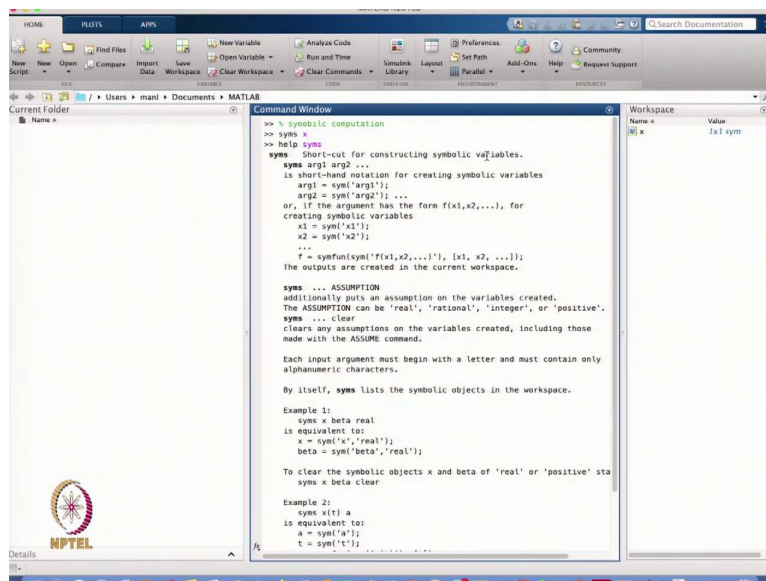


Scientific Computing Using Matlab
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Lecture No. 03
Symbolic Computation in Matlab

Hello viewers, welcome back to this course on scientific computing using MATLAB. So, this is the third lecture. So, in the previous two lectures we have started with the basics of the MATLAB. And now today we will talk about that how, because in the previous lectures we have done the numerical computation in the MATLAB. Today we will talk about how we can do the symbolic computation in the MATLAB. So, let us start with the MATLAB.

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```
>> \ symbolic computation
>> syms x
>> help syms
syms arg1 arg2 ...
is short-hand notation for creating symbolic variables
arg1 = sym('arg1');
arg2 = sym('arg2'); ...
or, if the argument has the form f(x1,x2,...), for
creating symbolic variables
x1 = sym('x1');
x2 = sym('x2');
...
f = symfun(sym('f(x1,x2,...)'), [x1, x2, ...]);
The outputs are created in the current workspace.

syms ... ASSUMPTION
additionally puts an assumption on the variables created.
The ASSUMPTION can be 'real', 'rational', 'integer', or 'positive'.
syms ... clear
clears any assumptions on the variables created, including those
made with the ASSUME command.

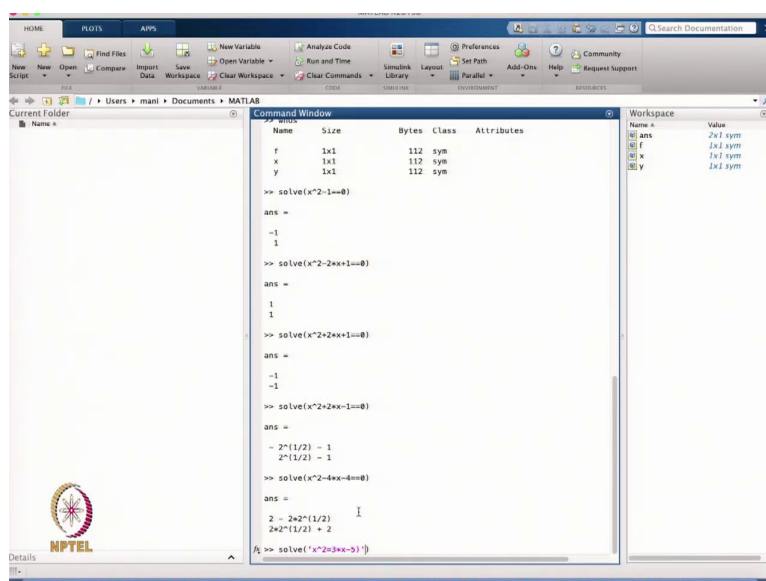
Each input argument must begin with a letter and must contain only
alphanumeric characters.

By itself, syms lists the symbolic objects in the workspace.

Example 1:
syms x beta real
is equivalent to:
x = sym('x','real');
beta = sym('beta','real');

To clear the symbolic objects x and beta of 'real' or 'positive' sta
syms x beta clear

Example 2:
syms x(t) a
is equivalent to:
a = sym('a');
t = sym('t');
```



```
>> syms
Name      Size      Bytes  Class  Attributes
f         1x1         112    sym
x         1x1         112    sym
y         1x1         112    sym

>> solve(x^2-1==0)
ans =
-1
1

>> solve(x^2-2*x+1==0)
ans =
1
1

>> solve(x^2+2*x+1==0)
ans =
-1
-1

>> solve(x^2+2*x-1==0)
ans =
-2^(1/2) - 1
2^(1/2) - 1

>> solve(x^2-4*x-4==0)
ans =
2 - 2*2^(1/2)
2*2^(1/2) + 2

>> solve('x^2+3*x-5')
```

So, today we will today talk about the symbolic computation. So, symbolic computation means that, now we can define variables as we usually do whenever we are doing the analysis or in the mathematics books. For example, I define, so this is the symbolic computation I will define, so syms means the symbolic, so help syms. So, it gives you that the syms is the shortcut of constructing symbolic variables. Now, I am defining that my variable x is symbolic.

So, similarly I can define the different variables like this one, here. So, I can define, so I have defined the variable x, now suppose I write $y = x^2$, so this is another variable and this variable is also in the symbolic form. Symbolic means $y = x^2$, because we know that in the mathematics we always write $y = x^2$ is a function, $y = \sin x$ is a function. Suppose, I define $y = \sin x$ is another function.

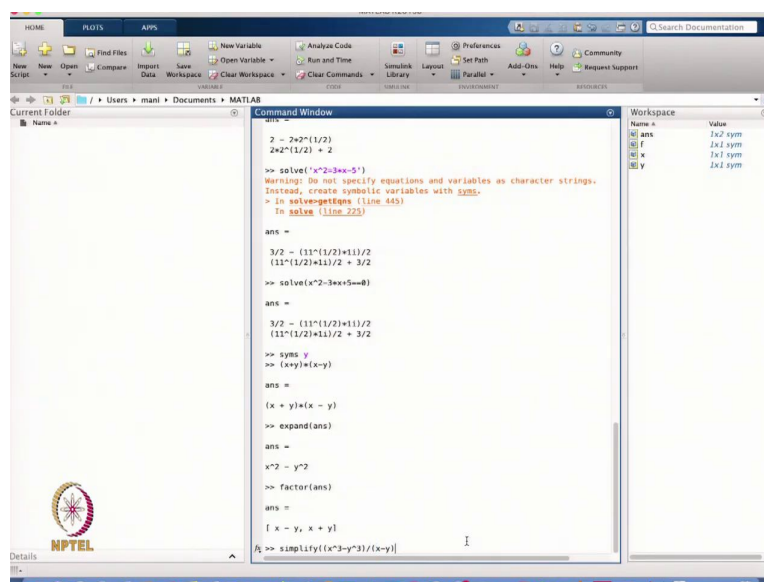
Now, if I want to check what this is, it is given that f is a 1 x 1 size function, bytes is 112 and class is symbolic. In the previous it was showing the double because of the numeric value. So, it is the, that is why it is showing that it is given in the double form bytes. So, x is also the symbolic with the 112 bytes, y is also the symbolic.

Now, suppose I want to define that, just I want to find the solve, I want to solve this quadratic equation. Suppose, I want to solve $x^2 - 1 = 0$. So, that gives you the answer. So, it solves this equation $x^2 - 1 = 0$ symbolically, and I get the answer minus 1 and 1. Suppose, I want to define another equation $x^2 - 2x + 1 = 0$. So, this is the equation I have defined, I want to solve this one. So, that is the root, so I am getting that is 1, 1.

So, similarly, I can define another equation just want to check the complex roots or maybe real roots. So, this is the root I am getting. Now, suppose I define like this, so this is the roots we are getting that is in the surd form, this is the -1, -2, the repeated roots.

Suppose now, I define another equation $x^2 - 4x - 4 = 0$, So, these are the other roots I am getting $2 - 2\sqrt{2}$, $2 + 2\sqrt{2}$. So, that is the value we are getting.

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The image shows a MATLAB interface with the Command Window and Workspace. The Command Window contains the following code and output:

```

2 - 2*sqrt(1/2)
2*sqrt(1/2) + 2
>> solve('x^2-3*x+5')
Warning: Do not specify equations and variables as character strings.
Instead, create symbolic variables with syms.
> In solvegetopts (line 445)
> In solve (line 225)
ans =
3/2 - (11^(1/2)+11)/2
(11^(1/2)+11)/2 + 3/2
>> solve(x^2-3*x+5==0)
ans =
3/2 - (11^(1/2)+11)/2
(11^(1/2)+11)/2 + 3/2
>> syms x y
>> (x+y)*(x-y)
ans =
(x + y)*(x - y)
>> expand(ans)
ans =
x^2 - y^2
>> factor(ans)
ans =
[ x - y, x + y]
>> simplify((x^3-y^3)/(x-y))

```

The Workspace shows the following variables:

Name	Value
ans	1x2 sym
f	1x1 sym
x	1x1 sym
y	1x1 sym

Suppose I want to solve another equation like this when I define the equation like this. Now, colon $x^2 = 3x - 5$. So, this one I would write with the semicolon, so that is also a warning, do not specify equation and variable as character string. So, that is the answer we are getting in the terms of this one, or maybe the same equation I can solve because here I am putting the semicolons, but the same equation I can solve by minus here, and plus here, equal to 0.

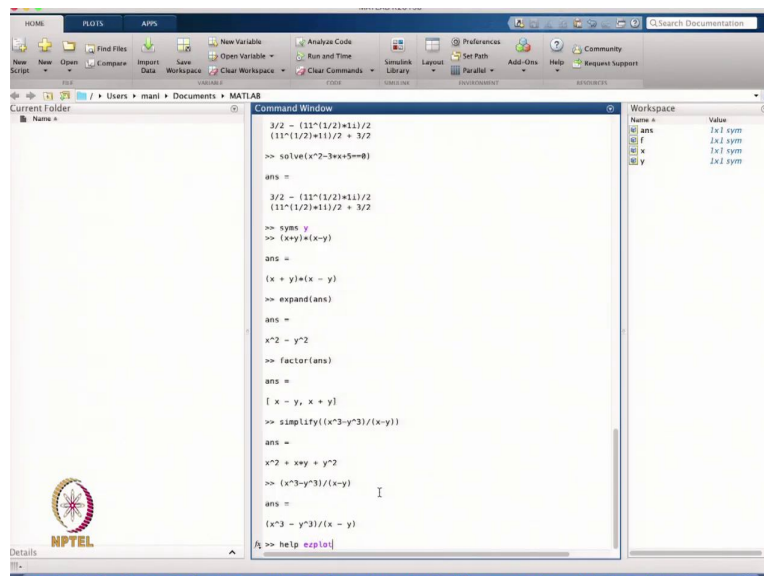
So, this is the equal to 0 we have to write double, two times we have to put $==$, and that gives you the solution. So, this is you can see that the solution these and these are the same, because this was another way we have written in which we are using only a single equal sign, but we are putting the equation in the semicolons. But here we are putting the equation in this form with the double equal sign.

Now, we can define maybe another equation I can write $x + y$, so I defined the another variable as sym y , so y is another variable I have defined, now I can take I can show that $(x + y) \times (x - y)$ and I want to see the answer. So, there is the answer, $(x + y) \times (x - y)$ or I can write this as expand, answer. So, that is, because I know that $(x + y) \times (x - y)$ can be written as $x^2 - y^2$.

So, I can write that whatever the answer we are getting, I can expand this one with the help of the command that is expand or maybe now what I can do, I can again factor the answer. So, that is the factor we got. So, that is the same one $(x + y) \times (x - y)$. So, expand means I will expand the answer in the compact form. And this is the factor, we can do the factorization of the given equation.

Maybe I can define another function using another command like I write, simplify, so I simplify define the command that $x^3 - y^3$ and then, so this is the bracket I have to put divided by $(x - y)$.

(Refer Slide Time: 7:36)



The image shows a MATLAB interface with the Command Window and Workspace. The Command Window contains the following code and output:

```

3/2 - (11^(1/2)+11)/2
(11^(1/2)+11)/2 + 3/2
>> solve(x^2-3*x+5==0)
ans =
3/2 - (11^(1/2)+11)/2
(11^(1/2)+11)/2 + 3/2
>> syms y
>> (x+y)*(x-y)
ans =
(x + y)*(x - y)
>> expand(ans)
ans =
x^2 - y^2
>> factor(ans)
ans =
[ x - y, x + y]
>> simplify((x^3-y^3)/(x-y))
ans =
x^2 + xy + y^2
>> (x^3-y^3)/(x-y)
ans =
(x^3 - y^3)/(x - y)
>> help expol

```

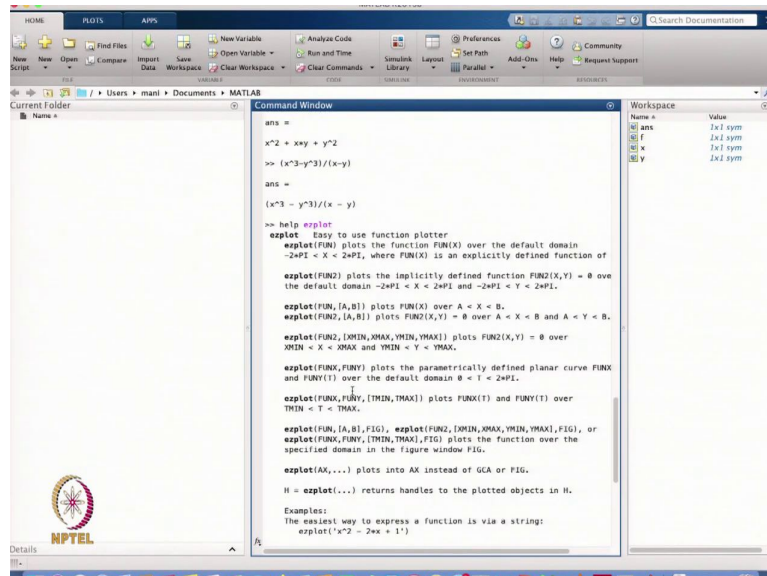
The Workspace on the right shows the following variables:

Name	Value
ans	1x1 sym
f	1x1 sym
x	1x1 sym
y	1x1 sym

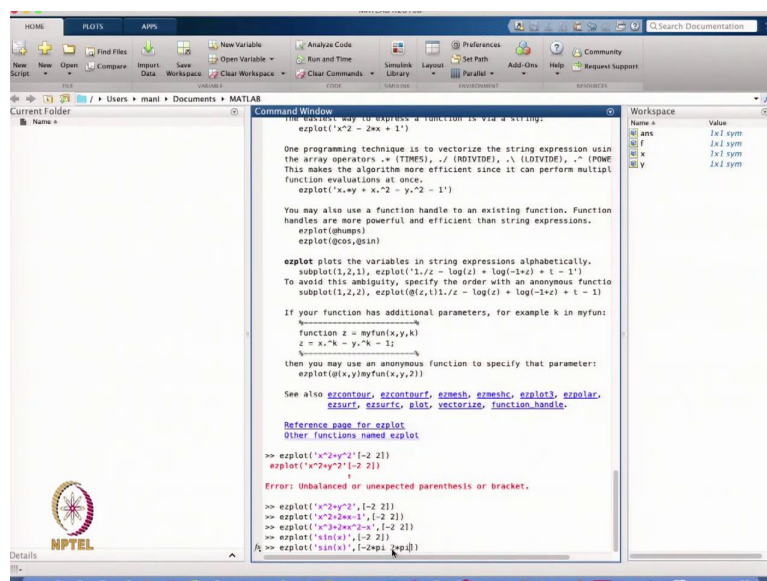
So, this is the command we have done. So, that gives you the $x^3 - y^3 / (x-y)$. So, the answer is $x^2 + y^2$. So, using the help of simplify, because otherwise if I write the command like this one and do not try to simplify, the answer will be this one; $x^3 - y^3 / (x-y)$.

But I want to, if I want to simplify, then it will cancel out whatever the common term is there and it will give you the answer after cancelling all the common terms. Now, this is the way we can define. Now, I can even plot the functions with the help of ezplot.

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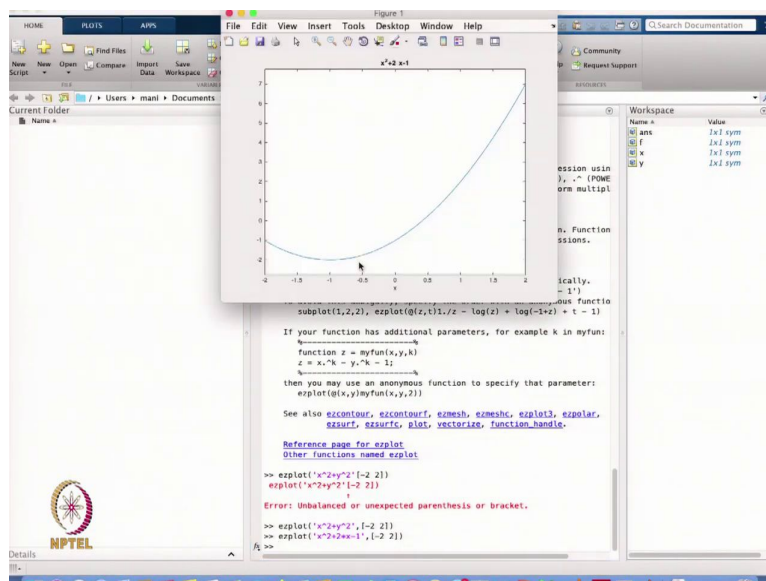
```
ans =
x^2 + x*y + y^2
>> (x^3-y^3)/(x-y)
ans =
(x^3 - y^3)/(x - y)
>> help ezplot
ezplot Easy to use function plotter
ezplot(FUN) plots the function FUN(x) over the default domain
-2*pi < x < 2*pi, where FUN(x) is an explicitly defined function of
ezplot(FUN2) plots the implicitly defined function FUN2(X,Y) = 0 over
the default domain -2*pi < X < 2*pi and -2*pi < Y < 2*pi.
ezplot(FUN,[A,B]) plots FUN(X) over A < X < B.
ezplot(FUN2,[A,B]) plots FUN2(X,Y) = 0 over A < X < B and A < Y < B.
ezplot(FUN2,[XMIN,XMAX,YMIN,YMAX]) plots FUN2(X,Y) = 0 over
XMIN < X < XMAX and YMIN < Y < YMAX.
ezplot(FUNX,FUNY) plots the parametrically defined planar curve FUNX
and FUNY(t) over the default domain 0 < t < 2*pi.
ezplot(FUNX,FUNY,[TMIN,TMAX]) plots FUNX(t) and FUNY(t) over
TMIN < t < TMAX.
ezplot(FUN,[A,B],FIG), ezplot(FUN2,[XMIN,XMAX,YMIN,YMAX],FIG), or
ezplot(FUNX,FUNY,[TMIN,TMAX],FIG) plots the function over the
specified domain in the figure window FIG.
ezplot(AX,...) plots into AX instead of GCA or FIG.
H = ezplot(...) returns handles to the plotted objects in H.
Examples:
The easiest way to express a function is via a string:
ezplot('x^2 - 2*x + 1')
```



```
The easiest way to express a function is via a string:
ezplot('x^2 - 2*x + 1')
One programming technique is to vectorize the string expression using
the array operators .* (TIMES), ./ (RIGHTDIV), \ (LEFTDIV), and \ (POND).
This makes the algorithm more efficient since it can perform multiple
function evaluations at once.
ezplot('x.*y',[-2,2],[-2,2])
You may also use a function handle to an existing function. Function
handles are more powerful and efficient than string expressions.
ezplot(@humps)
ezplot(@cos,@sin)
ezplot plots the variables in string expressions alphabetically.
subplot(1,2,1), ezplot(1./z - log(z) + log(1-z), t - 1')
To avoid this ambiguity, specify the order with an anonymous function
subplot(1,2,2), ezplot(@(z,1)/z - log(z) + log(1-z), t - 1)
If your function has additional parameters, for example k in myfun:
function z = myfun(x,y,k)
z = x.*k - y.*k - 1;
then you may use an anonymous function to specify that parameter:
ezplot(@(x,y)myfun(x,y,2))
See also ezcontour, ezcontourf, ezmesh, ezmeshc, ezplot3, ezpolar,
ezsurf, ezsurfs, plot, vectorize, function_handle.
Reference page for ezplot
Other functions named ezplot
>> ezplot('x^2+y^2',[-2 2])
ezplot('x^2+y^2',[-2 2])
Error: Unbalanced or unexpected parenthesis or bracket.
>> ezplot('x^2+y^2',[-2 2])
>> ezplot('x^2+2*x-1',[-2 2])
>> ezplot('x^2+2*x^2-x',[-2 2])
>> ezplot('sin(x)',[-2 2])
>> ezplot('sin(x)',[-2*pi,2*pi])
```

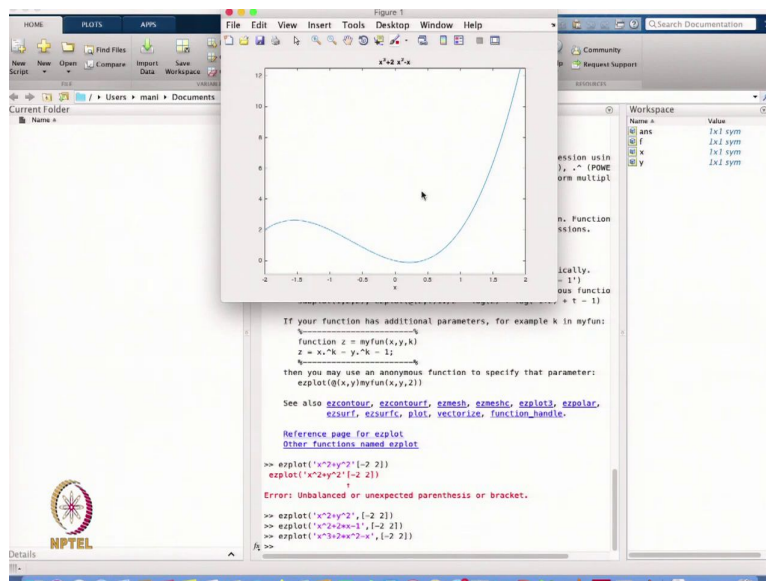
So, let us define this plotting. So, now I define help ezplot, so ezplot you can see that, ezplot is easy to use function operator, ez function plots the function x over the default domain. So, let us use this one. So, I write ezplot suppose I want to plot a circle. So, I define this one between the interval I define, so I take -2, 2 + 2, this one. So, there is a bracket ezplot $x^2 + y^2$ and then a comma we have to put.

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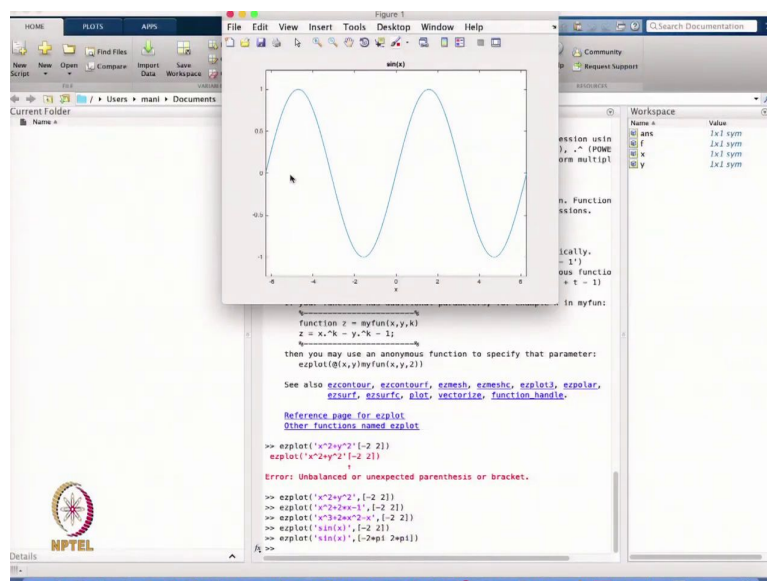
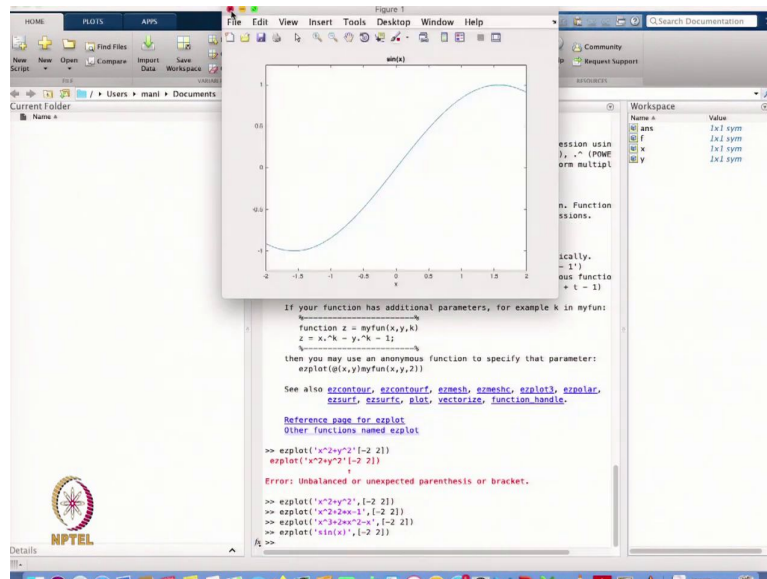
So, that is the ezplot. So, this is the plot it is showing $x^2 + y^2$, maybe this is a two dimension, so I just define the first dimension only. So, I just write $x^2 + 2 * x - 1$, so this is a plot. So, this is showing that this is a plot of $x^2 + 2 * x - 1$ or I can define the plot of x^2, x^3 , plus $x^2 - x$ in the same domain.

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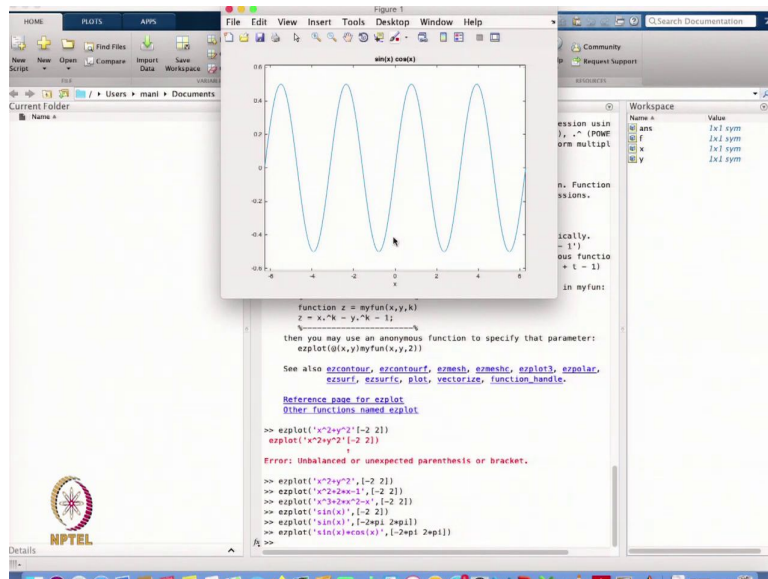
So, it will give you the plot of that function, because sometimes we want to plot, want to see the function, so we want to plot that function, so using this very simple command we can plot any function.

(Refer Slide Time: 10:25)



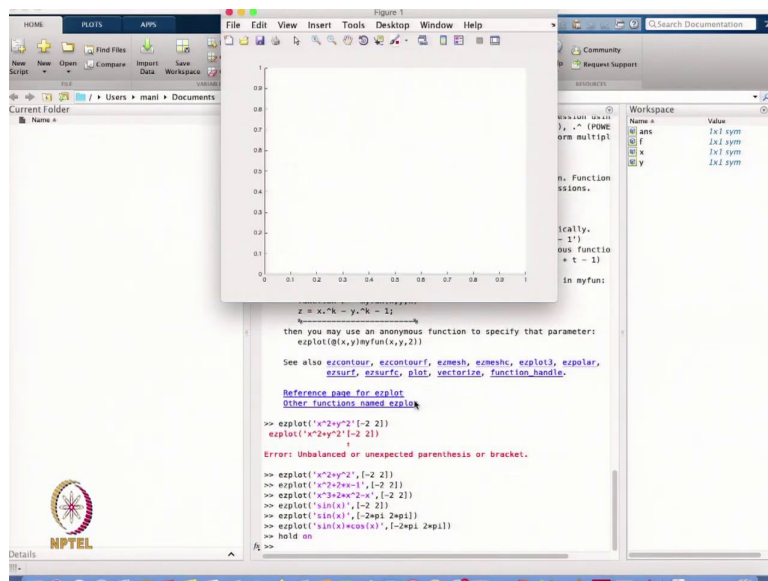
Suppose I want to plot, instead of this one I want to plot $\sin x$, so that gives you the value of the $\sin x$. I can change my, this domain, so I can take from -2π to 2π and that is the function we got, so this is the value of the $\sin x$ between -2π to 2π , so, this is one we can do.

(Refer Slide Time: 10:56)



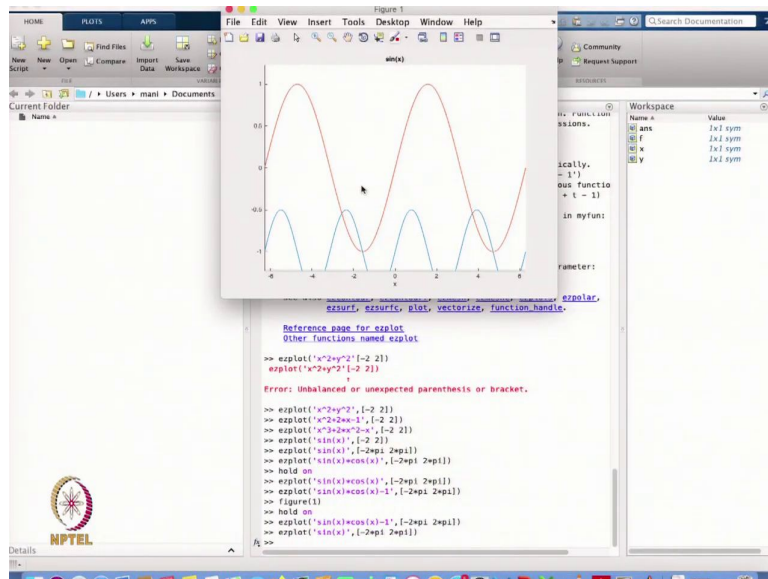
Maybe we can plot another function $\sin x * \cos x$. So, this is the value of the plot of the function $\sin x * \cos x$.

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And now, suppose I take the hold on, so hold on this graph. So, I can define now, the same command ezplot is coming.

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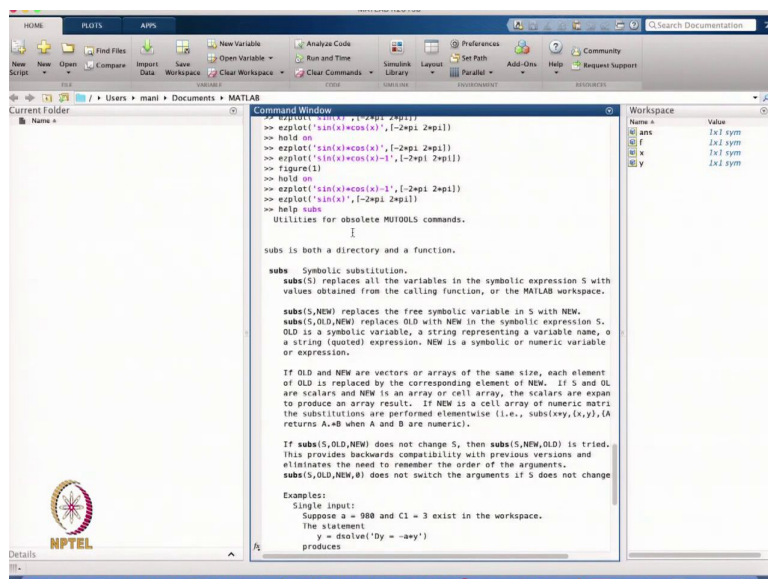


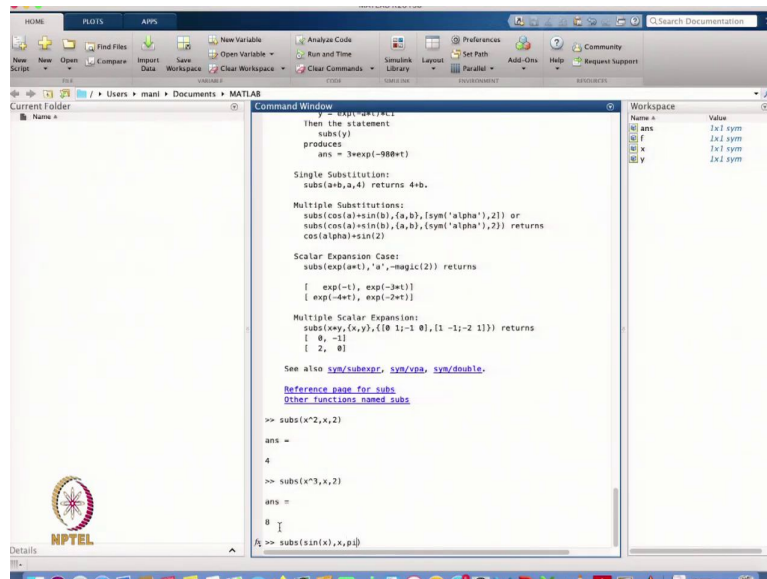
Now, I take another plot and instead of this I take -1, so that is the plot we are getting or maybe I can give the name to the figure. So, I first define the figure 1 and then I give the command hold on. So, this is the figure 1 and I just hold on, and then I plot the graph, that is the plot of the graph.

Then I define the other plot of the graph, just $\sin x$. So, this is another plot we are getting. So, that is a $\sin x$ and this is the $\sin x * \cos x$ using the help of hold on I can hold the same graph in the figure 1. So, I have given the name to this plot that is the figure 1. So, that one we can do, so that is the help of the, with the help of `ezplot` we can plot many functions.

Now, we do some other symbolic calculation because this type of calculation if you see earlier used to be done with the help of mathematica or the mapple, but now this is also possible in the MATLAB. So, MATLAB is being used now widely for both numerical computation as well as symbolic computation.

(Refer Slide Time: 13:15)





Now, suppose I take the help and then I write substitutes, so it shows you that, substitute is the symbolic substitution. So, in this case, suppose I have the equation, I write substitutes. And I write, suppose I write x^2 , this is my function, and I substitute $x = 2$ here, so, that gives the value the 4. So, how it is the, how we can define the value of the substitution, function substitution.

So, substitution is that, this is the function we have and that is the variable which is giving the value 2 and that is the answer we are getting. I can define the substitution of x^3 and, so that is the value of 2^3 . I can define the function $\sin x$ and then x , I can put maybe π , so that is the value 2. I can define π by 5, so, that is the value of the $\sin(\pi)$ by 5, or I can define the value of the function at π by 6 that is the half.

So, using this one, I can substitute the value of the variable in the given function or that is also possible that I define the function, first I define the function. Suppose, I define the function $x^2 - y^2$, so this is my function. And, now I write substitute if x is equal to 2, x , 2.

(Refer Slide Time: 15:06)

The screenshot shows the MATLAB Command Window with the following code and output:

```

See also sym/subexpr, sym/vpa, sym/double.
Reference page for subs
Other functions named subs

>> subs(x^2,x,2)
ans =
4

>> subs(x^3,x,2)
ans =
8

>> subs(sin(x),x,pi)
ans =
0

>> subs(sin(x),x,pi/5)
ans =
(2^(1/2)*i(5 - 5^(1/2))^(1/2))/4

>> subs(sin(x),x,pi/6)
ans =
1/2

>> ff=x^2-y^2
ff =
x^2 - y^2

%>> subs(ff,x,2)

```

The Workspace window on the right shows the following variables:

Name	Value
ans	1x1 sym
f	1x1 sym
ff	1x1 sym
x	1x1 sym
y	1x1 sym

The screenshot shows the MATLAB Command Window with the following code and output:

```

ff =
x^2 - y^2

>> subs(ff,x,2)
ans =
4 - y^2

>> subs(ans,y,3)
ans =
-5

>> ezplot(@log(x),[0 10])
ezplot(@log(x),[0 10])
Error: Unbalanced or unexpected parenthesis or bracket.

>> ezplot(@log(x),[0 10])
ezplot(@log(x),[0 10])
Error: Unbalanced or unexpected parenthesis or bracket.

>> ezplot(@log,10,[0 10])
ezplot(@log,10,[0 10])
Error: Unbalanced or unexpected parenthesis or bracket.

>> ezplotlog(x,[0 10])
ezplotlog(x,[0 10])
Error: Unbalanced or unexpected parenthesis or bracket.

>> ezplotlog(x)
ezplotlog(x)
Error: Unbalanced or unexpected parenthesis or bracket.

%>> ezplot('log(x)',[-2*pi 2*pi])

```

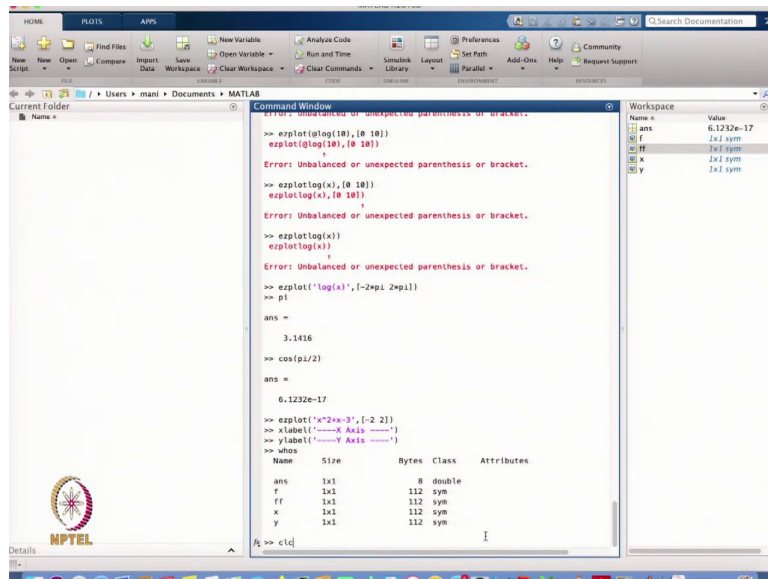
The Workspace window on the right shows the following variables:

Name	Value
ans	1x1 sym
f	1x1 sym
ff	1x1 sym
x	1x1 sym
y	1x1 sym

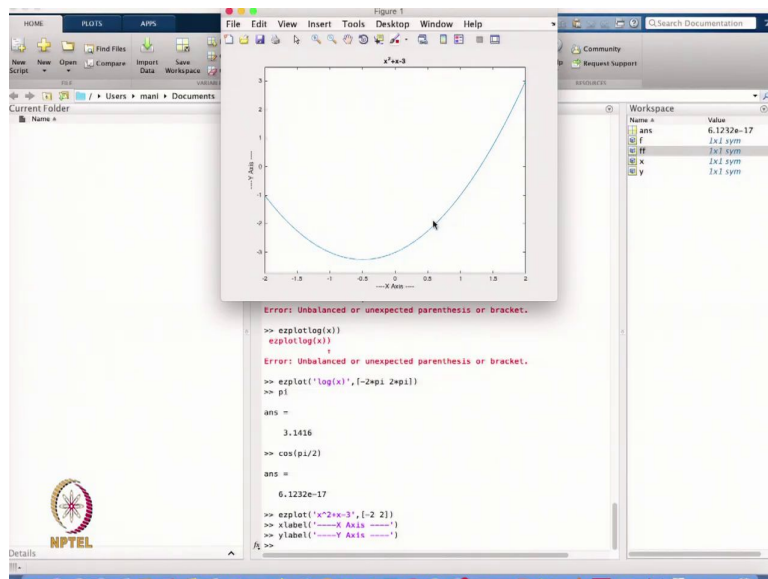
So, this gives you the value of the function that I have the function of ff and in that function I put instead of x, I put the x = 2 in the given function.

Now, I can define the same here and I can answer and then I put y = 3 and that is a -5, so it gives you 4-9, so, it is a -5. So, I can substitute the value of the different different variables in this one even if I can define these substitutions. Now, with the help of this symbolic computation, I can plot another something like I can plot ezplot at the rate maybe log x and define the plot on 0 to 10, suppose I want to plot this function.

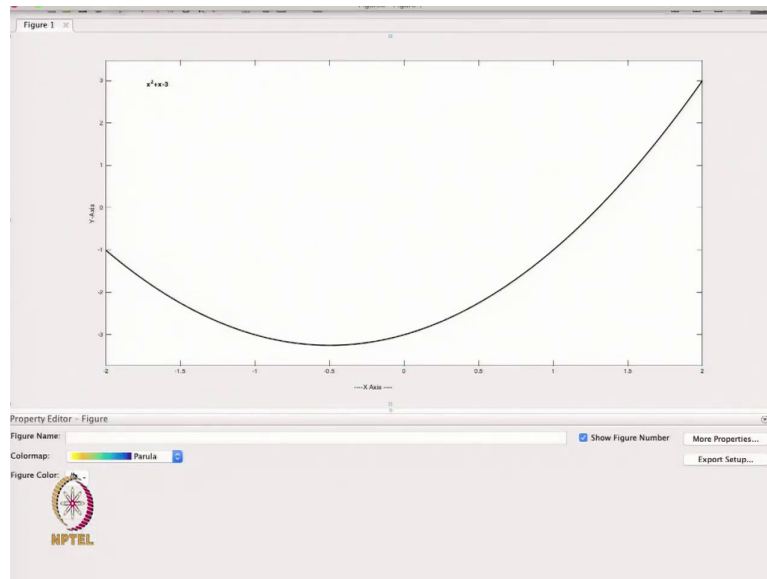
(Refer Slide Time: 16:19)



So, this is the, like this one I have done. So, I can use here my log x that is the value of the log x we have plotted. Now, maybe I can define another function. So, this is a π we can take from here. Now I want to define the π by suppose two values so that is the value we are getting. So, this is $\pi/2$ we know that this value is 0. So, in this case I am getting the value 0 and that is given by e^{-17} , so this is how we can define the little bit of symbolic computation then.
(Refer Slide Time: 17:07)



So, let us plot another variable or another function. Suppose I want to plot $x^2 + x - 3$ and I want this to be done in the interval from 0 to from -2. So, this is a plot we are getting.
(Refer Slide Time: 17:41)



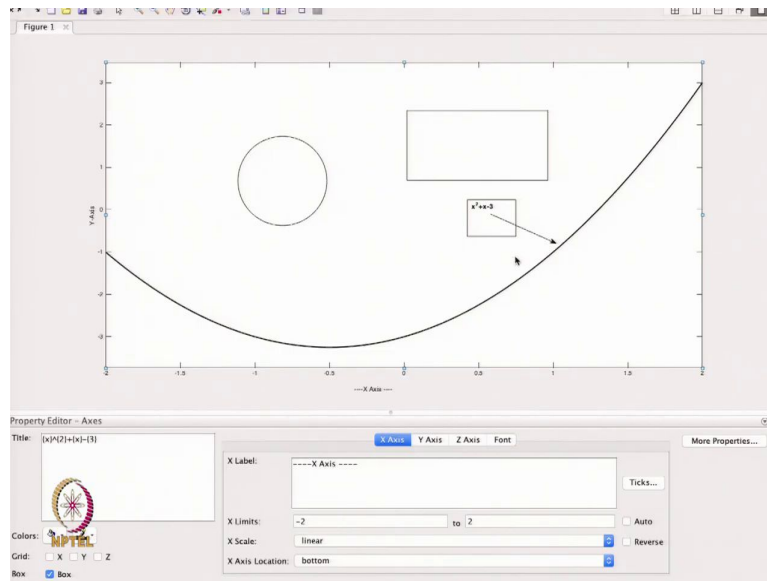
Now, I can also define the same way, title we have done, so x label. So, x label will give you this if I just write capital X axis I can define y label as y axis.

Now, we can see that this is a function even I can, because this is giving me the blue color, even so I want to change the color of this one, then maybe I can switch to edit, and then I can see that the figure properties with the help of the figure properties, you can see that I can change the color of this. So, that is the function we have. This is a graph of the function and it gives you that you suppose I want to change the color of this line, so I can put it as a black.

Now suppose I want to increase this because this is a very thin layer with the 0.5 line width. So, I can increase the width by 2. So this is the, that it becomes the bold one or dark one. The marker is there, I can even change the color of the marker. So, this is my function plot function.

And suppose, so this is the way we can take the axis, I can take the axis anywhere. Now, suppose this is my y axis, I can change the style of this y axis, this one, so, y axis even I can write here and I can write here like this one I can edit this one later on. So, this is my y axis, this is my x axis, I can take this one and then I can do the changes. So, this is my title I can move this here even inside the graph I can move this. So, with the help of editing you can edit your plot, what are the plot you are getting and suppose in the later on you want to make some changes, because in that case, you are not going to produce, reproduce the same plot. So, you can do the editing with the help of this edit of the plotting.

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Now, suppose I have done with this plot and I want to save this plot. So, this is the file and we can save, save this as. So, it will ask you where you want to plot. I suppose I want to plot it in the NPTEL and this is my, so I can write here I want to plot this one as figure 1. So figure 1, I want to plot in the name of figure. So, that one is saved now.

Suppose I want to use it later on in my paper and the latex or in word file, then I can also save this as. So, here we can, these are the different extensions we can see in our save our figure. So, this is BMP, EPS, JPEG. So, all these different styles we can, because in the latex, we generally use the EPS file, so I can save this function as a EPS. So, this function is now saved in your file with the EPS extension and another one is saved in the form of fig function. So, this is a different way we can define the given plot.

Even I can insert, suppose I want to insert, so I want to insert the arrow or the text arrow. So text arrow means suppose I want. So, this is the figure, so that is the figure, and here I can write by inserting the text arrow. Here I can write the value of the function, function is equal to $x^2 + x - 3$.

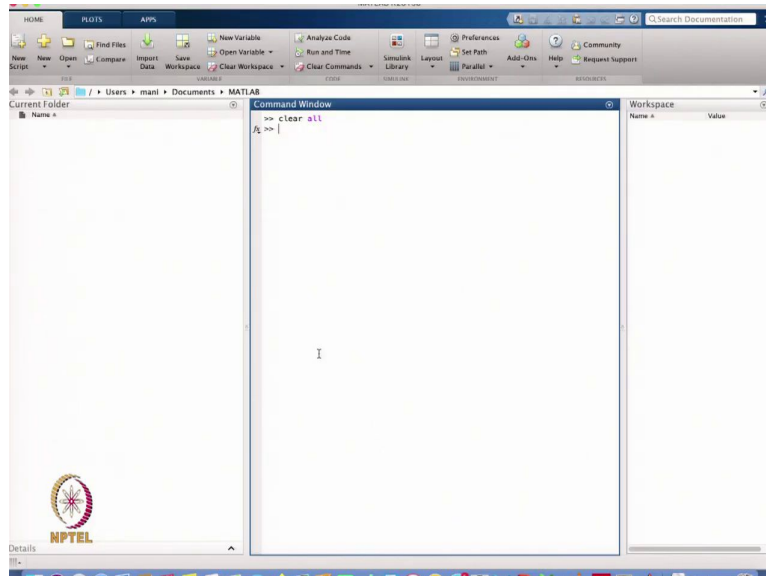
This is the function I am getting or maybe I can, I do not want to write this function here. Maybe I can delete this one or I can move this function, this here, this one that is the plotting, and this is the text arrow that is showing that this is because suppose we have a large number of plots in the same plot, we can tell them or classify them with the help of the arrow function, that which function represent the which functions.

This is all we can do with the, now we can insert the ellipse, we can insert the arrows or the line. So, suppose I want to insert the ellipse, so this is an ellipse so and getting, or maybe I can insert the circle here. Later on, we want to insert, and this is the view that maybe I can insert axis. So, axis I already inserted here, but if suppose we forgot to insert then later on we can insert these axis here.

Maybe I can insert a rectangle, so this is a rectangle or maybe I can move this rectangle from here. I can insert the rectangular box like this one, or maybe I can insert the, so this one we can play with this plot. Once we are done with the plot, then we can even make the changes we can play with this plot.

And we want, we can change whatever the changes we want to make later on, we can do that one. And then we can minimize this function and I, from here, I can delete this one. And now I can see what are the variables. So, all the variables today we are dealt with is the symbolic computation, the symbolic computation.

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And I can even clear my screen and I can clear all the variables. So, that is gone. So, this is all about the symbolic computation. This is a basic about the symbolic computation that we can do with the help of MATLAB. So, in the next lecture, we will go a little further with the basics of MATLAB. Thanks for watching. Thank you very much.