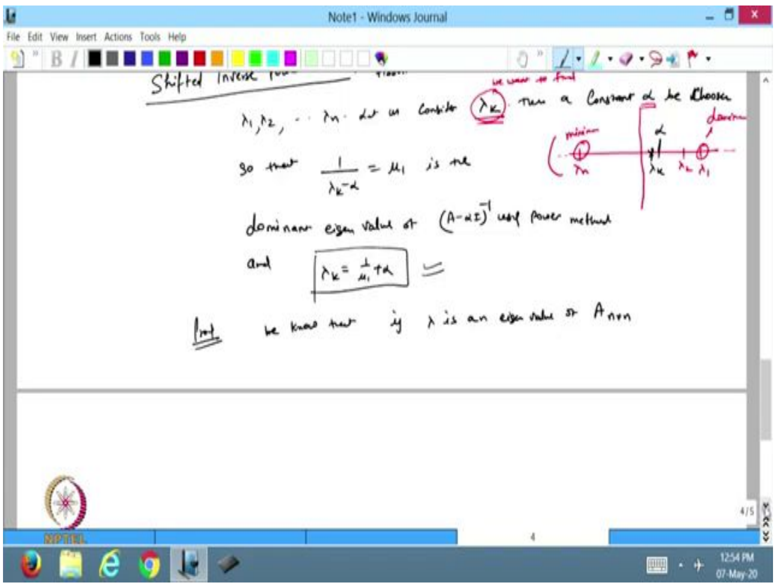

So, what I am doing here is that, so this is the experimentation so, proof is there, I know that we know that if λ is an Eigenvalue of matrix A.

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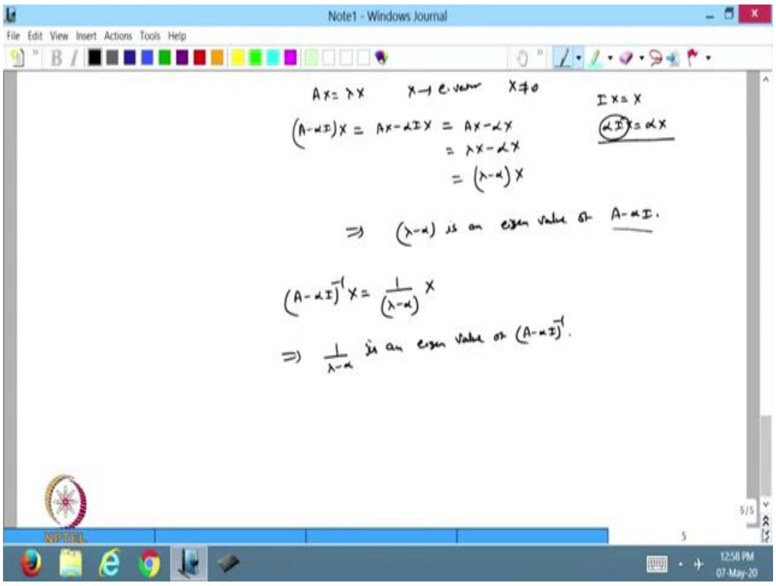

Then I can write as $Ax = \lambda x$ where, x is the corresponding Eigenvector and x is not equal to 0. Now, I also know that what about $(A - \alpha I)x$? This one I want to find, so this will be again Ax minus alpha i x and from here this will be equal to $Ax - \alpha x$ and also know that $Ix = x$ and $\alpha Ix = \alpha x$.

So, the matrix, any matrix αI the constant matrix has the Eigenvalue α so, from here I can say that this one I can write as $Ax - \alpha x = \lambda x - \alpha x = (\lambda - \alpha)x$ so, this can be written like this one. Because this should be a matrix, I should always put it here and then from here, I can choose this i here and then $\lambda - \alpha$ so, x I can now take it from here.

So, from here I can say that $\lambda - \alpha$ is in Eigen value of $A - \alpha I$ so, this is the Eigenvalue we are getting. So, now there is no need to write it here because this is already an axis vector, so αx and this say I can write no problem αx. now from a $\lambda - \alpha$ and x so, this is the Eigenvalue of the matrix this.

Now, from here I know that what about $(A - \alpha I)^{-1}$ and from the previous knowledge we know that $(A - \alpha I)^{-1}$. So, this one can be written as this from here

$(A-\alpha I)^{-1}x = \frac{1}{\lambda - \alpha}x$ so, that will be there. So, from here I can say that $\frac{1}{\lambda - \alpha}$ is in Eigen value of $(A-\alpha I)^{-1}$. So, that is the Eigenvalue.

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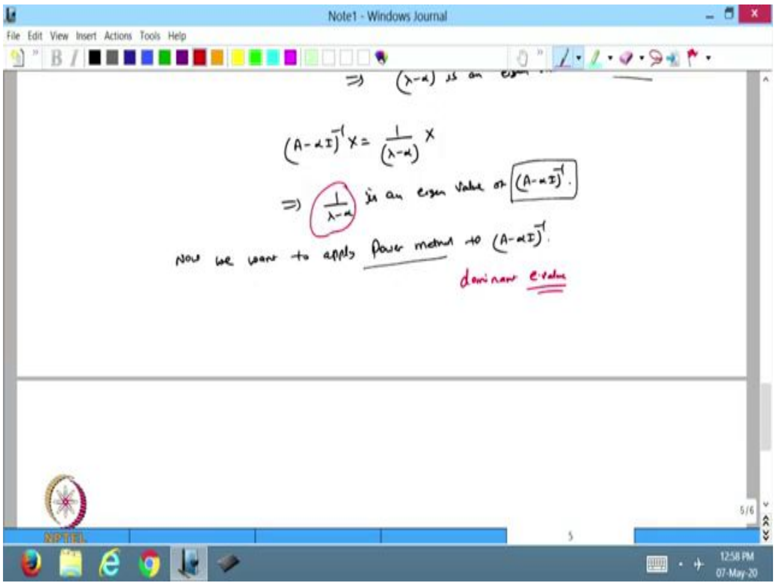

Now, what will happen? Now I want to apply the power method to this matrix. Now, we want to apply the power method to $(A-\alpha I)^{-1}$. Now, the question is that I know the power methods that give me the dominant Eigenvalue. So it is going to give me the dominant Eigenvalue now, how we can say that this is the dominant Eigenvalue so, that is the question.

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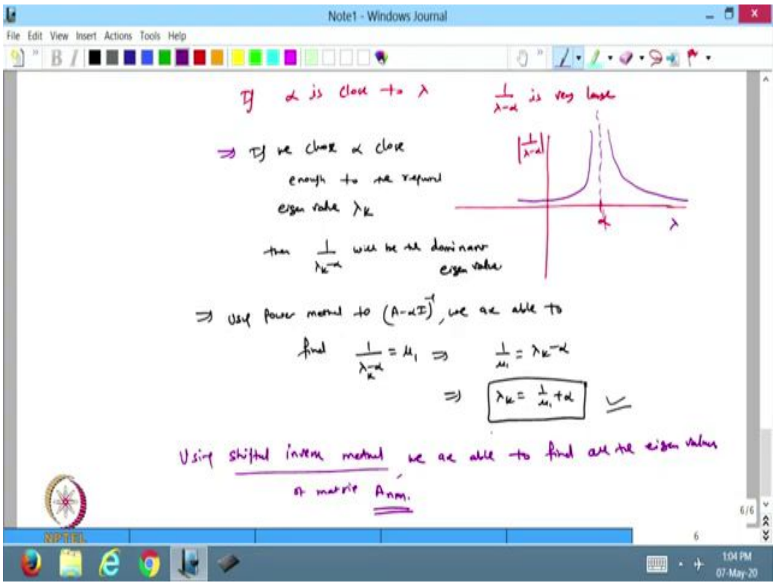

So, in this case we want to that now if α is closed to λ, we are taking this as a real valued the Eigenvalues so, if α is closed to λ then $\frac{1}{\lambda - \alpha}$ is very large in fact, if I want to plot a graph over this one so, suppose this is my alpha so, this is the value of and this is the suppose I take the, I just take the magnitude now, if you see, that this values is going to be like this one and this values is going to, the figure of this is going to be, because at $\frac{1}{\alpha}$ this is going to be infinity otherwise the value is this one. So, this is the value of λ or 1 over these values.

So, when the α is, so when the λ is closed to α, then this is going to be a value infinity because when the $\lambda = \infty$ and in that case otherwise if it is going to be away from this one then this value will be less. So, in that case we can see that this is the graph we can say then using this one so, which implies that, if we choose alpha close enough to the required Eigen value that is supposed in this case it is what we have taken? The λ_k so, this is the λ_k I have chosen, then $\frac{1}{\lambda_k - \alpha}$ will be the dominant Eigenvalue. So, that will be the dominant Eigenvalue.

So, with the help of using power method using power method to $(A - \lambda I)^{-1}$, we are able to find $\frac{1}{\lambda - \alpha}$ so, that is the λ_k I am taking so, suppose this is equal to I can call it what we called it? That is we calling it μ_1, so that is equal to the μ_1 so, this is the Eigenvalue of this so, from here I know that using from there I will get $\frac{1}{\mu_1} = \lambda_k - \alpha$ and from here my $\lambda_k = \frac{1}{\mu_1} + \alpha$.

So, now I am able to find a λ_k, where λ_k is the Eigenvalue of the matrix A. in between so, it may be in between any values. In fact now I can apply this to any values , it may be a least value of the maximum value also does not matter, so in this case I can apply this method to find any value of any Eigenvalue. So, that is the way, this is the way we can define this one and we can find the Eigenvalue. So, using this one, using this method that is called the shifted inverse method. So, using a shifted inverse method we are able to find all the Eigenvalues of the matrix of A matrix that is A n cross m.

So, not only we are able to find the least value or the dominant Eigenvalue but, with the help of shifted inverse method we are able to find any Eigenvalue so, we choose the α that is closed to any Eigenvalue and with the help of this shifted inverse method we will find that Eigen value so, that we will see that how we can use this methods to find the Eigenvalues and in the next lecture or may be after that we will be able to write the Mat lab code.

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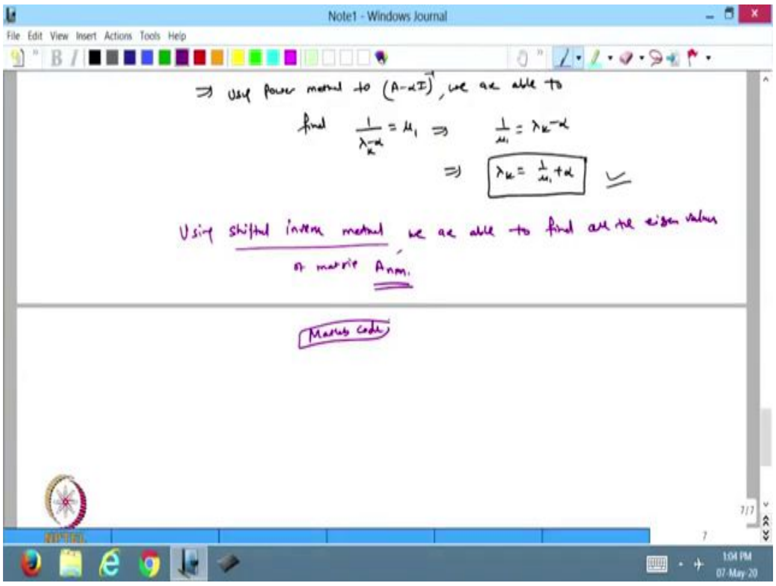

And then we will do that using the Mat lab code we should be able to find the Eigenvalues of any n cross m matrix. So, I think now we should stop here. So, today we have discussed continuing with the power method and then we have discussed the inverse power method and the last one is the shifted inverse power method. So, using the shifted inverse power method we are able to find all the Eigenvalues of the given matrix.

So, in the next lecture we will continue with this one and we will discuss other topics. So, thanks for watching this lecture, thanks very much.