Scientific Computing using Matlab

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Lecture 19
Continued

Hello viewers welcome back to the course on Scientific Computing using MATLAB. So, today we are going to discuss lecture 19. So, in the previous lecture we have started with some basics of the matrices. So, we will continue with this one. So today we will discuss with some other form of matrices.

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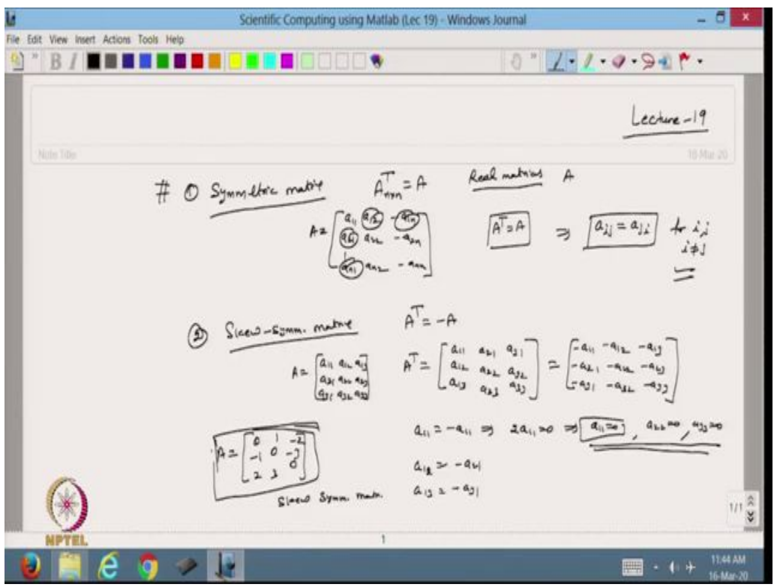

And so, we start with the maybe symmetric matrix. So, symmetric matrix means I have a matrix A, so that is a n x n matrix. Now, I take the transpose of that matrix. So, if that is equal to the matrix itself then it is called the symmetric matrix. So, in this case, what is going to do? So, if you see that my matrix A.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

And if I take the transpose A^T and that should be $A\text{-}A^T=0$. So, that gives me that my $a_{ij} = a_{ji}$. So, this element and this element should be the same, this element and this element should be the same. So, if this is true then for all ij such that $i \neq j$ because the diagonal elements is the same one, is the because except the diagonal elements all the elements should be the same.

So, this condition is true other than the diagonal elements, so then it is called the symmetric matrix. Second one is we also call it skew symmetric. So, in the skew symmetric matrix $A^T= -A$. So, in this case what is happening? Suppose I have a matrix A as given above.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

Now this is $A^T= -A$, So from here one thing is true that $2\, a_{11} = 0$. So that gives me the $a_{11}=0$.

So, the diagonal elements, but now here we are talking about real matrices. So, real matrix means all the elements of the matrix a are real. So, in this case from here I can see that my diagonal elements $a_{11}=a_{22}=a_{33}=0$. So, in the skew symmetric matrix the first condition is that the diagonal elements should be 0, we are talking about the real matrix.

And then from here I can see that my $a_{12}= -a_{21}$; my $a_{13}= -a_{31}$ and so on. So, this is an example of the skew symmetric matrix.

$$A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix}.$$

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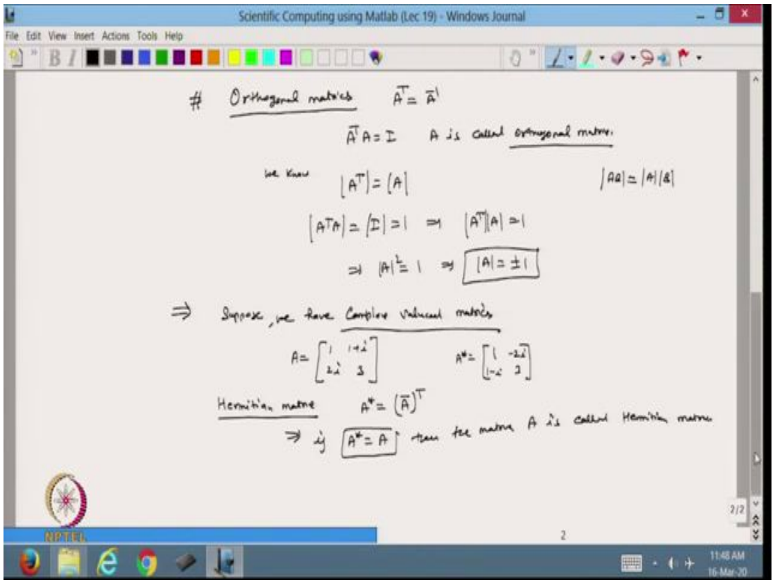

Then we defined another matrix as an orthogonal matrix so orthogonal matrix means whenever I have $A^T = A^{-1}$. It means $A^T A=I$. So, in that case the matrix A is called an orthogonal matrix. So, if you see the column vector of the matrix, they will be orthogonal to each other. So, that is they called the orthogonal matrix.

We also know that $|A^T|=|A|$. So, from here I can write $|A^T A=I|=|A^2|=|A|^2= \pm 1$.

So, if I have the orthogonal matrix then the determinant of the orthogonal matrix is always either ±1. So, that is the way we can find the determinant of an orthogonal matrix. So, this is a condition about the orthogonal matrix. Now, we can just go a little bit ahead and then suppose we have complex valued matrices

$$A = \begin{bmatrix} 1 & 1+i \\ 2i & 3 \end{bmatrix}.$$

So in this the elements, value of the elements is a complex number also. So, this is a complex valued matrix or complex matrices. So, in this case, if I want to define the symmetric matrix, there is a condition for the Hermitian matrix. So, Hermitian means, in this case I will define

$$A^{\cdot} = \left(\overline{A}\right)^{T}$$

$$A = \begin{bmatrix} 1+i & -2i \\ 2-2i & 4 \end{bmatrix}, \quad \overline{A} = \begin{bmatrix} 1-i & +2i \\ 2+2i & 4 \end{bmatrix}$$

$$A^{\cdot} = \begin{bmatrix} 1-i & 2+2i \\ 2i & 4 \end{bmatrix}$$

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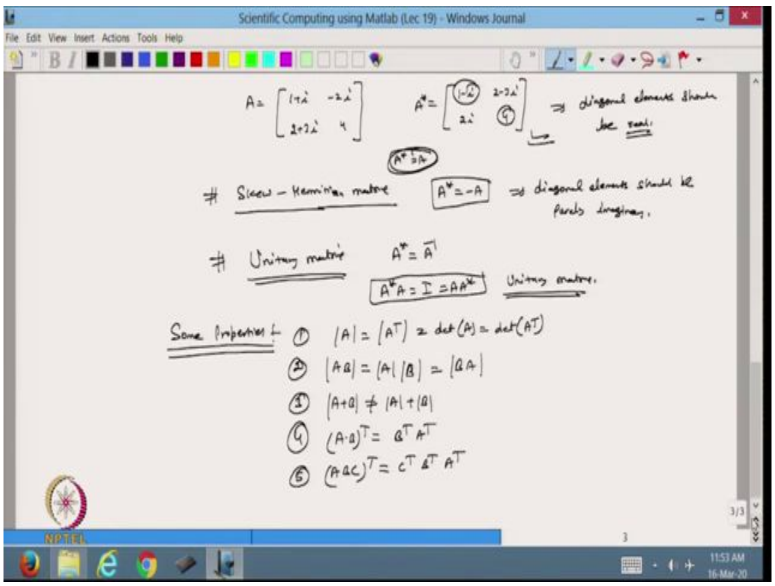

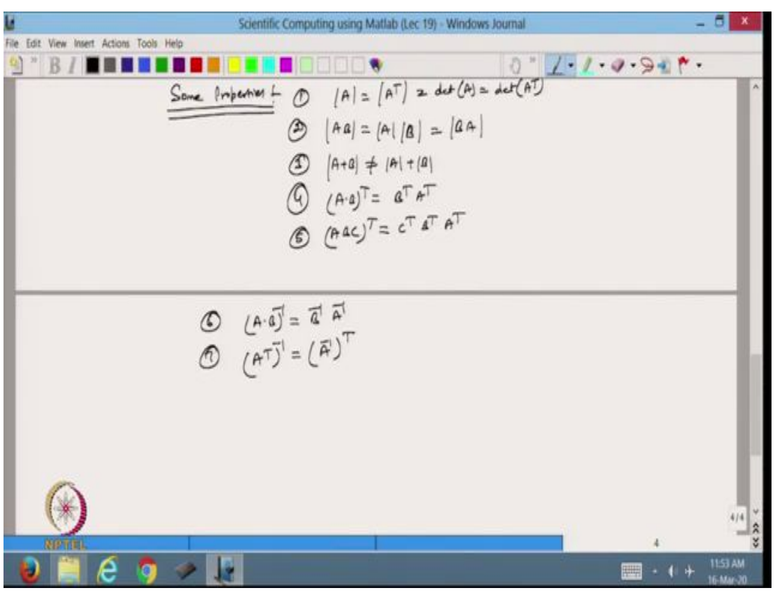

So, if A^*=A (Hermitian matrix) then one thing is true that this diagonal element should be the same, the first of all. So, if I take the conjugate, if the diagonal elements are complex then definitely its conjugate will be different from the previous one. So, from here one condition is true that the diagonal elements should be real.

So, the diagonal element should be purely real only then we can construct a Hermitian matrix. The next is skew Hermitian. So in the skew Hermitian, so now I can in the skew Hermitian the same thing is there whenever A^*= -A then it is a skew Hermitian matrix. So in this case the diagonal elements should be purely imaginary. So it should be purely imaginary. In the real sense we have seen that the diagonal elements should be 0.

So, you can say that the real part should be 0 so it should be pure imaginary and the third one is a unitary matrix. So, the unitary matrix is analogous to the orthogonal matrix. So, this is whenever A^*= A^{-1}. So, then it is called the unity matrix, okay. So, this is the same way as the symmetric matrix, skew symmetric and orthogonal.

So, this is all about some matrices. Now we define some properties. (determinant of A = |A|)

$$|A| = |A^T|$$
$$|AB| = |A|\,|B|$$
$$|A+B| \neq |A| + |B|$$
$$(AB)^T = B^T A^T$$

Fifth one is I can take maybe more than 2.

$$(ABC)^T = C^T B^T A^T$$
$$(AB)^{-1} = B^{-1} A^{-1}$$
$$\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$$

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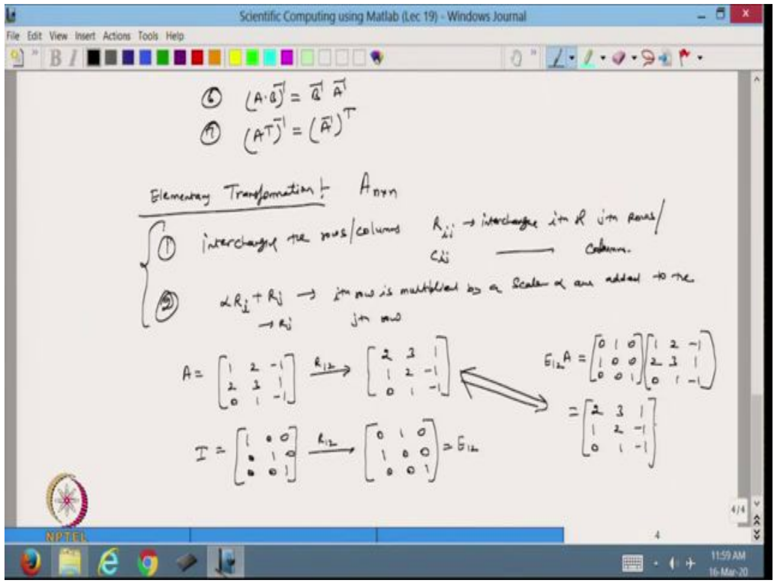

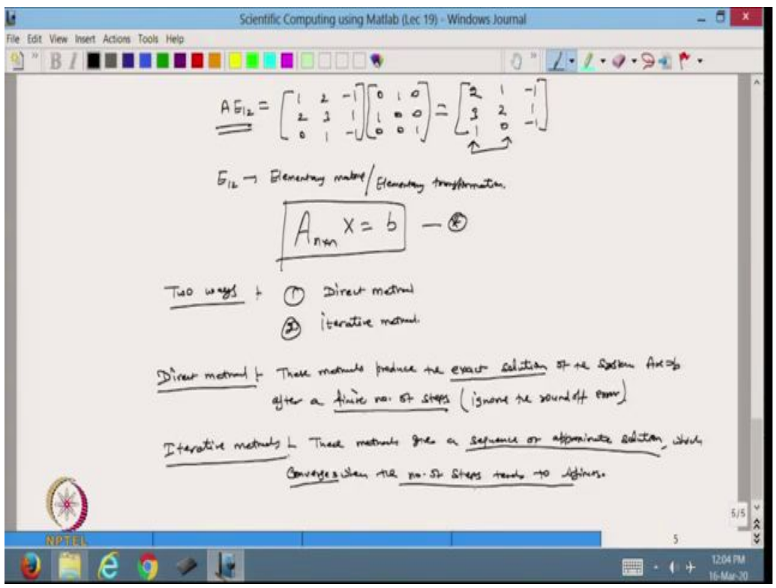

Now we start with the main course and that will deal starting with the elementary operation, elementary transformations. So what are the elementary transformations? So suppose I have a matrix A that is supposed I am taking the square matrix, n x n. Now what I can do is that the first one is interchanging the rows or columns. So suppose I have a row represented by R.

So, I can write R_{ij}, so it means I am interchanging ith and jth rows or if I write C_{ij} so that is the columns. Second one is that if I write like αR_i+R_j. So in this case what I am doing? I am multiplying the ith row by a scalar α and added to the jth row. So, the effect will be up to the jth row. So in this case, it is interchanging the rows and this is multiplying some rows and adding or subtracting to the other row.

So these are the 2 operations or called the elementary operations or the, and the transformation is called elementary transformation.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ 0 & 1 & -1 \end{bmatrix} \rightarrow R_{12} \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

It means I am applying the elementary operation, interchanging the first and the second row. So this one I can write as so I will interchange the first and second row. But what I can do is that I take a unit matrix, identity matrix I. So, the identity matrix is this

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_{12} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

So, E means elementary matrix after interchanging the rows 1 and 2. Now, what I do is that I will apply this E_{12} x A. So, I will pre-multiply the matrix A with E_{12}. So let us see what will happen.

$$E_{12}A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

Now, I multiply by the second the first will come. So, this is if you see, then this is the same as this one. So instead of applying the operation directly, if I take the elementary operations on the unit matrix and get the elementary transformation or elementary matrix and P multiply, so I will get the same output as we are getting from here. So this is what we have done is this one.

Now, suppose instead of pre-multiplying, I will post-multiply.

$$AE_{12} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So, this will come. So, in this case you will see that first and the second columns have been interchanged.

$$\begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

So, that is called a post multiplying and pre multiplying the matrix with the elementary matrices. So, this E12 we call it elementary matrix or elementary transformation.

Because in linear algebra we know that the linear transformation can be represented by the matrix. So, this is called elementary transformation. So this we can apply so now we start with our main syllabus so let us, so onward we have a system $A_{nxn}x=b$ and we want to solve this system, okay. So, let us write the system as a star. Now there are two ways to solve this one, to solve this system, so two ways.

The first one is the direct method. So this is the direct method. The second one is an iterative method. So let us write the definition of direct method. So, these methods produce the exact solution of the system Ax=b after a finite number of steps. So, they give you an exact solution almost, if I ignore the round-off error.

So you know that the round-off error is always there. But suppose I ignore this one then this method provides the exact solution of the system after a finite number of steps. So, that is called my direct method and what about the iterative method? So, these types of methods, so these methods give a sequence of approximate solutions which converge when the number of steps tends to infinity. So, that is called the iterative methods. So in the iterative methods we have a sequence of approximate solutions and that solution converges when the number of steps tends to infinity.

So, in this case at every iteration the solution is becoming more accurate and when the number of steps tends to infinity only then you will get the exact solution. So, that is called iterative methods.

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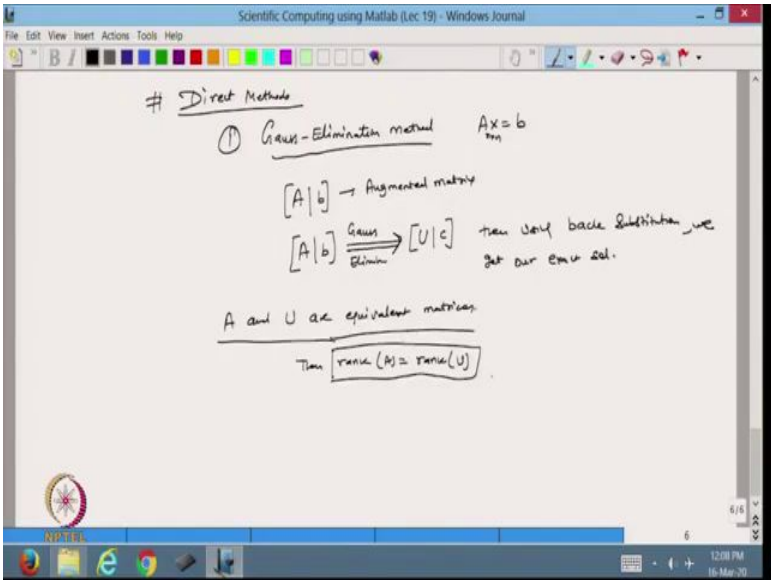

So, let us start with the first one is the direct method. So the first one we are dealing with here is the Gauss-Elimination, Gauss-Elimination method. So, in the Gauss-Elimination method, what we are going to do is have a system Ax = b and this is n x n matrix. Now, what I do is I will write matrix A and then it is the right hand side vector. So, this matrix is called an augmented matrix. So, this is the augmented matrix. Now, what I will do is that I will apply my elementary operations on this matrix and I will convert this matrix into another matrix, that is I can call it U .

So, this is converting a matrix into the upper triangular matrix and that is the right hand side vector. So, we are converting this system from this augmented system to the augmented system using my Gauss-Elimination And then using back substitution we get our exact solution. So, in this case we get our exact solution.

And from our knowledge of matrices we know that then the matrix A and U are equivalent matrices. So A and U will be the equivalent matrices. So that is the Gauss-Elimination method. So, in this case we know that the A and U are equivalent matrices, then the rank of A is the same as a rank of U.

So, that is also another condition of the equivalent matrices, that the rank of A is equal to the rank of U. Because in the equivalent matrices only the rank will be the same and the other nature of the matrix will not change, if A is a singular matrix then U will be also singular. If A is non-singular U will be non-singular.

So, applying the elementary operation, we cannot change the nature of the matrix only that we are reducing the matrix into this some easier form that is upper triangular matrix or the lower triangular matrix and then we can we are able to solve the system with the help of back substitution if it is upper triangular or the forward substitution if it is a lower triangular, but the rank will be same.

So, this is the Gauss-Elimination method. So, we will stop today and in the next lecture we will continue with the Gauss-Elimination method for this one. So thanks for viewing, thanks very much.