**Scientific Computing Using Matlab**
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**Lecture 12**
**Regula-Falsi and Secant Method for Solving Non-Linear Equations**

Hello viewers. Welcome back to this course. So, today we will continue with the lecture 12. So, in the previous class we have started with the iterative methods, the fixed point iteration.

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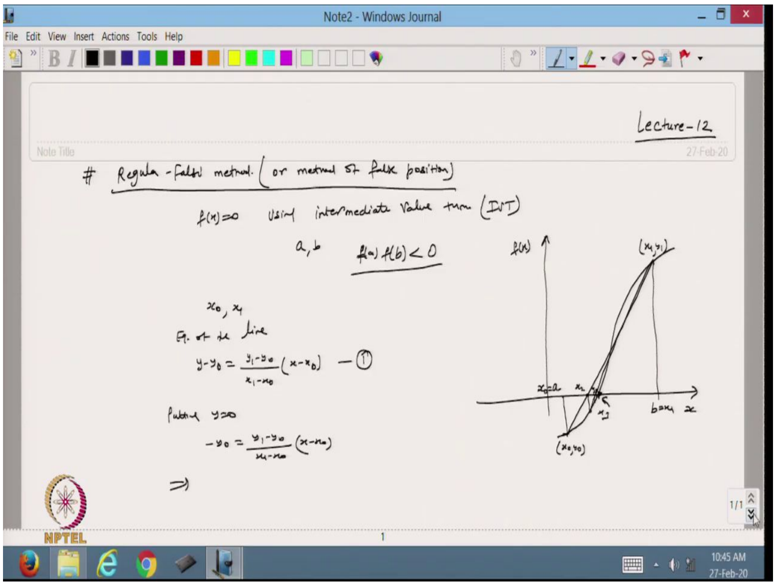

Now, this lecture will start with another method and that is called the Regula-Falsi method or this method is also known as method of false position. So, what is the meaning of this Regula-Falsi method. So, I suppose I have a function f(x) = 0 so this function is given to me and I want to find the root of this function.

Finding the root of this function now using inter suppose I get a value of [a b] and I will find that f(a) f(b) < 0. So, it means that the root lies between a and b. So, what I do is suppose that it is my function that is the x- axis, this is f(x) and suppose my function goes like this one.

So, what do I do is that I will start with a so let it this is my a and I will start with this b. So, I call it $x_0$ and I will call it $x_1$. So, based on this one I will try to so this is my equation and that is the line we are passing for this one. So, what I will do is that I will see where this line is cutting the x axis so this is the place which is cutting the x axis that is the place. Now, I will call this place another approximation and I will call it $x_2$.

Now, based on this $x_2$ now I will take another line passing through this and then I will get see the things where it is cutting the x axis so that is the point it is $x_3$ and so on and ultimately we are reaching because that is the place this is the root of the equation and we are reaching towards this one in the iterative method in the iterative way so that is called the Regula-Falsi method.

So, in this case so I start with $x_0$ and $x_1$ and I will write the equation so equation of the line this will be $y - y_1$ so that is equal to so in this case I have the value of this one and the value of this one is so I can write here $y_1 - y_0$ and then $x_1 - x_0$ , $x - x_0$ and maybe I can take this equation. So, $y - y_0$ so this is my $x_0$ so that point is my $x_0 - y_0$ and this is my point $x_1$ $y_1$.

So, based on this one this is the equation of the line $y - y_0 = \frac{y1 - y0}{(x1 - x0)}(x - x_0)$. So, that is the equation of the line. Now, I want to see that where this line is cutting the x axis. So, by putting y = 0 in this case so I will get this point value. So, if I put y = 0, I will get

$$- y_0 = \frac{y1 - y0}{(x1 - x0)}(x - x_0)$$

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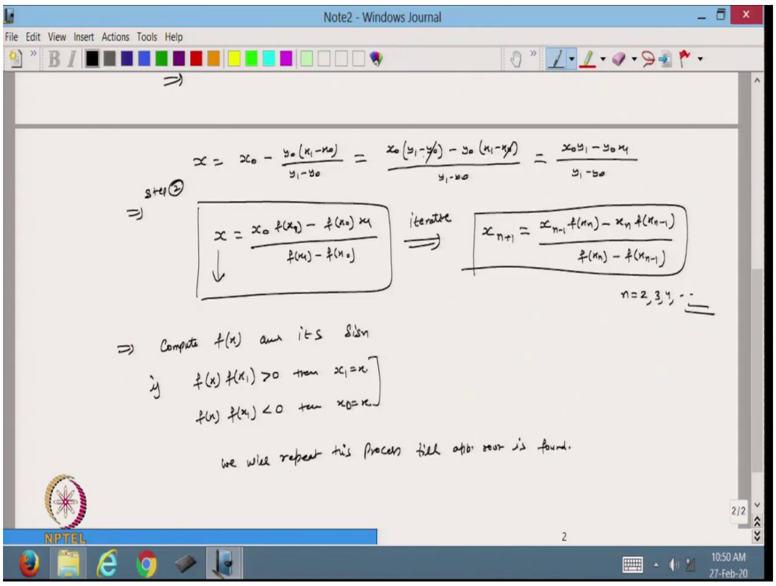


So, based on this one I can write from here my x can be written as so I will write from here it

$$x = \frac{(x_0\ f(x_1) - f(x_0)x_1)}{f(x_1) - f(x_0)}$$

In this case I need two initial approximations like in the bisection also I need two initial approximations.

But in the iterative process fixed point iteration I need only one approximation to start the process, but here I need to x naught and x1 and based on this one so this methods can be written at in the iterative process

$$x_{n+1} = \frac{\left(x_{n-1}\ f(x_n) - f\left(x_{n-1}\right)x_n\right)}{f(x_n) - f\left(x_{n-1}\right)}$$

So, that is my iterative process where my n is so in this case if I put the n = 2, 3, 4 and so on because I have put 2 then I need 2 and 1 so it will be the 3 or maybe we can start with 1 in that case it will be $x_1$ and $x_0$ and will get $x_2$. So, based on this one that is the iterative method we can have. Now, what we will do in this case is find the value of a and b.

So, this process we can say that this is step number 1. Finding the value a and b and then after that I will get the value of x so this is my step 2 so step 2 based on this one. Now compute my f (x) and check its sign:

$$f(x)\, f(x_1) > 0 \ then\ x_1 = x$$
$$f(x)\, f(x_1) < 0 \ then\ x_0 = x$$

So, once I do this one this one we will repeat this process till an approximate root is found. So, in this case that is called the Regula-Falsi method.

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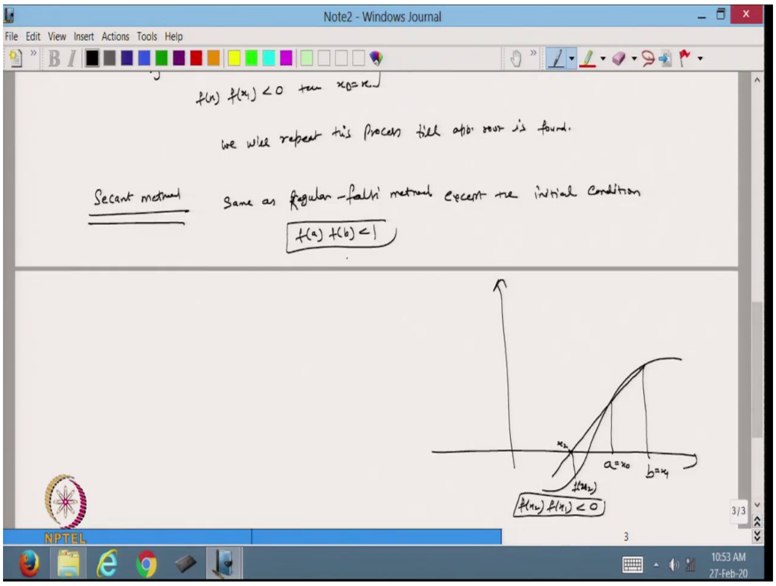


Another method is based on the same way, but that is another method and that is called the Secant method. So, it is the same as the Regula-Falsi method except the initial condition because in the Regula-Falsi we have to find f(a)  f(b) of opposite sign and then based on the intermediate value theorem we can start with the process, but in this case, no need of this one. So, it works the same way so let us write this one.

So, suppose I have a function like this and suppose in this case I will write take a function value  f(a) and this is another value  f(b) and I do not care whether f(a) and f(b) are of the opposite sign because in the Regula-Falsi I was taking my f(a) here and f(b) here so that was the opposite sign, but in this it does not matter. So, what I do I will find the value of Secant and I will check that where this value of the secant is cutting the x axis.

So, this is the place it is cutting the x axis. So, I can take this one and I will get this value that is my $x_2$ and you see that after this $x_2$ now I get the value of $f(x_2)$ and this is so I can choose any of this value into $x_2$. So, from here you can see that now my $f(x_2)\ f(x_1)$ are of the opposite sign. So, in this case now after that it will be the same verbally as the Regula-Falsi method.

So, except for the initial condition, this Secant method is the same as the Regula-Falsi method. So, there is no need to repeat this one only we have to change the initial approximation that whatever the initial point I will want to take we can choose that one so that is the thing.

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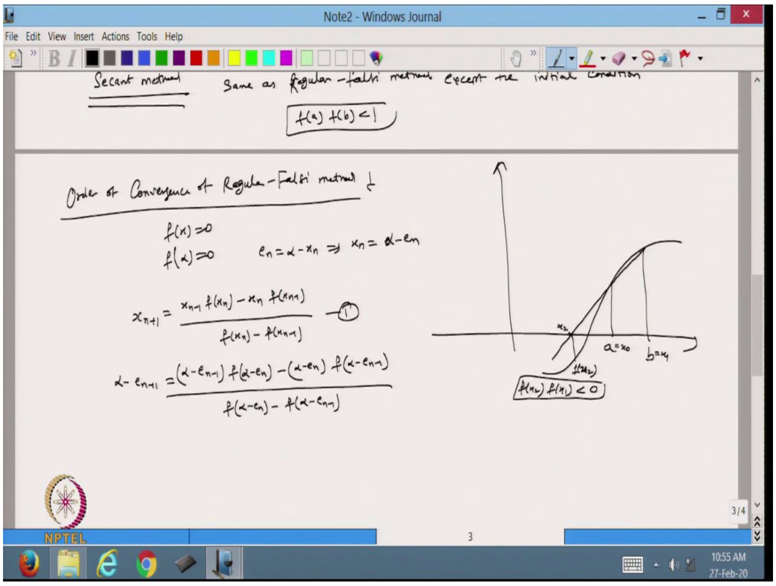


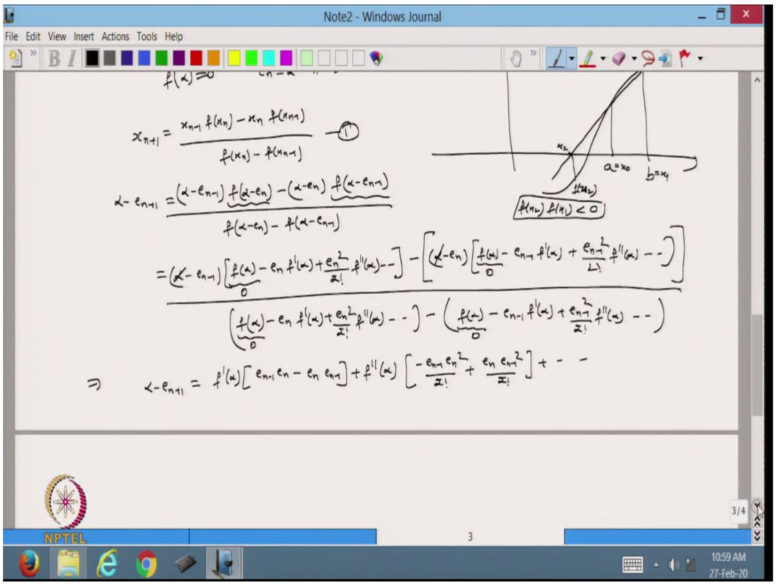


So, from here now I want to find the order of convergence of Regula-Falsi. So, how we can find the order of convergence of the Regula-Falsi method. So, I have my equation f(x)=0 and based on this one I can choose my $x_0$ so that alpha is the exact root so we can have the exact root here. Now, I can take my $e_n$ is equal to the error at the nth step. So, my Regula-Falsi method is this one: ($x_n = \alpha - e_n$)

$$x_{n+1} = \frac{(x_{n-1}\ f(x_n) - x_n f(x_{n-1}))}{f(x_n) - f(x_{n-1})}$$

$$x_{n+1} = \frac{(x_{n-1}\ f(\alpha - e_n) - x_n f(\alpha - e_{n-1}))}{f(\alpha - e_n) - f(\alpha - e_{n-1})}$$

$$\alpha - e_{n+1} = \frac{\left(\left(\alpha - e_{n-1}\right) f(\alpha - e_n) - (\alpha - e_n) f\left(\alpha - e_{n-1}\right)\right)}{f(\alpha - e_n) - f\left(\alpha - e_{n-1}\right)}$$

Using the Taylor series expansion as we have done in the previous case also.

$$f(\alpha - e_n) = f(\alpha) - e_n f'(\alpha) + \frac{e_n^2}{2!} f''(\alpha) - \frac{e_n^3}{3!} f'''(\alpha) + -\ldots$$

After substituting, we get

$$\alpha - e_{n+1} = \frac{\left(-\alpha f'(\alpha) + \frac{f''(\alpha)}{2}\left[\alpha\left(e_{n-1} + e_n\right) - e_{n-1} e_n\right]\right)}{-f'(\alpha) + \frac{f''(\alpha)}{2}\left(e_{n-1} + e_n\right)}$$

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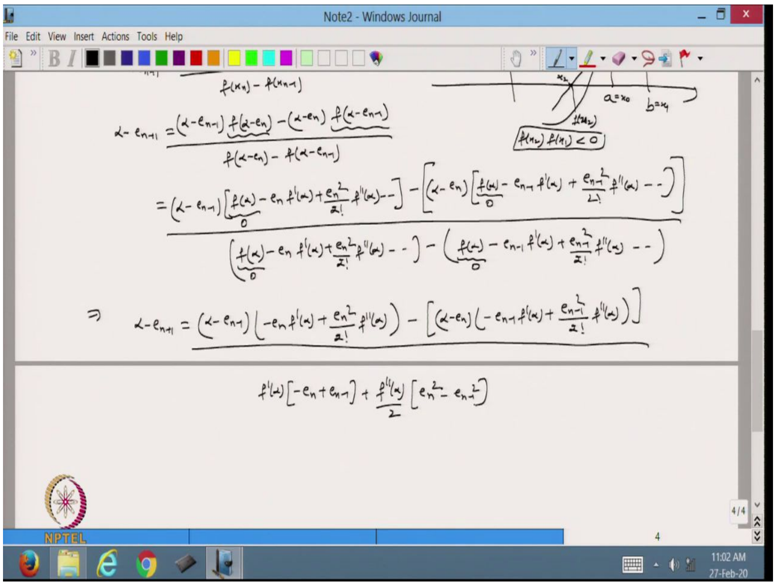

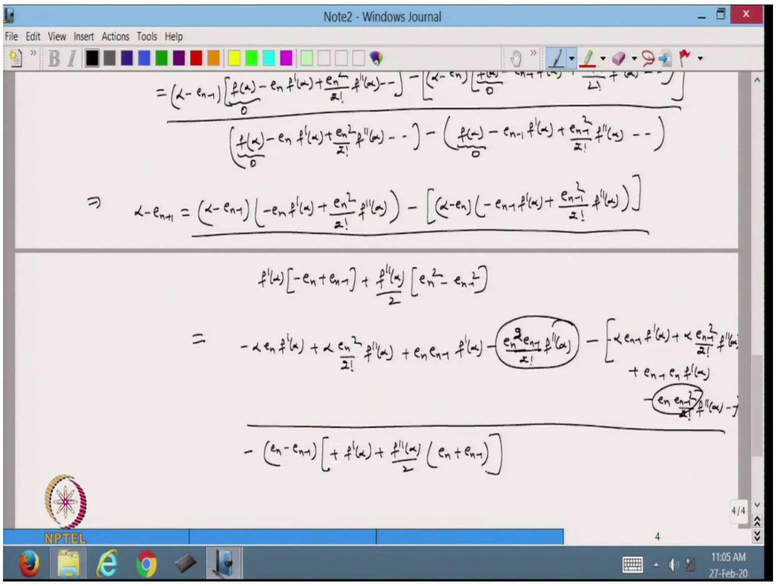


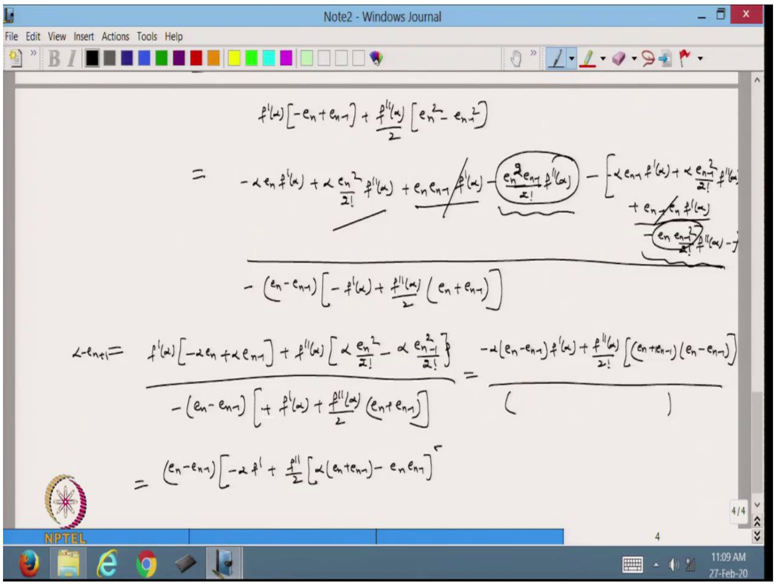


$$\alpha - e_{n+1} = \frac{(-\alpha + k(\alpha(e_{n-1} + e_n) - e_{n-1}e_n))}{-1 + k(e_{n-1} + e_n)}, \quad k = \frac{f''(\alpha)}{2f'(\alpha)}$$

So, this we will ignore all the powers 3 powers and more than that so we will ignore this one minus the same way I will get:

$$\alpha - e_{n+1} = (\alpha - k(\alpha(e_{n-1} + e_n) - e_{n-1}e_n))(1 - k(e_{n-1} + e_n)\ )^{-1}$$

So, this one is divided by this factor so from here I can choose my factor. From here I can find out. So, this one I can take common so this one I can write as f dash So, this I got and based on this one so from here I can write that

So, we get

$$\alpha - e_{n+1} = (\alpha - k(\alpha(e_{n-1} + e_n) - e_{n-1}e_n))\left(1 + k(e_{n-1} + e_n) + k^2(e_{n-1} + e_n)^2 + \ldots\right)$$

By ignoring the third and higher order terms in $e_{n-1}$ and $e_n$ , we get

$$\alpha - e_{n+1} = \alpha + k\, e_n e_{n-1}$$

After cancelling α, we get

$$e_{n+1} = k\, e_n e_{n-1} \text{ -----------------(1)}$$

$$k = -\frac{f''(\alpha)}{2f'(\alpha)}$$

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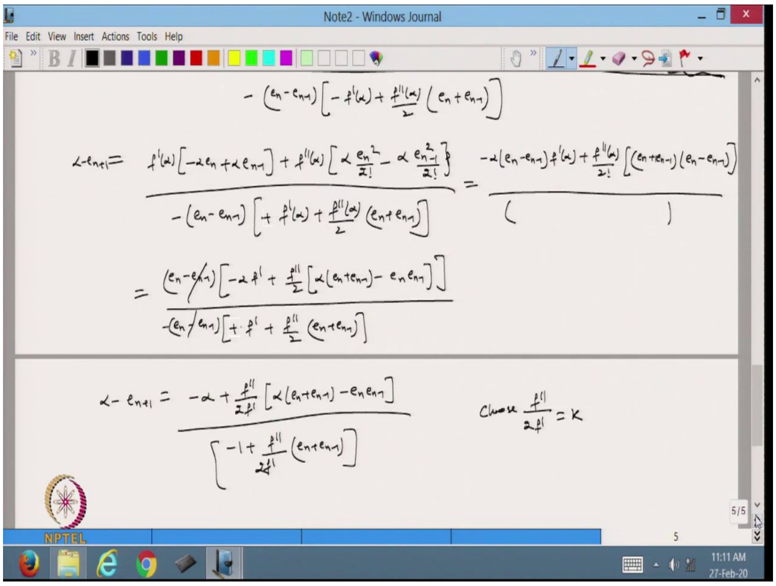

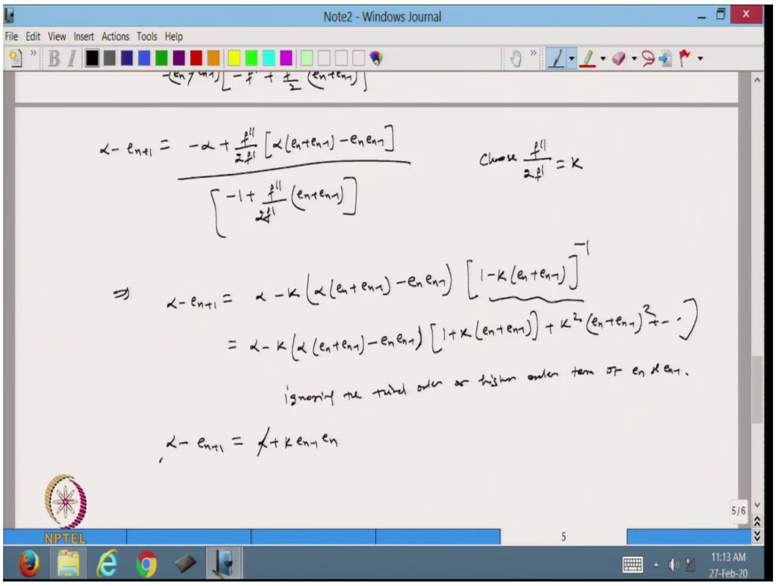

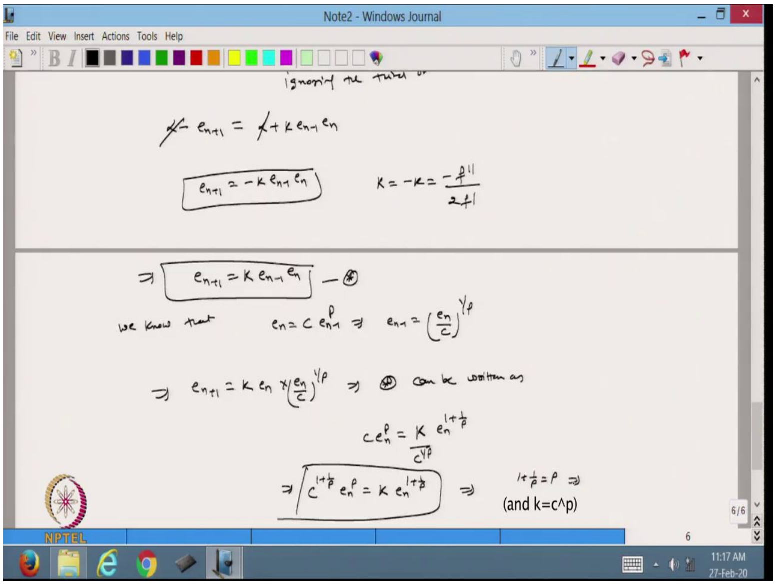

We know that

$$e_n = C\, e_{n-1}^{\,p}$$

So, we can write as

$$e_{n-1} = \left(\frac{e_n}{C}\right)^{\frac{1}{p}}$$

So, above equation (1) can be written as

$$C e_n^{p} = k\, e_n \left(\frac{e_n}{C}\right)^{\frac{1}{p}}$$

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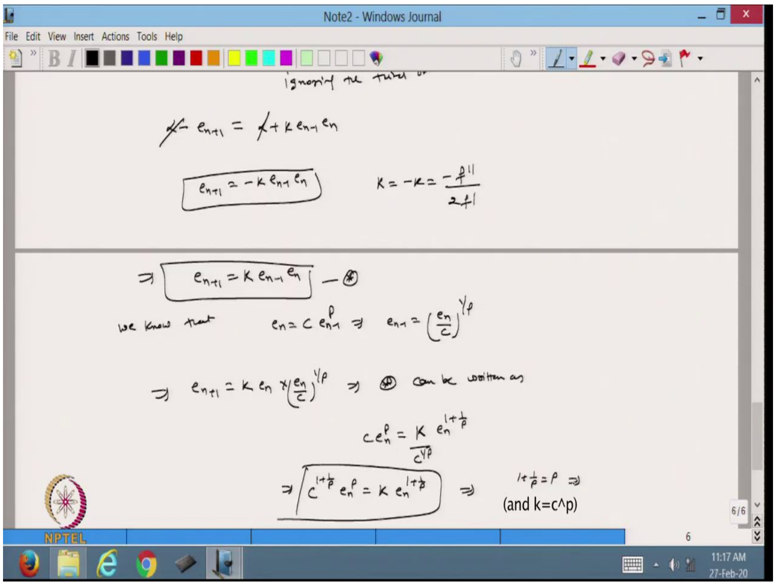


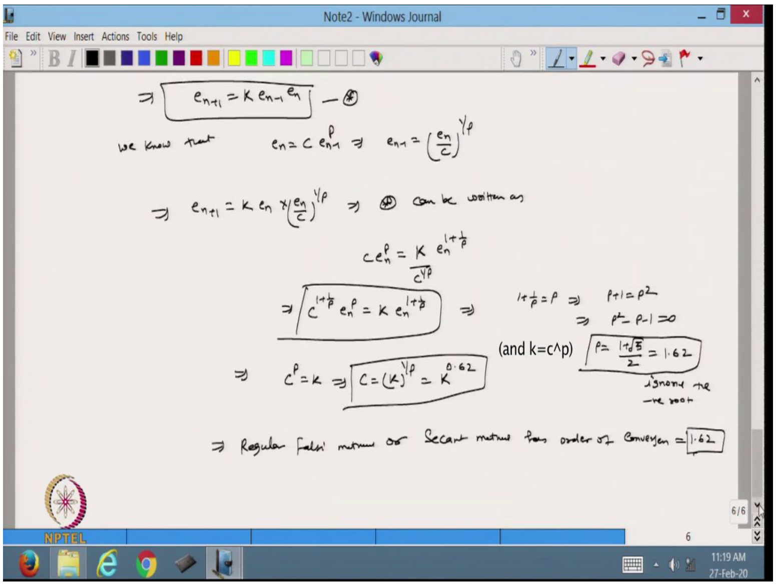


So, from here I can write

$$C^{1+\frac{1}{p}} e_n^{p} = k\, e_n^{1+\frac{1}{p}}$$

We get

$$p = 1 + \frac{1}{p} \; and \;\; C^{p} = k$$

I will get a quadratic in p so from here I can write

$$p^2 - p - 1 = 0.$$

On solving , we get

$$p = \frac{\left(1+\sqrt{5}\right)}{2}, \; \frac{\left(1-\sqrt{5}\right)}{2}.$$

Since p>0 so we get p=1.62

So, for the convergence I will choose only the positive value of p and that value of the p is coming this one. So, that is the order of convergence is 1.62. So, based on this one I can say that the Regula-Falsi method or Secant method has order of convergence that is equal to 1.62.

So, it is more than the linear convergence, but less than the quadratic convergence so it is in between the convergence we can have. So that is the order of convergence we get for the Regula-Falsi method. So, in this lecture we have discussed the Regula-Falsi method, the Secant method and then it is order of convergence and it shows that the order of convergence in this case is 1.62 as compared to the order of convergence in the bisection that was 1.

And in the case of fixed point iteration it was the 1. So, in this Regula-Falsi method we can call the high order method as compared to the iterative method fixed point iteration method. So thanks in this lecture so thanks for viewing thanks very much.