Introduction to Methods of Applied Mathematics
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Lecture - 06
Second Order Linear Differential Equations with Variable Coefficients

Hello viewers, welcome back to this course. So today we are going to start with the lecture number 6.

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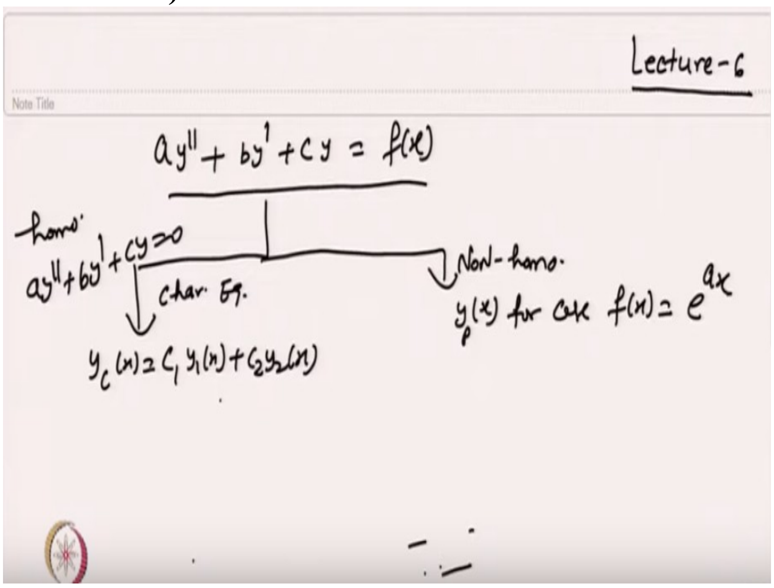

So in the previous lecture we have started dealing with second order non-homogeneous linear differential equation with constant coefficients. And in that case we have discussed that how we can solve. So this system we have solved with first we have solved with the homogeneous part and that is written as a y double dash + b y dash + cy = 0.

And from here we have solved this equation with the help of characteristic equation and then we found the complimentary solution that can be written as c 1 y 1 x + c 2 y 2 x. So in this case y 1 x and y 2 x they are the linearly independent solution of this equation. So once we know the solution of the corresponding homogeneous equation then we have started with solving non- homogeneous part.

So this is the homogeneous part. And in that case we are going to find out the particular solution and that we have represented by y p x. Now we have found the y p x for cases when f x was some exponential function a x.

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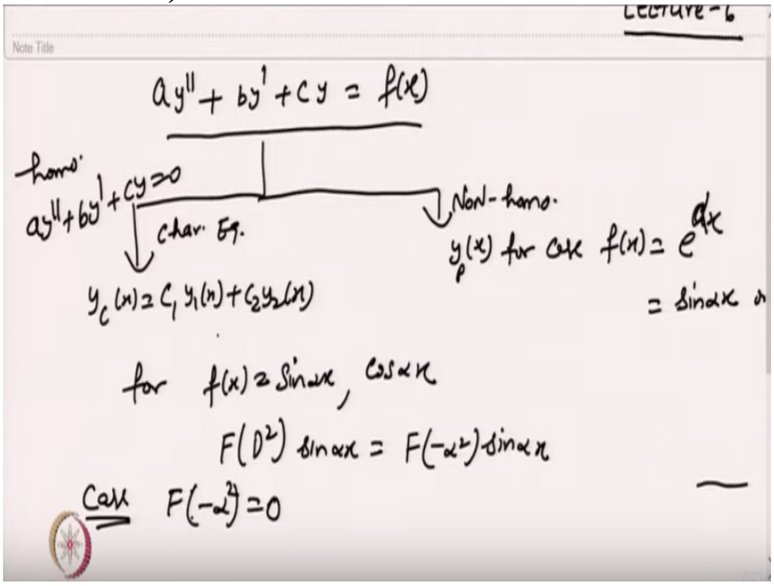

And also in the previous one we have started with a trigonometric function which has sin alpha x or this is also alpha you just take because a we are taking as a coefficient of y double dash. So sin alpha x or cos alpha x. So in that case we have already solved that how we can solve this type of differential equation. Now for the sin, for f x is equal to sin alpha x cos alpha x in the last class we have discussed that we get F D square sin alpha x and we put instead of D square we put F of minus alpha square sin alpha x and then we solved.

So in the last class we have solved one example also. Now I am taking the case, so this is the case I am taking. What about when F of minus alpha square becomes zero. So this is the same case as we have also done for the exponential case. When e raised to power alpha x and alpha is the root of the corresponding characteristic equation. So then how we can solve this one? So in this case, so let us do one example.

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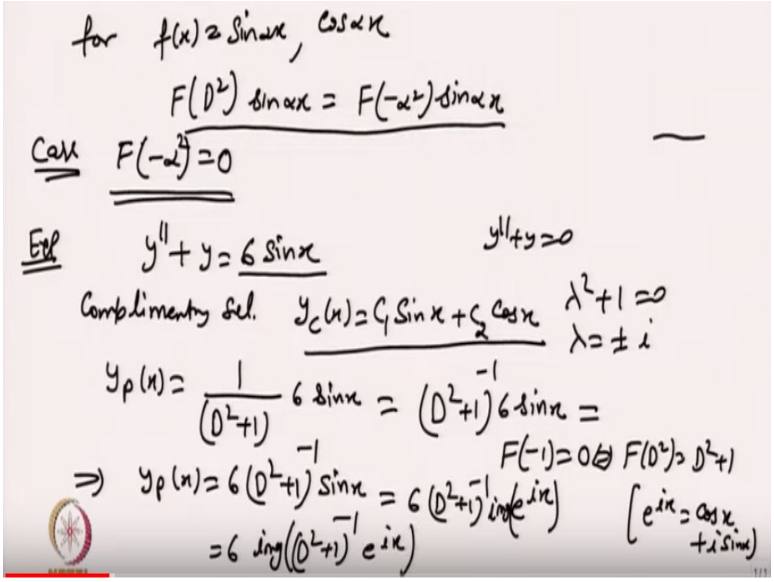

So this can be done with the example. So let us take one example. So example I am going to solve that suppose we have y double dash plus y is equal to 6 and sin x. So in this case, I know that my complimentary solution y c x will be because the homogeneous part will be y double dash plus y equal to zero from here I will get the characteristic equation D square sorry I will get the characteristic equation lambda square + 1 = 0.

And from here I will get the lambda is equal to plus minus iota. So in that case I will get the solution. So my complimentary solution will be c 1 sin x + c 2 cos x. Now so this is my complimentary solution. And you can see from here that sin x is appearing in the solution and sin x is also the function on the right hand side. So in this case, now I want to find the particular solution.

So how to find out the particular solution, so that we are going to discuss. So my particular solution y p x in this case will be 1/D square + 1 6 sin x. Because some time also we can write this one as generally we write this as – 1 6 sin x. But in the some book some just to solve this one this inverse we also represent by this way. So this one I want to solve.

Now if I put and apply the same formula what we have done here in this case if I put minus 1 instead of D square then this become zero. So this is the case when F(- 1) become 0 because my F(D square) is D square + 1. So when I put this instead of D square I put -1 then it become 0. So this is the case from here. Now, so after doing this one so how to solve this type of equation. So there is a always some tricks to solve this type of equation.

So in this equation what I will do is that I will take D square plus 1 inverse 6 I can write here and sin x. So this I generally what we do is that I will write this as same and the sin x I will take e i x because e i x I know that e i x is cos x plus i sin x. And I want to solve for the sin x. So I will put imaginary part of e i x and this further can be reduced to 6 D square plus 1 inverse and from here I can take this imaginary part here. So I can write 6 and the imaginary part of this e i x. this one I can do.

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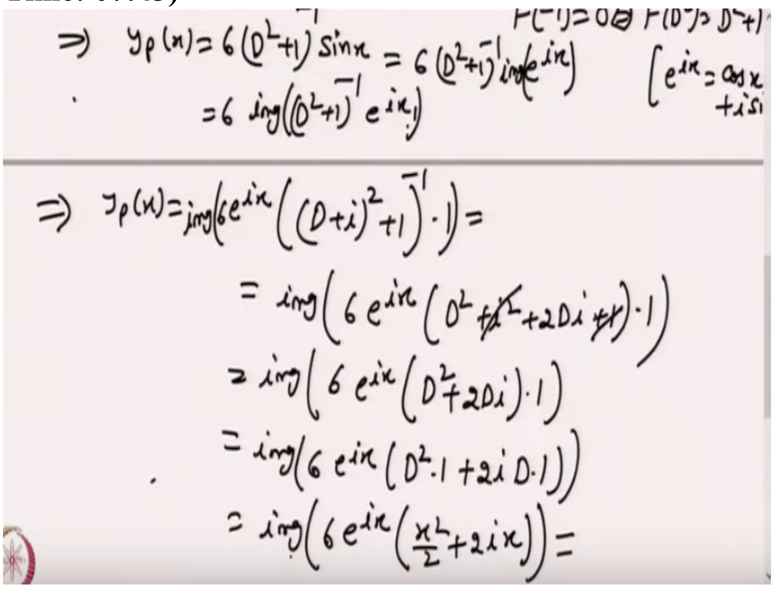

Now so after doing this one from here what I will get is that my y p x become 6 and now I will apply the formula taking multiply by 1. What we have done that taking the product of two function and then operating it with some operator. So what we do is that in this case I will apply the previous formula and then I will take e i x on the left hand side and my operator will become D + i whole square plus 1 into 1.

So in this case this can be written as e i x and imaginary part is also there. So you always do not forget to write imaginary part. So I will write here 6 and then imaginary part. And in this case also I will write imaginary part of 6 e i x and this one I can expand. So it become D square + i square + 2 D i + 1 and operating on 1. So this is my operator. So this becomes 6 e i x and this D square will be there.

This i square is – 1 so this will cancel out with this. So we will get 2D i operating on 1. Now, so this is my operator which can further be reduced to imaginary part of 6 e i x and now this is the one so from here I can take this function as D square 1 plus 2i D 1. This one can be written like this one. So I will write in this case imaginary part 6 e i x and D square of 1 is if I am taking the one time integration it will be x.

So another time it will be x square by 2 plus 2 i into x. So this will become. So after doing this one I just want to take the imaginary part of 6 e i x and this become so further I can reduce this one.

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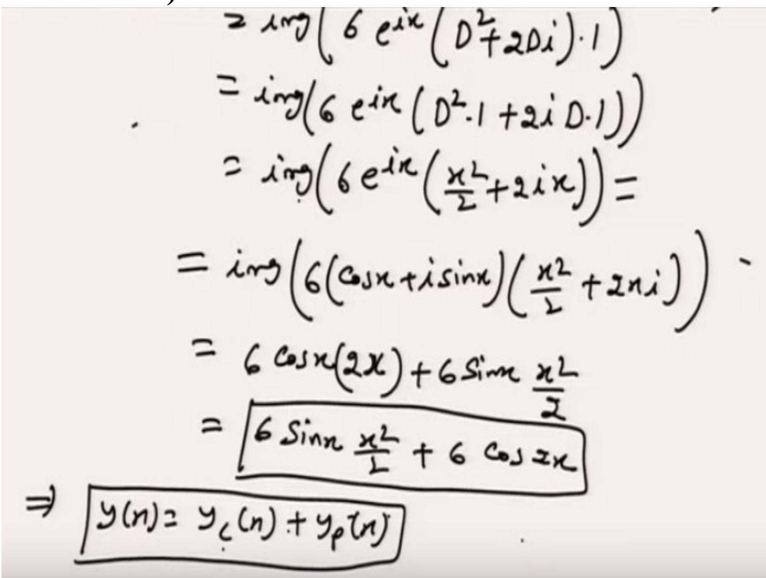

So this equation can be further reduced. Now it becomes 6 and this is cos x plus i sin x and multiply by x square by 2 plus 2 x i. So this one I want to do. Now I want to find the imaginary part. So now multiply. So if I take multiply by 6 cos x multiplied by x square by 2 then this is the real part. But if I take 6 cos x into 2 x then it will be a imaginary part. So it becomes 6 cos x into 2 x and then from here i sin x will multiply by this one.

So it will be 6 sin x x square by 2. And the other part here will be i into i that will become the real part of that one. So from here so from here what I will get is now then sin x is x square by 2 plus 6 cos 2 x. So this is my particular solution and then from here I can write my general solution y x. So this will be y c x plus y p x. This can be it. Now the thing is that okay, no there is a one mistake I have done.

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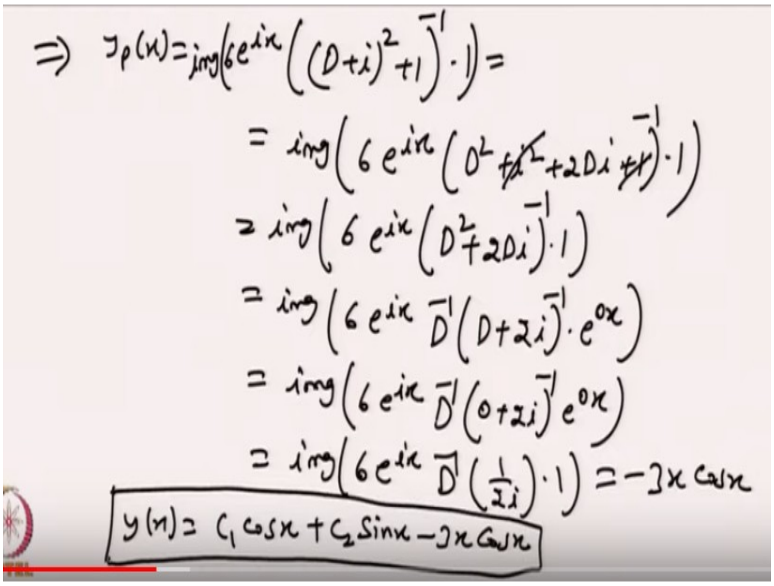

It was the inverse also here. So we have to go back and just erase this one because it is the inverse function. So we have to do the inverse here. So let us do again. So this will be imaginary part of and from here I can take my D inverse common. So inside I will get D + 2i and that is the inverse of 1. And 1 I can put as 0 x. So from here I will get imaginary part of 6 e i x and D inverse and this one I can solve just what we have solved for the exponential function.

So I put instead of D I put the 0 here. So I will get 0 + 2i inverse into e 0 x because one can be done like this one. So from here I will get imaginary part of 6 e i x and D inverse and this will becomes 1/2i into y. So if you solve it further you will get 3 x cos x. So that will be the your solution. So in that case my general solution will be c 1 cos x + c 2 sin x – 3x cos x. So that is your general solution for this particular problem.

Now this is the case we have solved for the equation when the right hand side function is a sin function. Now we go further and take the next case.

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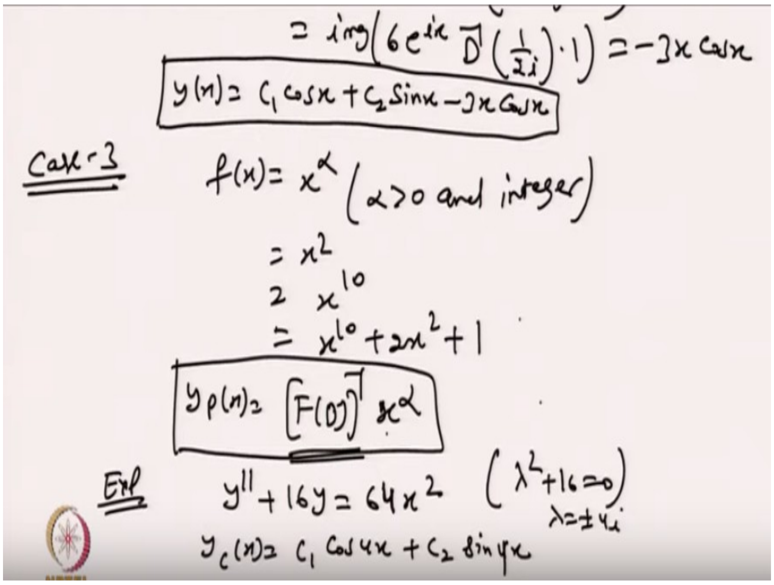

This is case number 3 when my right hand side function f x in the equation is x raised to power some alpha where the alpha is positive and integer. So like on the right hand side I may have x square, I may have x raised to power 10, I may have x power 10 plus 2 x square plus 1 like this one. So when the a polynomial of this type on the right hand side then how we can solve this one.

So in that case my particular solution will be F D inverse x raised to power alpha. Now x raised to power alpha is that and this is the integral operator. So how we can solve this type of solution when we have y p x. So let us do one example and based on that example we can check or we can show that how this type of function on the right hand side can be solved. So let us solve this example.

So let us take y double dash + 16 y = 64 x square. So 64 x square. So in this case my y c x will be c 1 cos 4 x + c 2 sin 4 x. Because I know that my characteristic equation will be lambda square + 16 = 0 and from here I get the lambda is equal plus minus 4i and then I can find the solution based on that one. So from here I want to find the particular solution.

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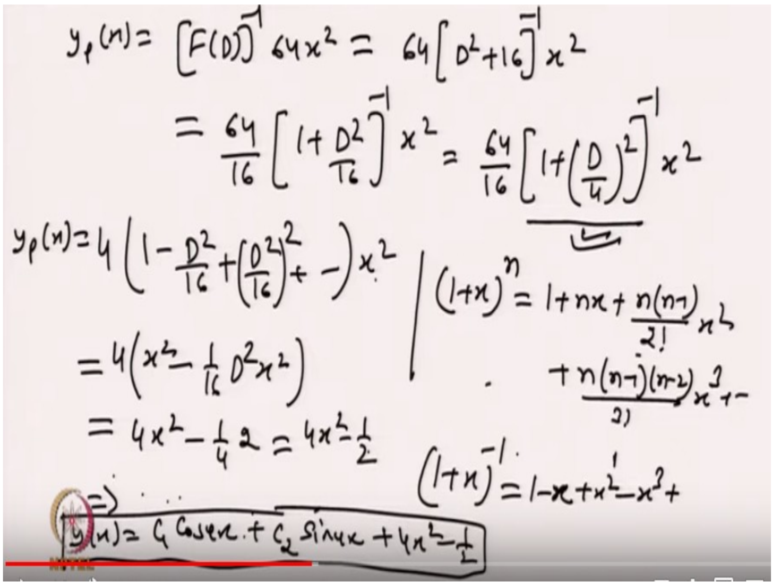

So how we can find the particular solution. So in the particular solution I will get my F D inverse into 64 x square. So that is my solution on the right hand side. This can be written as I can take 64 here and my F D in this case will be D square plus 16 inverse x square. So this again can be written as 64 and I can take from here the 16 I can take common. So if I take the 16 common with the inverse so it will become 16 here and inside I can write D square over 16 inverse x square.

Now this further can be reduced and this can become 64 by 16 1 plus D by 4 square inverse x square. Now what you do is that this one I can solve with the help of the binomial expansion. Like I know that the binomial expansion of 1 plus x raised to power minus n, minus n or plus n I can just do with the any value of n. So this can be written as 1 + nx + n(n – 1) by 2 factorial x square + n (n – 1) (n – 2) by 3 factorial x cube and so on.

So what you do is that we expand this inverse with the help of this binomial expansion. So once I so after doing this one so from here my y p x can be written as so it will be 4, so this will be 4. And then this I expand so I will expand it by the formula. So it will be D square by 16 from here. Then I can write this one as, so in this case my n is – 1. So 1 + x minus 1. So what you will get from here.

You will get 1 – x because instead of n I put the – 1 and this will be – 1 and – 2 this will cancel out. So this will become x square. Again this become x cube and so on. So from here I can apply this formula. So this will become D square over 16 square by this formula and so on operating on x square. Now this will keep going because this is a infinite series.

But I know that x square is the function and if I take the first derivative it will be 2x, second derivative it will be 2 and after that it will become 0. So in this case this infinite series will truncate till D square. After that they no need to take the terms of belonging to D square more than the power of D square. So it becomes plus. So I can operate this on the x square.

So it becomes x square minus 1 by 16 D square x square and that is it. So after that because it becomes the power of 4. So that operating on x square it become 0. So from here I will get 4 x square minus and this will be 1/4 and it will be 2. So this will be 4 x square minus half. So once I get 4 x square minus half so from here I can solve my equation y x = c 1 cos 4 x + c 2 sin 4 x and then this solution y 4 x square – 1/2.

So that is the solution of the equation when the right hand side function is 64 x square. So this type of terminology we can use whenever we have a function on the right hand side and that is the polynomial.

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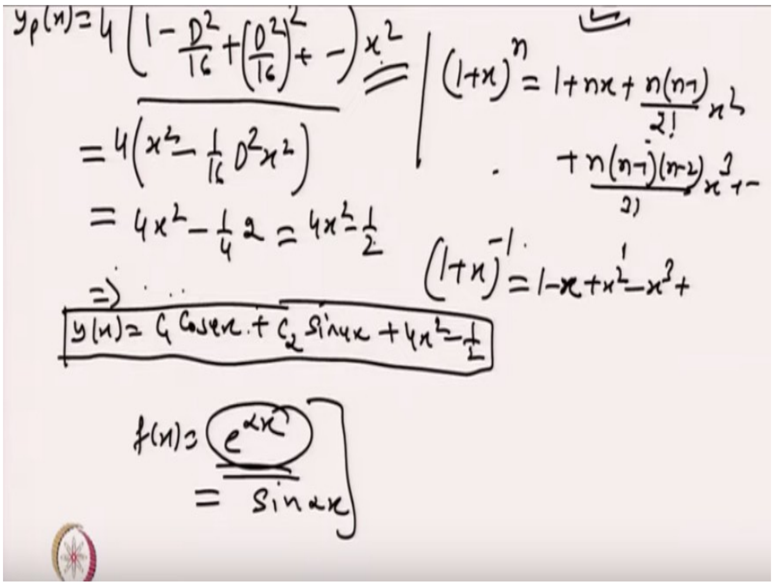

This same things we can apply the question comes that why cannot we apply the same thing for the other function like the exponential function or the sin function. Because in this case we know that we have a infinite series. And suppose I take the function f x on the right hand side is alpha x and I operate this series on this function. But this is exponential function.

So if you keep taking the derivative of this exponential function it is not going to become zero. Does not matter that you are which order you are taking the derivative of this function. So this function the derivative of this function never become zero. So in that case we are unable to stop or truncate this series. So that becomes very difficult to find out.

So that is why we cannot apply this method for this type of function or we cannot apply this type of terminology for the function whenever the trigonometric function is there. So in that case also this sin alpha function it keep going on whenever we take the derivative and is not going to become zero. So that is why these two functions we cannot apply this terminology but whenever we have a polynomial of this type whose derivative after some time become zero in that case we can apply this formula.

So this one is over. Now after that now till now I am able to solve the differential equation whose coefficients are constant and the right hand side function is either the exponential or sin, cos function or some polynomial.

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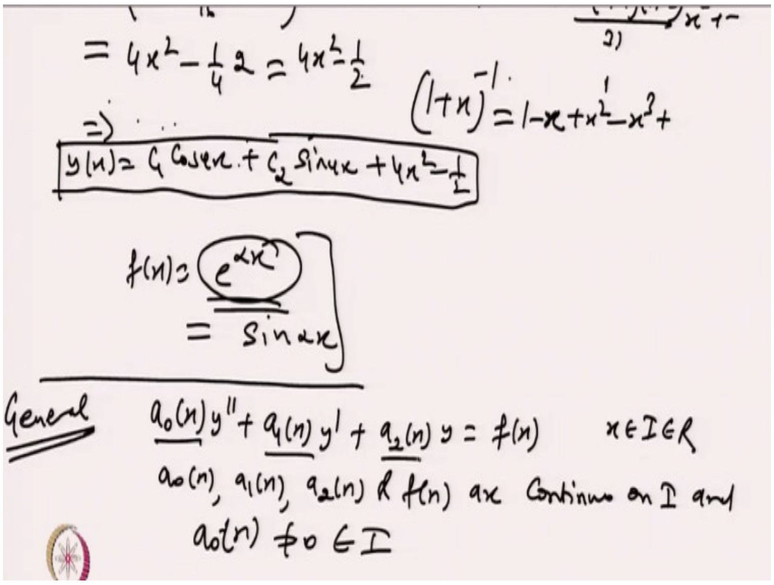

But what about if I want to solve a differential equation such that I have some general form a 2 x y is equal to some function f x where my x belongs to some interval, belongs to real line and my function a naught x, a 1 x, a 2 x and my f x are continuous on I and a naught x is never zero belongs to I. So in that case I have now a general, so this is the general second order linear differential or non-homogeneous differential equation. So this is my functions and now I do not know that how we can solve this type of equation.

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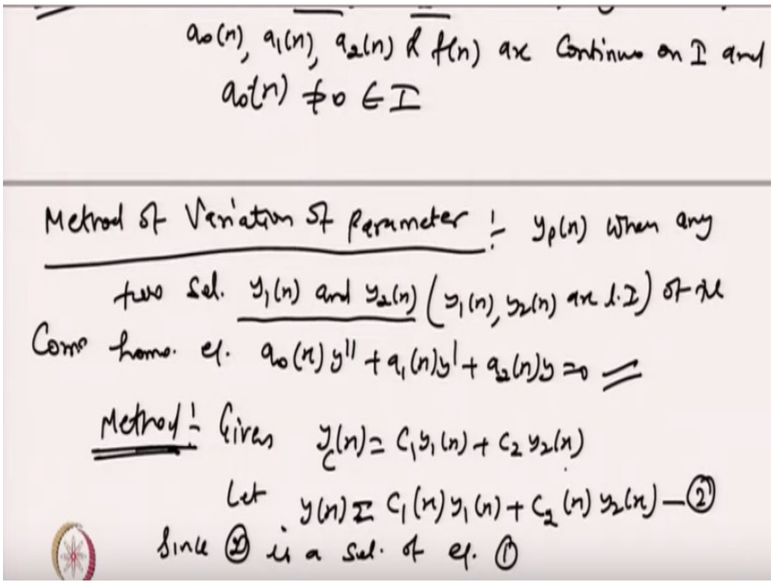

But there is a very famous method, that method of variation of parameter. So this method is used to find particular solution of the equation let us give it a name. So I will get it name a equation number 1. So this method of variation of parameter is used to find the particular solution of general equation 1. So this is when any two solutions y 1 x and y 2 x. So this y 1 x and y 2 x are linearly independent.

So this we can find that the particular solution can find when any two solution y 1 x and y 2 x of the homogeneous, corresponding homogeneous equation a naught x y double dash + a 1 x y dash + a 2 x y = 0 is known to us. So this is the very strong condition that y 1 x and y 2 x which are linearly independent and the solution of this equation should known to us.

Only then if I know this solution I can proceed to find out the particular solution of any differential equation with any f x. because f x is exponential then I can I know that how to find out but this is in this case we are taking the general form. So a naught x, a 1 x and a 2 x. This is the function of x. in the earlier cases we are taking the function where this a naught, a 1, a 2 are the constant function.

So this is the condition. So this is the method that how we can proceed to find out the particular solution and this is known to me. Now let y p x is my solution so I should write not here y p just y x. I just write y x. Let I take y x is a solution of so I know that if I know the value of y 1 x, y 2 x I can write my complimentary solution as c 1 y 1 x + c 2 y 2 x. So this is known to me, right. So from here so this is given.

I just write it is given to me. So now I will take that let my solution y x I take as c 1, so we just write down 1 here. It is a in this case it was a constant, arbitrary constant c 1 and c 2. But let us assume that c 1 is a function of x and this is y 1 x plus c 2 is also a function of x and this is y 2 x. So this is I assuming that my c 1 x and c 2 x are the function of x instead of c 1, c 2 that we have taken here as a constant.

So as the method say the method of variation of parameter. So I can call it that c 1 and c 2 are the parameters and this is varying that it is a function of x. Now, so let I take that this is 2.

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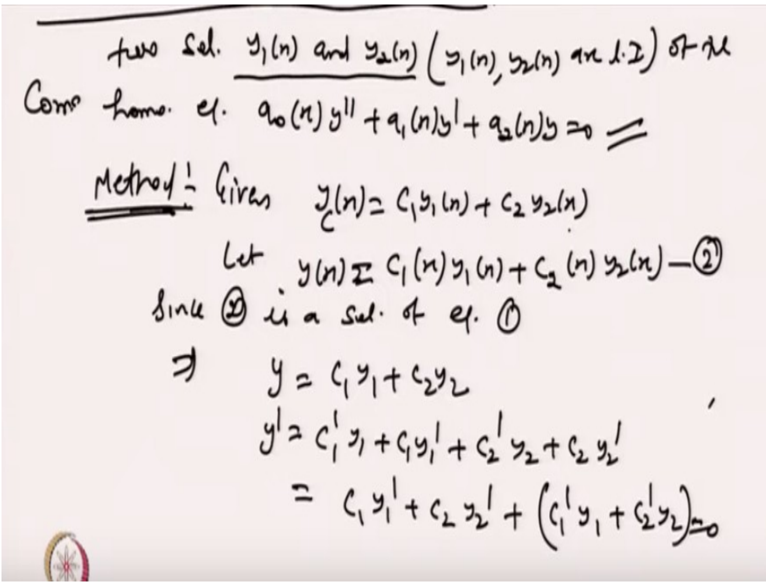

So 2 is the solution of since 2 is a solution of equation 1 then now for just for the simplicity to do the some manipulation I am taking the derivative. I will take the derivative. I will write this equation number 2 as y is equal to c 1 y 1 + c 2 y 2 because we have to take the derivative, first derivative, second derivative. So that is why I am writing the equation number 2 in this form.

But it is understood that y is a function of x, c 1 is a function of x, y 1 and y 2 are the function of x and c 2 is also a function of x. So that is understood. Now from here I take y dash, so y dash will be the first derivative and I apply this one. So it becomes c 1 dash y 1 + c 1 y 1 dash + c 2 dash y 2 + c 2 y 2 dash. So from here I will just write here c 1 y 1 dash + c 2 y 2 dash + c 1 dash y 1 + c 2 dash y 2. So this factor I just take and make this factor is equal to zero.

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Let $y(x) = c_1(x) y_1(x) + c_2(x) y_2(x)$ — (2)

Since (2) is a sol. of eq. (1)

$\Rightarrow y = c_1 y_1 + c_2 y_2$

$y' = c_1' y_1 + c_1 y_1' + c_2' y_2 + c_2 y_2'$

$y' = c_1 y_1' + c_2 y_2' \quad \boxed{(c_1' y_1 + c_2' y_2) = 0}$ — (3)

$y'' = c_1' y_1' + c_1 y_1'' + c_2' y_2' + c_2 y_2''$

Eq. (1) becomes

$$a_0(x)\left[c_1' y_1' + c_1 y_1'' + c_2' y_2' + c_2 y_2''\right] + a_1(x)\left[c_1 y_1' + c_2 y_2'\right] + a_2(x)\left[c_1 y_1 + c_2 y_2\right] = f(x)$$

So from here I take equation that this is zero. So I call it third. So I will remove this one, okay. So this come here. So my y dash become this one and y double dash become c 1 dash and then y 1 dash + c 1 y 1 double dash + c 2 dash y 2 dash + c 2 y 2 double dash. Now I substitute the value of y, this y, y dash and y double dash in the equation number 2, in the equation number 1.

So equation number 1 becomes a naught x and then I put this my value. So c 1 dash y 1 dash + c 1 y 1 double dash + c 2 dash y 2 dash + c 2 y 2 double dash + a 1 x and then I will put this one c 1 y 1 dash + c 2 y 2 dash + a 2 x and then I will put c 1 y 1 + c 2 y 2 is equal to my f x.

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$$\Rightarrow \quad y = c_1 y_1 + c_2 y_2$$

$$y' = c_1' y_1 + c_1 y_1' + c_2' y_2 + c_2 y_2'$$

$$y' = c_1 y_1' + c_2 y_2' \qquad \boxed{(c_1' y_1 + c_2' y_2) = 0} \; - (3)$$

$$y'' = c_1' y_1' + c_1 y_1'' + c_2' y_2' + c_2 y_2''$$

Eq. (1) becomes

$$a_0(x)\left[c_1' y_1' + c_1 y_1'' + c_2' y_2' + c_2 y_2''\right] + a_1(x)\left[c_1 y_1' + c_2 y_2'\right] + a_2(x)\left[c_1 y_1 + c_2 y_2\right] = f(x)$$

$$\Rightarrow c_1\left[a_0(x) y_1'' + a_1(x) y_1' + a_2(x) y_1\right] + c_2\left[a_0(x) y_2'' + a_1(x) y_2' + a_2(x) y_2\right] + a_0(x)\left[c_1' y_1' + c_2' y_2'\right] = f(x)$$

So I will substitute this one. So from here I will just simplify this one and I will (collect) the terms corresponding to c 1 and corresponding to c 2. So I will just (collect) the terms corresponding to c 1. So c 1 becomes, it becomes a naught x y 1 double dash and then c 1 can be taken from here. So it will be a 1 x and then y 1 dash and from here it will be a 2 x and y 1. So this is the corresponding c 1 we have collected.

Now I take the terms corresponding to c 2, again the same. So it will be a naught x y 2 double dash plus from here it will be a 1 x y 2 dash plus a 2 x and y 2. So this become plus the remaining term I will write here. So remaining terms this is the term I am getting. So from here I will get a naught x. From here I will get c 1 dash y 1 dash plus c 2 dash y 2 dash. So this is the terms, remaining term I am collecting and then become it is equal to f of x.

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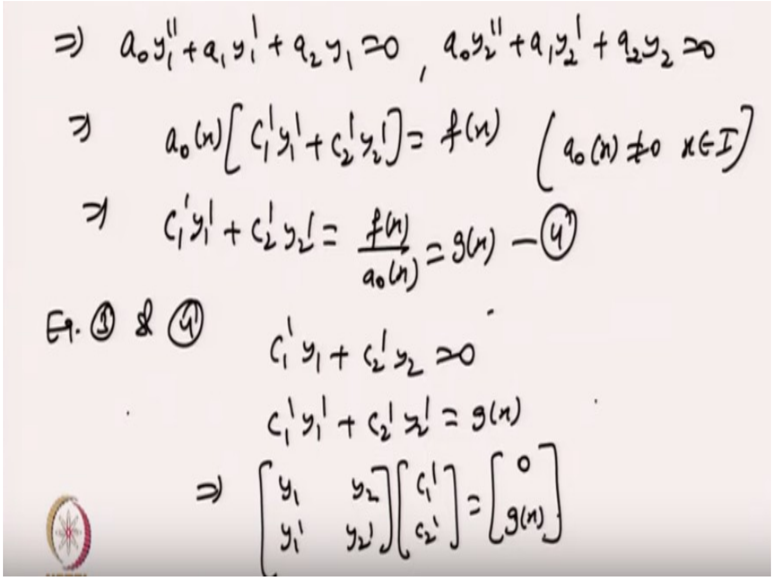

Now, so once I get this value, now what about this function, this equation. What about this equation? Now if you see here that y 1 x are the solution of the corresponding homogeneous differential equation and y 2 are the solution of the corresponding homogeneous equation. It means that my a naught y 1 dash + a 1 y 1 dash + a 2 y 1 is zero. Because that is the solution of the homogeneous part.

Similarly, my a 2 y 2 dash + a 1 y 2 dash + a 2 y 2 that is also equal to zero. So I will put this part equal to zero and this part equal to zero. So from here I can write that I will be left with only a naught x and then c 1 dash y 1 dash + c 2 dash y 2 dash is equal to my f x. Now I have assumed that my a naught x is not equal to 0. So this is we have assumed that is not equal to 0 where x belongs to the bounded interval whatever we have taken.

So from here I can write this as c 1 dash y 1 dash + c 2 dash y 2 dash is equal to my f x divided by a naught x. So we call this as a new function. Let us call it g x. So I will call it equation number 4. So now I will solve, so from the equation number 3 and 4. The equation third was c 1 dash y 1 c 2 dash y 2 equal to zero and another equation is this one y 1 dash + c 2 dash y 2 dash is equal to my g x.

So this is if you see that this is system of equation and from here I can write this system as y 1 y 2 y 1 dash y 2 dash. So this is my matrix. Here I will call it c 1 dash c 2 dash and right hand side function become zero and g x. Now this is the system of equation. So I can solve this system because I want to find here c 1 dash and c 2 dash.

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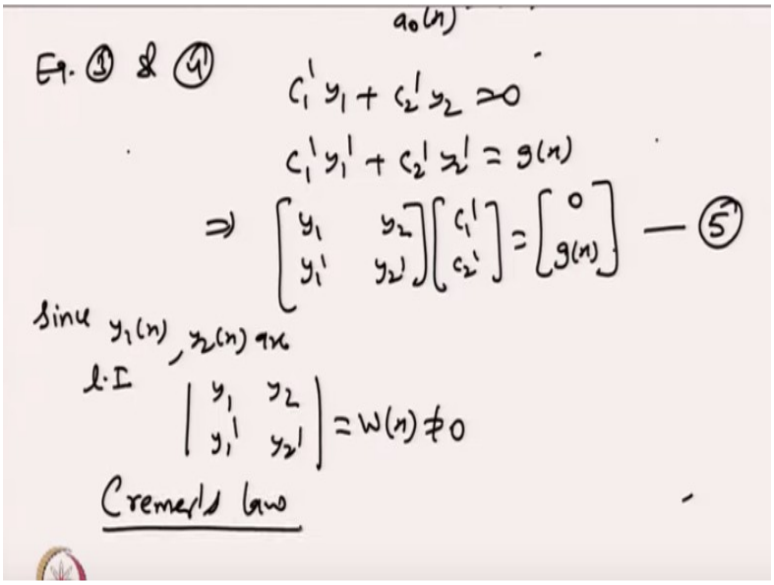

So from here if I want to find c 1 dash and c 2 dash I will take the help of this matrices and now what about this y 1. I know that y 1 x and y 2 x they are linearly independent. So from here I can say that since y 1 x and y 2 x are linearly independent it means that y 1 y 2 y 1 dash y 2 dash so this is I know that this is Wronskian and this is not equal to zero. So if this Wronskian is not equal to zero then I can solve this system with the help of Cramer's law.

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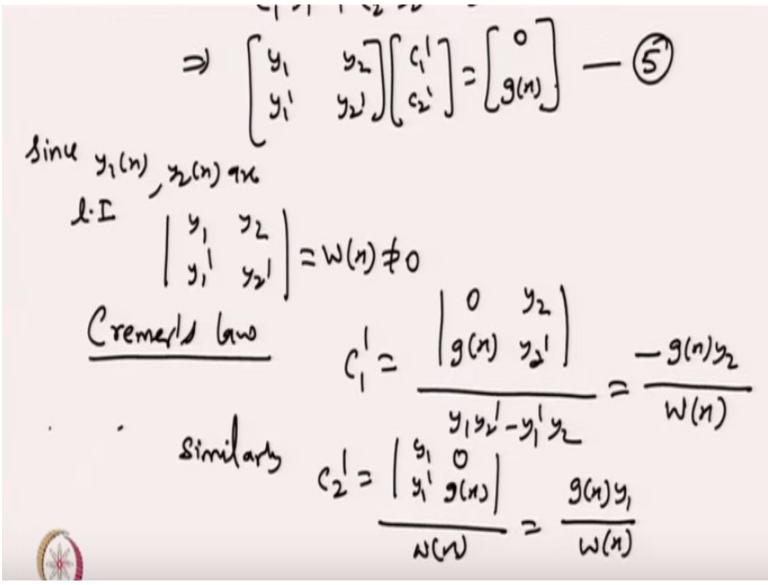

So if I apply the Cramer's law I can find the solution of this one. So my in that case c 1 dash will be just substituting the right hand side vector at the first column. So the solution becomes zero g x y 2 y 2 dash divided by the determinant of this matrix and that determinant I know that will be y 1 y 2 dash – y 1 dash y 2. and this is I know that this is Wronskian.

So from here I can write this one as so if I take the determinant of this one. So it will be minus g x y 2 divided by the Wronskian that is a function of x. So from here I will get the c 1 dash. Similarly, my c 2 dash I can find out with the help of Cramer’s law. So this will be y 1 y 2 dash and this will be 0 g x and divided by the Wronskian as a determinant of the given matrix. So from here I will get this side g x y 1 divided by the Wronskian w x.

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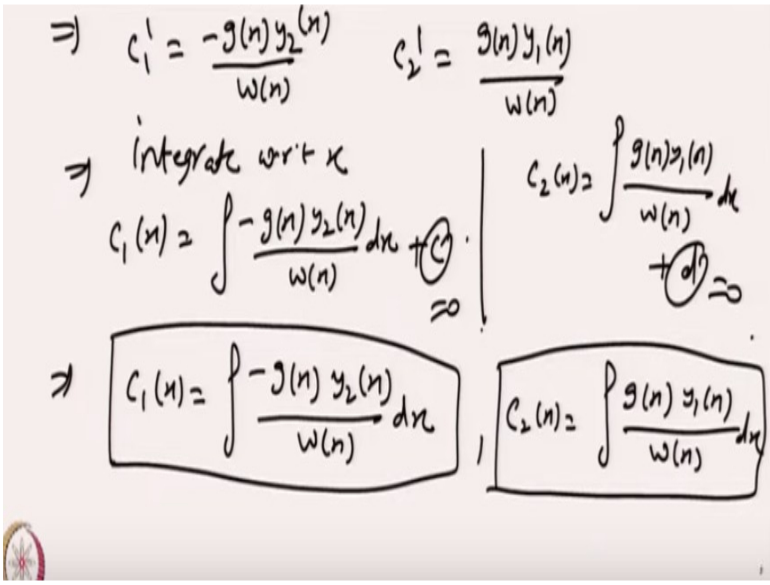

So from here, now I am able to find the value of c 1 dash becomes – g x y 2 as so I know that y 2 is a function of x. So I can write here as this one and c 2 dash is also I know that the function of x. So it will be y 1 x and w x. Now I want to find the value of c 1. So from here I want to find the value of c 1 because I wanted to find out what will be the c 1 x and c 2 x. So I want to find the value of c 1.

So I will integrate both side, this. So what you write here? Integrate with respect to x both side. So on the left hand side I will get c 1 x, now I will add c 1 x on the left hand side. And on the right hand side I will get – g x y 2 x divided by w x d x. this is the integration so + c I can take as a integration, constant of integration and also I will get my c 2 x from here integration it will be g x y 1 x and w x d x plus constant of integration d.

Now in this case I need the solution, only one solution I needed so I will take this as 0 and this as 0. So from here I will get my c 1 x is equal to – g x y 2 x because g x is known to me, y 2 x is known to me, w x is known to me. So that become my c 1 x and from here I will get c 2 x that will be g x y 1 x over w x d x. So this is I can find out once I know the value of g x, y 1 x, y 2 x, w x and then once I get this value.

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$\Rightarrow$ integrate w.r.t x

$$c_1(x) = \int \frac{-g(x)\,y_2(x)}{w(x)}\,dx + c, \quad c = 0 \qquad c_2(x) = \int \frac{g(x)\,y_1(x)}{w(x)}\,dx + d, \quad d = 0$$

$$\Rightarrow \boxed{c_1(x) = \int \frac{-g(x)\,y_2(x)}{w(x)}\,dx}, \quad \boxed{c_2(x) = \int \frac{g(x)\,y_1(x)}{w(x)}\,dx}$$

$$\boxed{y_p(x) = -\int \frac{g(t)\,y_2(t)\,y_1(x)}{w(t)}\,dt + \int \frac{g(t)\,y_1(t)}{w(t)}\,y_2(x)\,dt}$$

So from here after finding the value of c 1 and c 2 x I can find my solution. So that solution I am looking for was the particular solution. So in this case I will get, so this is my c 1 x. So I can find my I can write my here g x y 2 x w x d x, so this is my c 1 x. And now I am writing here in the term of x. So I should just take so I can write this as t and y 1 x dt plus I can write here my g t y 1 t w t into y 2 x d t. So from here I can write, so that is my particular solution.

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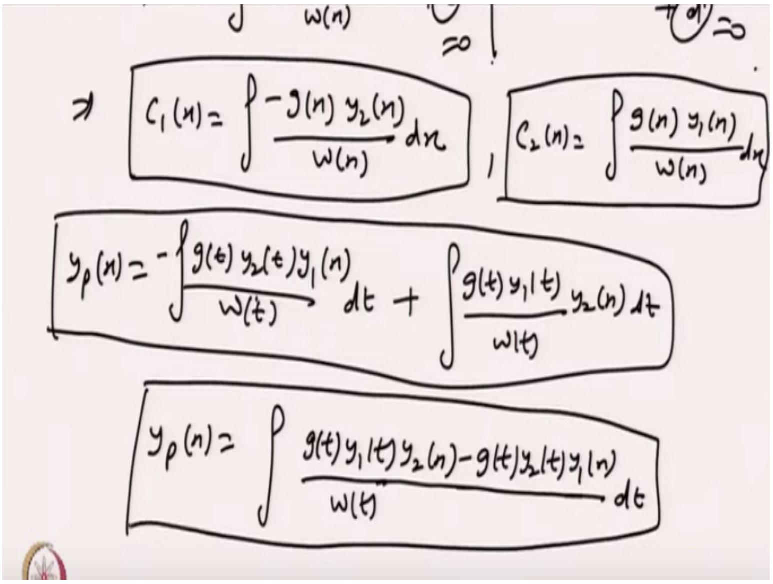

So this particular solution further you can make it and you can write this as integration so I can write from here that g t y 1 t from here w t y 2 x this one minus g t y 2 t y 1 x d t. So that is my particular solution.

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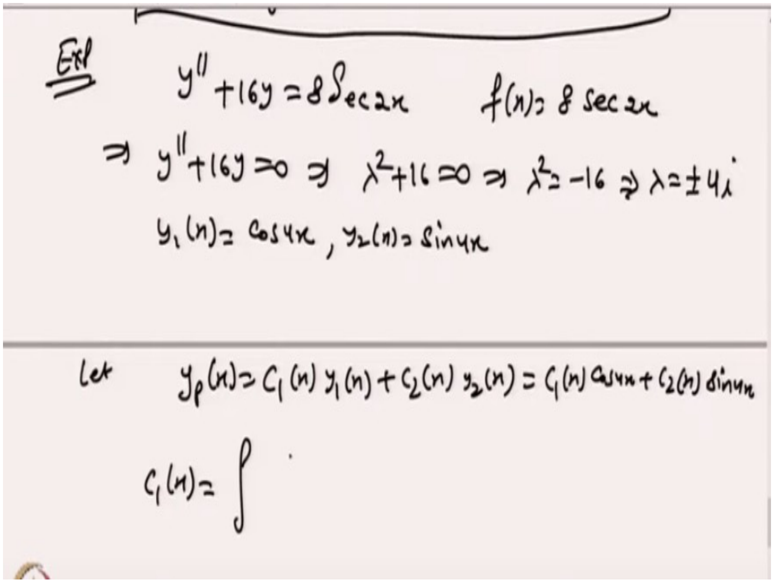

So now whatever the function f x we are taking on the right hand side that function is a continuous so that is integrable in the given interval so from here I can find this integration and then we get the particular solution. So let us do an example. So let us take one example. So I want to solve this one. So let us take one example based on this one, y double dash + 16 y = 8 sec 2x. So let us take this example.

Now in this case, the left hand side is a linear equation with constant coefficient. So constant a naught is my 1 and a naught is 1 a 1 is 16. but the right hand side function f x in this case is 8 sec 2x. And till now we have not done any methods in which if I have the function f x on the right hand side and that is a secant function I know the method to find out the particular solution.

Because till now we have done the particular solutions that whenever the function on the right hand side is a exponential function or sin x or cos x or some polynomial. So this if I want to solve, if I want to solve and want to find the particular solution then I have to apply the method of variation of parameter. But to start with the method of variation of parameter I should know the two solution for the corresponding homogeneous differential equation.

So in this case I know that the homogeneous part will be 16 y double dash + 16 y = 0. So from here I know that the characteristic equation will be lambda square + 16 = 0. And from here I can find out that the lambda square will be – 16 and that gives me lambda is equal to plus minus 4i. So from here I know that my y 1 x will be cos 4x and my y 2 x is sin 4 x.

Because this is the solutions we are using to find out the complimentary solutions. So this y x is known to me now because that was the initial condition to start with that. If I want to start with the method of variation of parameter we should know that the two solution y 1 x and y 2 x and they should be linearly independent. So they are the linearly independent.

So from here I know that let my particular solution y p x will c 1 x y 1 x + c 2 x y 2 x. So from here c 1 x cos 4x + c 2 x sin 4x. Now I know that formula to find out c 1 x. So let us find the c 1 x. So c 1 x I know that c 1 x can be written as so what was the c 1 x? This is my c 1 x, - g x y 2 x over w x.

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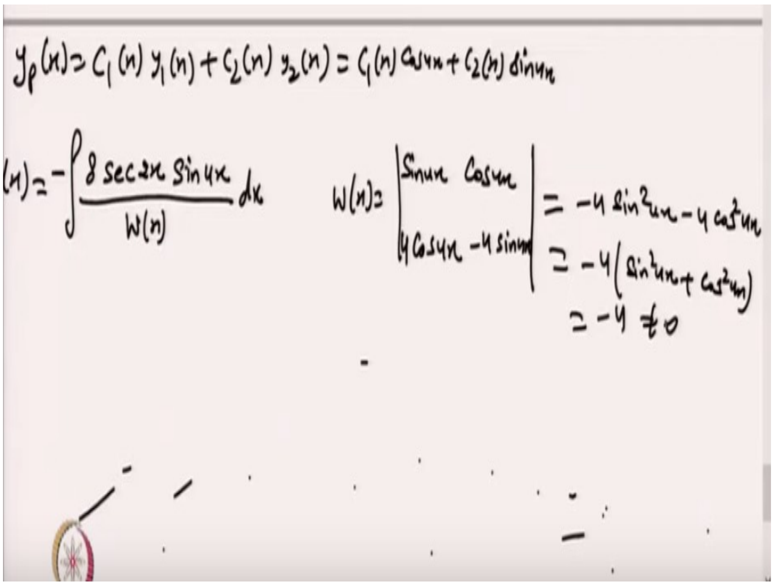

So minus g x is the function on the right hand side. So this function on the right hand side is 8 sec 2x. So it will be 8 sec 2x and y 2 x is sin 4x. So this will be sin 4x divided by the Wronskian x d x. Now the Wronskian I know that this can be found with the help of, so this is the two function I have; sin 4x, cos 4x and taking the derivative of this one. So it will be 4 cos 4x and this will be – 4 sin 4x.

So from here I can find out this one. So this will become -4 sin square 4x – 4 cos square 4x and from here I can take – 4 common and this will become sin square 4x + cos square 4x and this is - 4 . So it is never zero because they are already linearly independent. So this is my Wronskian.

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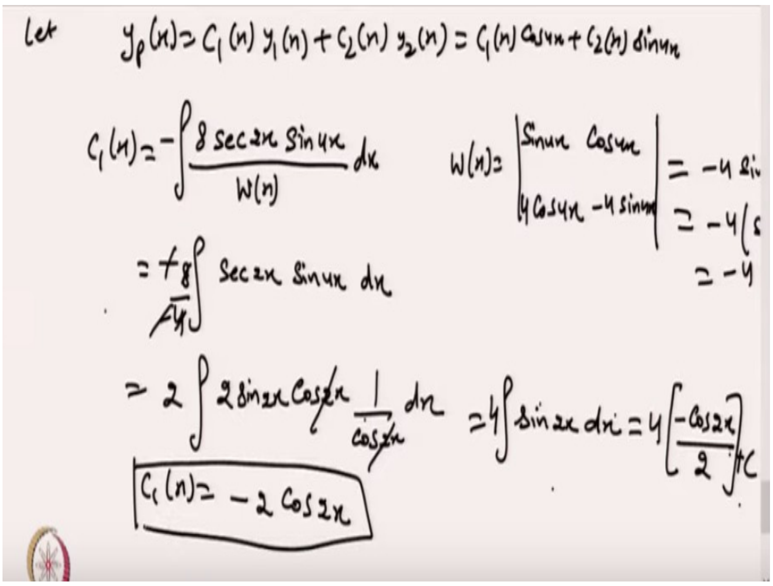

So once I know the value of the Wronskian my c 1 becomes – 8 I can take outside – 4 also I can take outside. From here inside I will get secant 2x sin 4x d x. Now I know that the half angle formula for the trigonometric function. So this sin 4x can be written, from here this will cancel out and so it will be 2. So from here I can add sin 4x can be written as 2 sin 2x cos 2x into secant.

Secant can be written as cos 2x and that is dx. So this will cancel out and from here I will get 4 sin 2x dx. And this will be 4 the sin the derivative will be – cos integration will be – cos 2x by 2 plus constant of integration. So constant of integration because in this case we are looking for only one solution. So this c I will take 0 so from here my c 1 x will be – 2 cos 2x. So this will be my c 1 x. now the same way I can find out c 2 x.

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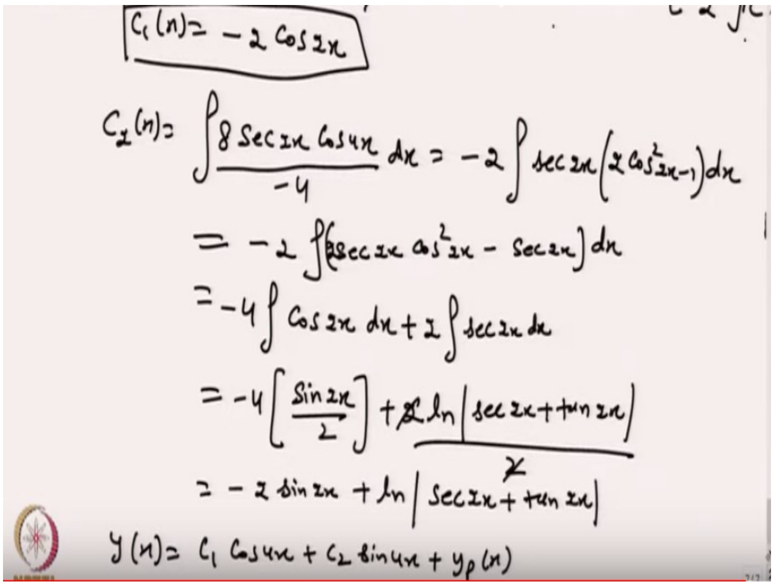

So my c 2 x will be again integration. My g x is 8 secant 2x into y 1 x. So y 1 x is cos 4x divided by – 4 dx. So that is my c 2 x. So in this case this will be – 2 and it will be secant 2x and the cos 4x can be written as 2 cos square 2x -1 by dx. So this can be written as this one. So from here divided by so minus for this one. Now this become – 2 secant 2x into cos square 2x minus so this one is 2 I am writing here minus secant 2x dx.

So from here it becomes – 4. So this will you will have only square will cancel out. So you will get only cos 2x because 1 over secant is 1 over cos 2x. So cos 2x will cancel out. You will get only cos 2x here, dx minus so this -2 will go inside and becomes 2 and integration of secant 2x dx. So from here my solution become now, so cos 2x integration is sin 2x by 2 plus 2 and then log secant 2x plus tan 2x divided by 2.

So this will you will get is – 2 sin 2x plus this will cancel out and this will be ln secant 2x+ tan 2x. So once I know the value of c 1 and c 2 then I can write my general solution.

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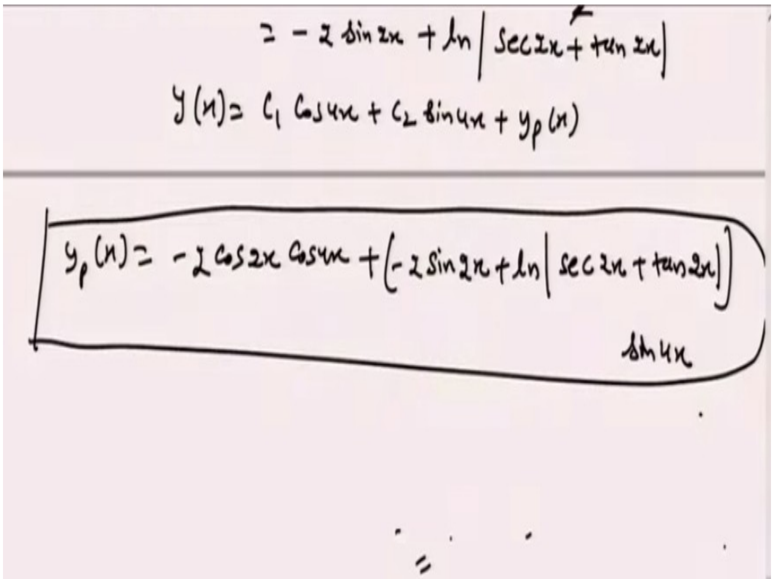

So general solution will be in this case c 1 cos 4x + c 2 sin 4x + y p x where my y p x will be c 1 is – 2 cos 2x. So – 2 cos 2x cos 4x + (-2 sin 2x + ln sec 2x + tan sin 4x). So that is my particular solution. So this is the solution of a second order linear differential equation whose right hand side is a function that is a secant which is not of the form the case 1, case 2 and case 3 we have solved.

So this is the way we can find out the solution for any general function f x. So thanks for watching. Thank you.