Introduction to Methods of Applied Mathematics
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Lecture - 04
Second Order Linear Differential Equations with Constant Coefficients

Welcome viewers back to this course. So today we are going to discuss lecture 4. Till now in the lecture 1, 2, and 3 we have discussed about the how to how we can solve the first order differential equation, the linear and few nonlinear equation. Then in the last lecture, we have started doing the second order linear differential equation. Then we have defined the linearly independent solution Wronskian.

So today I am going to start with the linear second order differential equation. And we first we will deal with the homogeneous differential equation and then we will go for the non-homogeneous differential equation. So let us start with the linear equation with constant coefficient.

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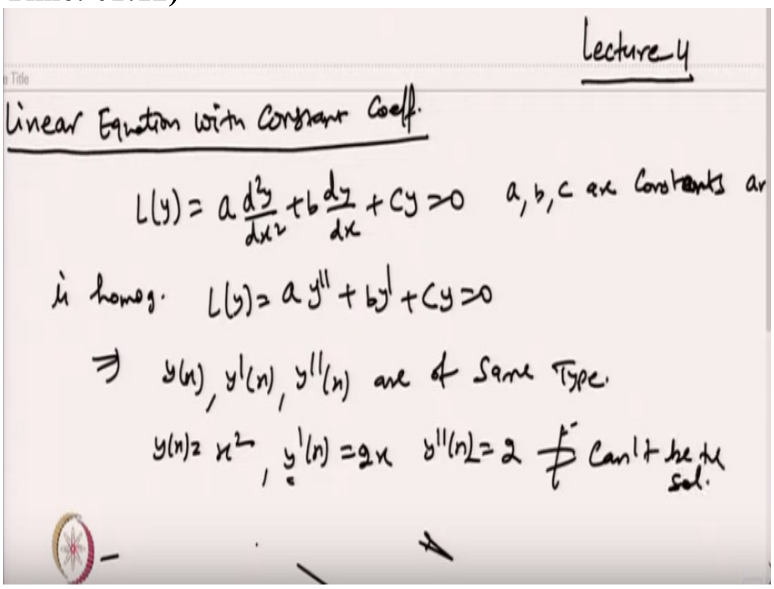

So I will start with the simple differential equation. So let us we have our differential equation. L is the operator, differential operator as we have defined in the last class. So let us take this one as a d square y/dx square + b dy/dx + cy = 0. So in this case, what I am doing is I am taking my a, b, and c are constants. And this equation is homogeneous because the right hand side function I have taken is equal to zero.

So this differential equation generally we also write Ly = a y double dash + by dash + cy = 0. Because it is very difficult to write each and every time dy/dx. So we generally for the shortcut we can write this equation as this also. Now so my a, b, c are constant and this is equal to zero. Now I want to solve this differential equation as we have also discussed in the last lecture.

So basically if I want to solve this differential equation, I need a function.. yx that is a solution of this differential equation says that, if I take the y multiply by some constant plus the derivative multiply by some constant plus the second derivative multiply with a constant I want that that should be equal to zero. So I need a function yx which has yx, y dash x and y double dash x are of same type.

Same type means their derivatives I am taking then also they should be of same type. For example, I started with yx equal to some polynomials or I take this that yx is equal to x square. Then I know that its derivative y dash x will be 2x and y double dash x will be 2 only. So this is a quadratic, this is a linear function, and this is 2 the constant function. So in this case this is of different type. So in this case this cannot be the solution of this differential equation.

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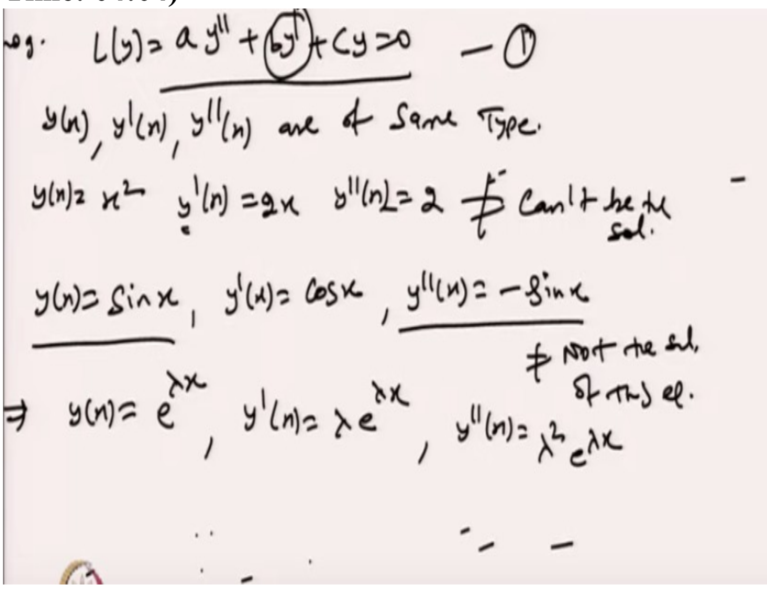

So what we do is now I will try to take some another function. So let us take the another function. So I let choose I take that yx = sin x. So suppose this sin x is a function I want to choose. So in this case I know that its derivative y dash x will be cos x and y double dash x will again will be – sin x.. So in this case if I take the function yx = sin x its second derivatives of similar type.

But the first derivative is the cos x. So if I put this one in this equation, so this is my equation, I call it equation number 1. So for the second derivative, for the first derivative the cos function will come here in this case. So if I take the combine all together this function multiply by some constant I cannot get the value equal to zero. So in this case this cannot be also not the solution of this equation.

Because if I do not have the first derivative in this equation then this function is a very good function and I can have the solution of this type. But in this case, I have the first derivative also. So sin x is not the good solution, good approximation or a good function as a solution of this equation. Now so then I go further and I choose that let yx is equal to I choose some exponential function e raised to power lambda x.

Then I know that if I take the derivative of this function, it will it will e raise to power lambda x. And second derivative of this function is equal to lambda square e raised to power lambda x. So in this case, this function, its first derivative and its second derivative, are of similar type. Just multiply by some constant that is a lambda. So it means that this can be a good choice for the solution of this differential equation.

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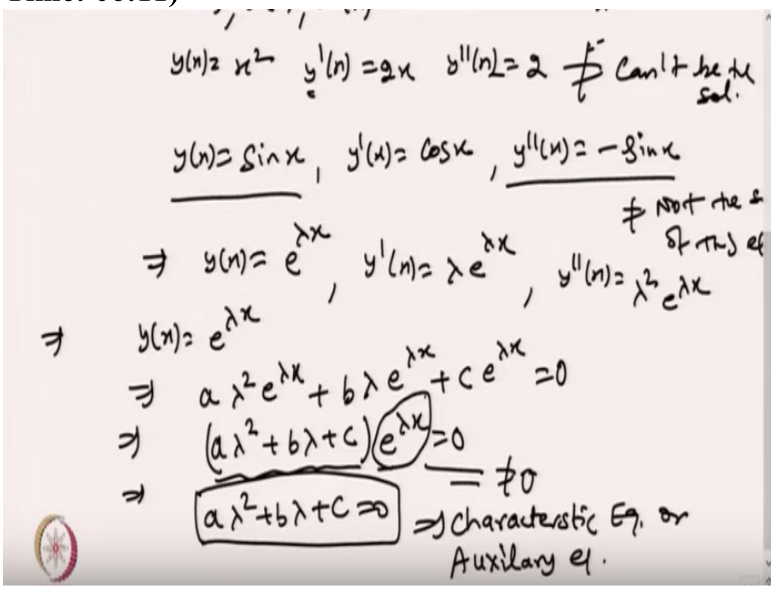

So let me choose, we start with y x equal to e raised to power lambda x and then I substitute this in the equation number 1, then I will get that it should satisfy the equation because this is a solution. So on substitution, I will get a, I should get a and then the second derivative will be lambda square e raised to power lambda x plus b lambda e raised to power lambda x plus x (c) e raised to power lambda x equal to zero.

And then from here we can find, we can take this lambda square + b lambda + c together and taking e lambda x outside then become this one. So we have also discussed that in the previous lecture, then this function is not equal to zero. So it cannot be zero. So we take this quadratic a lambda square + b lambda + c = 0. This is called the characteristic equation or the so this is called characteristic equation or sometime also we call it auxiliary equation.

So now this is the quadratic equation because we are dealing with the second order linear differential equation. So it has the roots because this quadratic equation I can find the roots of this quadratic equation by the methods we already know that how to find the roots of this one.

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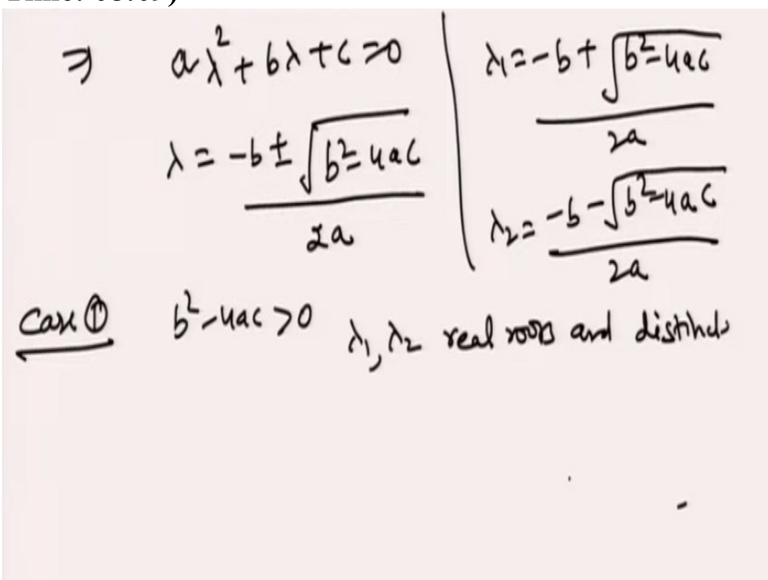

So now so I have this a lambda square + b lambda + c = 0. And then I know that its roots can be found with the help of -b, the formula standard formula b square – 4ac by 2. So in this case, I always go for two roots. So lambda one is - b + b square - 4ac= 2a and lambda 2 = - b – b square – 4ac by 2a. So 4 ac. So I always I am going to have two roots.

Now depends upon that the roots will be real roots, it will be distinct roots, it will be complex roots, it will be repeated roots, everything depend upon this factor b square - 4ac. So I will take the case number one. So when I take that my b square -4ac is positive. It means in this case I will have two roots. So that will be lambda 1 and lambda 2. So this will be real roots and distinct because in this case I will put the value of that one.

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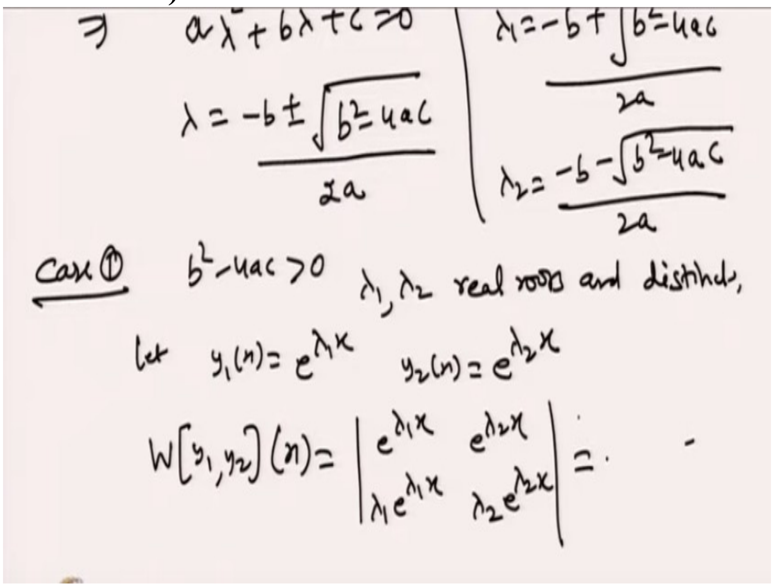

So it will be plus and minus so it will be distinct roots. So if these are they are the distinct root so let I have two solution now. So now let y 1 x I choose e raised to power lambda 1 x and y 2 x I choose e raised to power lambda 2 x because these roots are real and distinct.

Now I want because I know that the general solution of this differential equation will have two arbitrary constants and also the y 1 and y 2 x should be linearly independent only then I can define the general solution of this differential equation. So now in this case I got these two solutions. So let us see that what will happen about the Wronskian of y 1 and y 2 x.

So in this case, my Wronskian will e lambda 1 x, e lambda 2 x. It will be lambda 1 e lambda 1 x and lambda 2 e raised to power lambda 2 x. As we have discussed the Wronskian in the last class.

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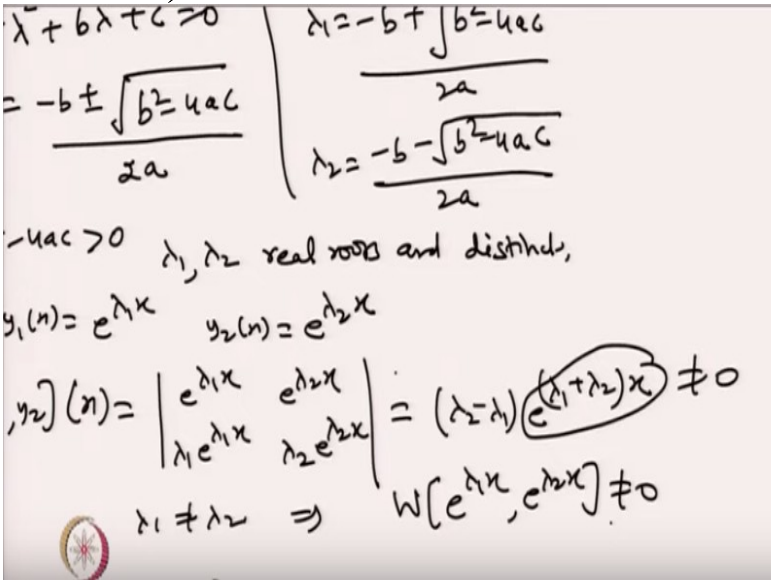

So if I go find out the determinant of this one. So it will becomes lambda 2 – lambda 1. This is lambda 2 - lambda 1 e raised to power lambda 1 + lambda 2 x. Because this will be multiply minus this one. Now in this case I know that this part is never zero and since lambda 1 is not equal to lambda 2, which implies that the Wronskian e lambda 1 x and e lambda 2 x is never zero.

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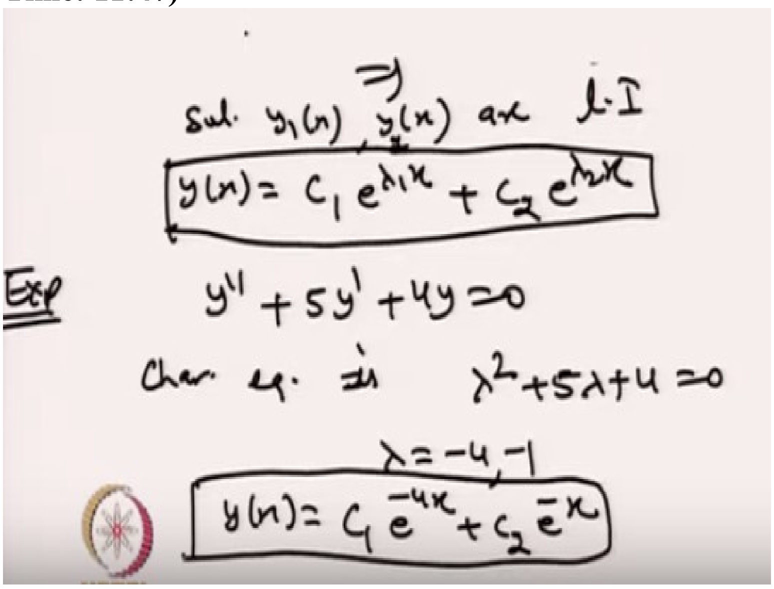

So if the Wronskian is never zero for all value of x, which implies that the solutions y 1 x and y 2 x are linearly independent. So if they are linearly independent, I can define the general solution of the equation number 1 and that will be y x is equal to some constant c 1 e raised to power lambda 1 x plus c 2 e raised to power lambda 2 x. So that is my general solution for the equation number 1. So this is the case 1 and so if I want to do the example, so let us take one example.

So I start with the differential equation y double dash + 5 y dash + 4y = 0. So in this case I want to write because I can write the directly the characteristic equation. So the characteristic equation is so I can write directly, lambda square + 5 lambda + 4 = 0. So from here I will get lambda is equal to -4 and -1. So these roots are real roots and they are distinct. So now I can write the general solution of this differential equation.

So this would be c 1 e raised to power -4 x plus c 2 e raised to power minus x and they will be linearly independent. So this is the general solution of the differential equation. Now I will move little bit further and then I take the case number 2.

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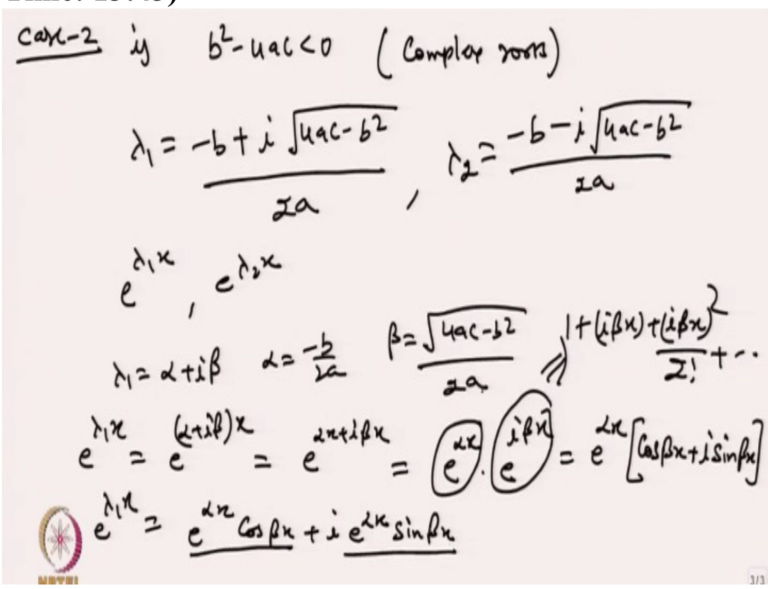

So case 2, I take that if b square – 4ac is negative. So in this case I will have complex roots. And the complex roots will be that the lambda 1 will be -b plus iota 4ac – b square under the root by 2a and the lambda 2 will be - b - iota. So this iota is a complex number 4 ac – b square under the root by 2a. Now in this case we have the 2 solutions, 2 roots lambda 1, lambda 2 and they are complex roots.

So we know that if I want to solve find out the solution then my solution will be e raised to power lambda 1 x and e raised to power lambda 2 x. Now so we first we have to define that how we can define the exponential function for a complex number.

Because this is a complex number. So let us say that I want to define that what is the value of e lambda 1 x. So e lambda 1 x will be in this case, so let us take lambda 1. So this will be e - b + iota. So I what I call it? So let us just to make the calculation little bit simple. I will take this as lambda 1 I will make as a alpha + beta where my alpha is – b/2a and beta is 4ac – b square under the root 2a. So this is my alpha beta.

Then I wanted to find what is my lambda 1 x. So it will be alpha + i beta into x. Now this is exponential. So it will becomes alpha x plus i beta x. And we know that I can separate this one as alpha x into exponential i beta x. Now this is a real function. It is a real value. Now what about this one?

So we know that this can be written as e alpha x and by the formula of the Euler for the Euler form of the complex numbers I can define this one as cos beta x + i sin beta x. Because this is exponential and I know that I can expand this exponential as 1 + i beta x + i beta x square by 2 factorial and so on by the exponential, the exponential series and then we separate the things. So this will reduce to this form.

So cos beta x + I sin beta x. So from here now what I do I separate this function as a real part and the complex part. So in this case I will have e alpha x cos beta x + I times e alpha x and sin beta x. So that is my e lambda 1 x. Now the differential equation we are dealing with we need the real valued function as a solution of this equation.

But in this case we are getting the complex valued functions as the solution of differential equation. So we have to find out that how we can use this complex valued function to find out the real valued solution of the differential equation.

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$$e^{\lambda_1 x} = e^{(\alpha+i\beta)x} = e^{\alpha x + i\beta x} = e^{\alpha x}\cdot e^{i\beta x} = e^{\alpha x}[\cos\beta x + i\sin\beta x]$$

$$e^{\lambda_1 x} = \underline{e^{\alpha x}\cos\beta x} + i\,\underline{e^{\alpha x}\sin\beta x}$$

$$y_1(x) = e^{\lambda_1 x} = U(x) + i\,V(x)$$

Sub. in eq. ①

$$L(y_1) = a\big(U(x)+iV(x)\big)'' + b\big(U(x)+iV(x)\big)' + c\big(U(x)+iV(x)\big) = 0$$

$$\Rightarrow a[U''(x)+iV''(x)] + b[U'(x)+iV'(x)] + c\big(U(x)+iV(x)\big) = 0$$

$$\Rightarrow \big(aU''(x)+bU'(x)+cU(x)\big) + i\big(aV''(x)+bV'(x)+cV(x)\big) = 0$$

$$\Rightarrow \begin{cases} aU''(x)+bU'(x)+cU(x)=0 \\ aV''(x)+bV'(x)+cV(x)=0 \end{cases}$$

Now so let us call it this as a su…m. I will take it as ux and I will call it i some vx. Now this is my y 1 x that is equal to e lambda 1 x. So let us put this one in the differential equation 1 because this is the solution of that equation. So in this case, if I substitute this one in equation 1, substituting in equation 1, I will get ux + i vx. So this will be equal to L(y 1).

So second double derivative multiply by a and then b ux + i vx first derivative + c times ux + iv x and that is equal to zero because this is the solution of the differential equation. So it should satisfy the differential equation. So from here, if I further simplify this one and take the derivatives, so this will becomes a u double dash x + iv double dash x + b u dash x + i v dash x + c and then the same ux + iv x that is equal to zero.

Now I separate the real part and the imaginary part. So from here I will get a u double dash x + b v dash x + c not v it is u, u dash x + c ux. So this is the real part plus iota av double dash x + b v dash x + c vx = 0. Now I know that the complex number a complex function is equal to zero, it means its real part and the imaginary part should be equal to zero.

So from here I will get a u double dash x + b u dash x + c u x = 0 from here. And putting this imaginary part equal to zero I will get a v double dash x + b v dash x + c v x = 0. So now you can see that from the complex solutions, I will get now two real solution, which are satisfying the my differential equation and that is u and v.

So from the if the roots are complex roots then from even from the one of the root that is lambda 1 we are getting two real valued solution of the differential equation and that was our purpose to find out the real valued solution.

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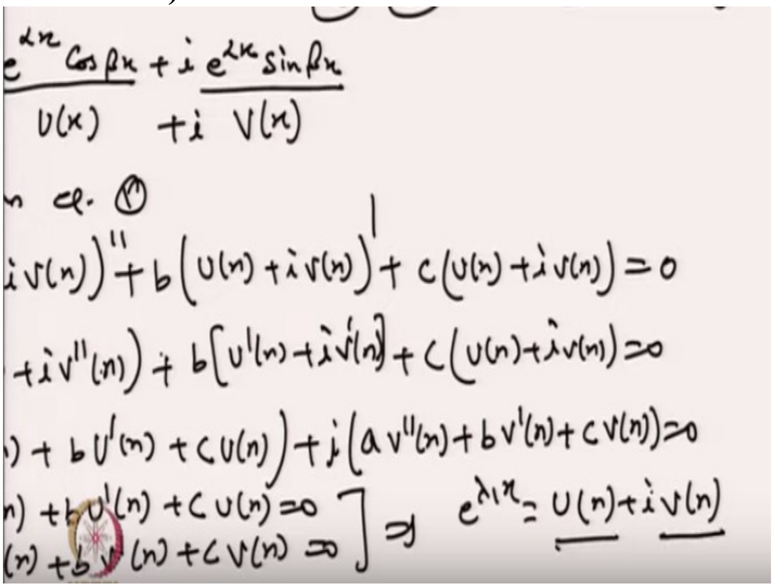

So this is the and so from here I can say that if my e raised to power lambda 1 x was u x + i v x it was u x + i v x then this is one of the solution. And this is another real valued solution of the differential equation. Now I want to check that they are two solution. So whether these are linearly independent or not because I want to define the general solution of the differential equation. So let us check that whether these are the general solution or linearly independent solution or not.

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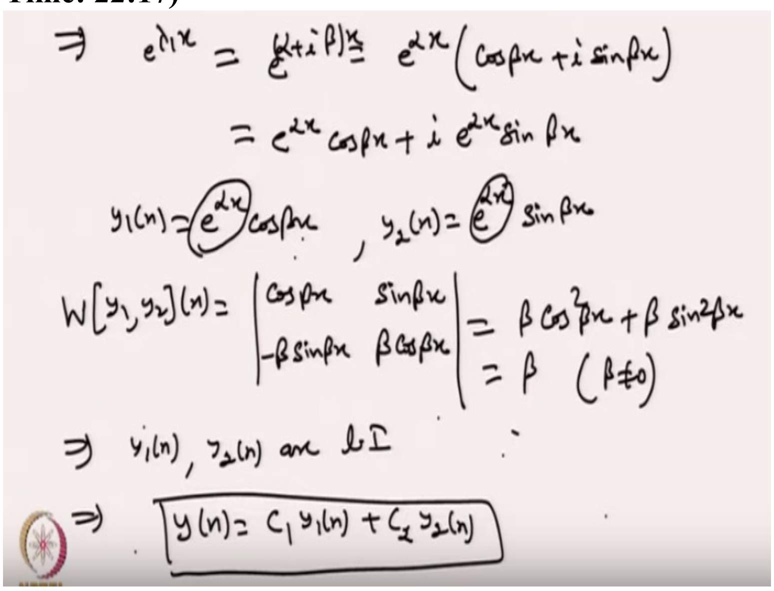

So in this case, what we do now I have e raised to power lambda 1 x. So that can be written as e alpha + i beta that we have defined into x. So it is e alpha x and then I have defined cos beta x + i sin beta x. So from here I got e alpha x cos beta x + i times e alpha x sin beta x. So what I do now I choose the two solution. So y 1 x I choose e, the y 1 I will choose e alpha x into cos beta x.

So this is the first solution I am choosing and then I choose another solution y 2 x, that is e alpha x into sin beta x. Now I want to find the Wronskian. So this is y 1 and y 2 x. So this one I am finding out. Now this e alpha x and e alpha x this has common. So I can skip this one from here just to ease the calculation and I can just check what about this cos beta x and sin beta x.

Because this is the common so I can take the common outside. So because this is the common function, so if these two function are linearly independent then we are done. So in this case it will be - beta sin beta x and this will be beta cos beta x. So from here if I take the determinant, so this will become beta cos square beta x plus beta sin square beta x and this is beta into 1. So this become beta.

And beta is always not equal to zero because if beta is zero then we have a real solutions. So in this case we are considering that we are going to have the complex roots so my beta is never equal to zero. So from here my Wronskian is not equal to zero. So from that I can say that y 1 x and y 2 x are linearly independent. Now if I have the linearly independent functions and that is the solution of the given differential equation.

So from here I can define my general solution and that will be c 1 y 1 x + c 2 y 2 x. So then I can define my general solution for my differential equation. Now so let us do one example based on this one.

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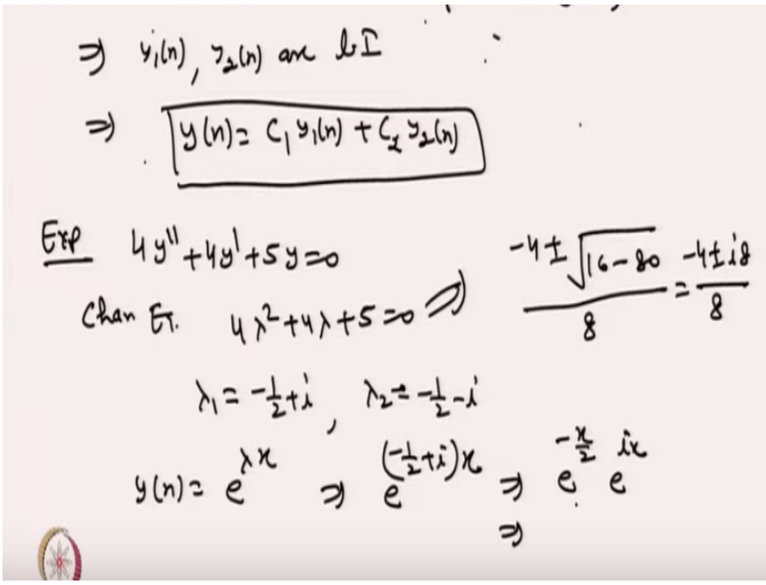

So example let us take one example 4 y double dash + 4 y dash + 5 y = 0. So this is my second order linear differential equation with constant coefficient. Then from here, I can define my characteristic equation. So characteristic equation will be 4 lambda square + 4 lambda + 5 = 0. So in this case, if I want to find the roots of this equation, so I will get the roots.

So lambda 1 I am getting minus half plus iota and lambda 2 I am getting minus half minus iota because from here I can define my roots minus b minus 4 plus minus b square is 16 minus 4 ac. So it will be 80 by 2a it will be 8. So this will becomes minus 4 plus minus iota. So it will be 64 8 by 8. So it will be minus 1/2 + iota and - iota. Now I get the two complex solutions for the lambda 1 and lambda 2 two of the complex numbers.

Then from the theory we know that my solution in this case so my y 1 x will be now my solution is this one. So my y x solution was e raised to power lambda x. So from here I will define e raised to power minus half plus iota x. Just I am choosing only one, lambda 1. So from here I will get e minus x by two into e i x.

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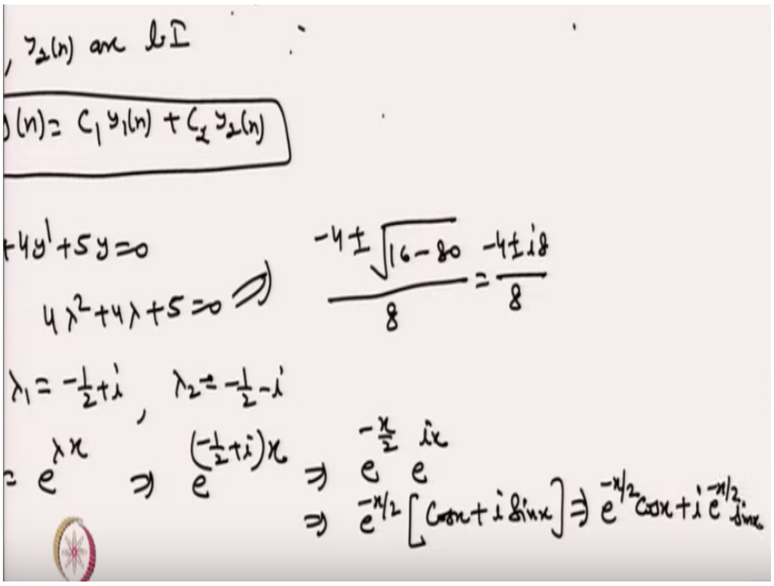

From here I will get that it becomes e –x/2 and e raised to power ix becomes cos x + i sin x. So from here I will get e raised to power –x/2 cos x + i times e raised to power –x/2 sin x. So this is my complex function and from there so just now we have done that from this one I can find out the two linearly independent solution.

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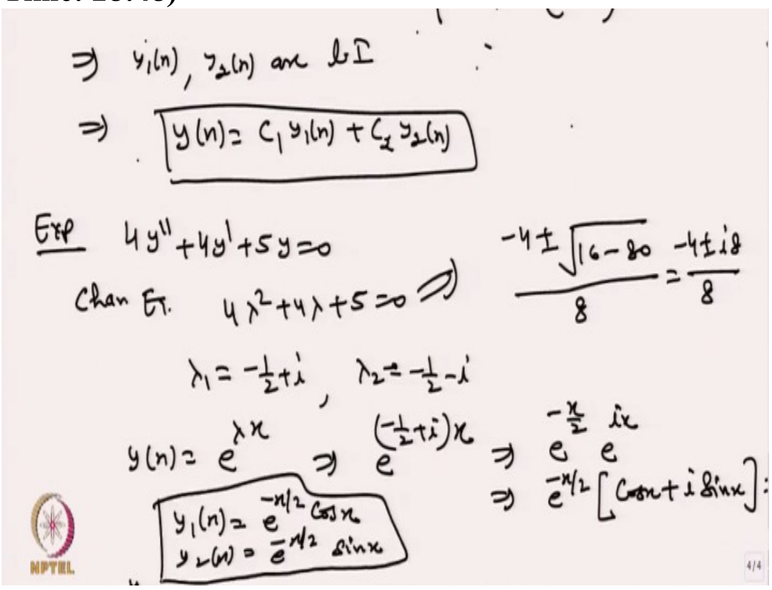

So from here I can directly write my y 1 x will be e –x/2 cos x and my another solution y 2 x will be e raised to power –x/2 sin x. So in this case, they do not need to find out that what is going to happen if we choose lambda 2, the same thing will come again. So from here I will get two linearly independent solution.

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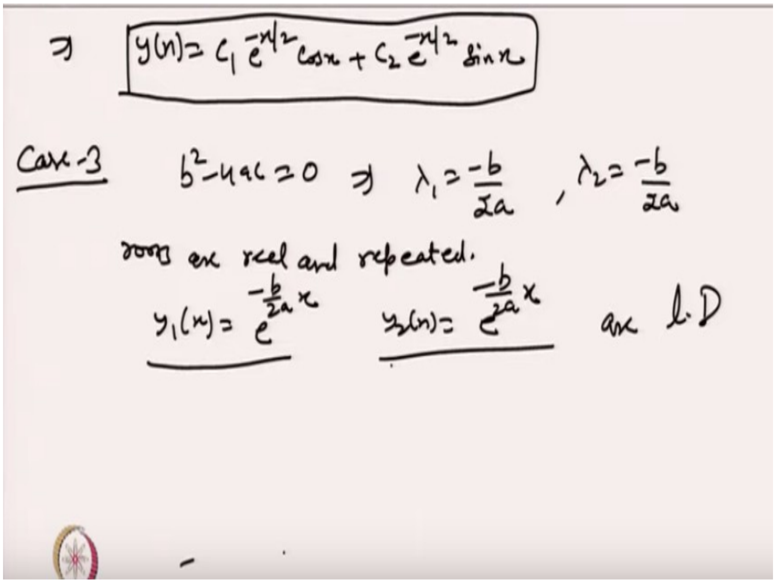

And once I get this 2 linearly independent solution I can write my general solution for the differential equation. So this will be c 1 and e raised to power –x/2 cos x + c 2 e raised to power –x/2 sin x. So that will be the general solution of the given differential equation. So because if I have the general solution then by giving the initial (value) problem I can find out the value of c 1 and C 2 and then in that case we will get the unique solution of the given equation.

Now so after doing this one I will go for the case number 3. So this one we are doing only for the homogeneous case with the constant coefficient. So in the case number 3 it may happen that b square -4 ac that is in the when we find out the roots of the characteristic equation, so this becomes equal to zero. So in that case what will happen, my lambda 1 will be – b/2a and my lambda 2 will be – b/2a.

So in this case the roots are real and repeated. Now if I write the solution corresponding to this two then I can write my y 1 x = e raised to power – b/2a x and my y 2 x will be e raised to power – b/2a x. But the problem comes with this one that this solution and these solutions are linearly dependent. But to find out the general solution of the equation I need two linearly independent solutions.

But in this case we are getting only linear dependent. So for what we will do that? For this one I will find out the, so with the help of this I want to define. So with the help of the one solution because one solution is known to me.

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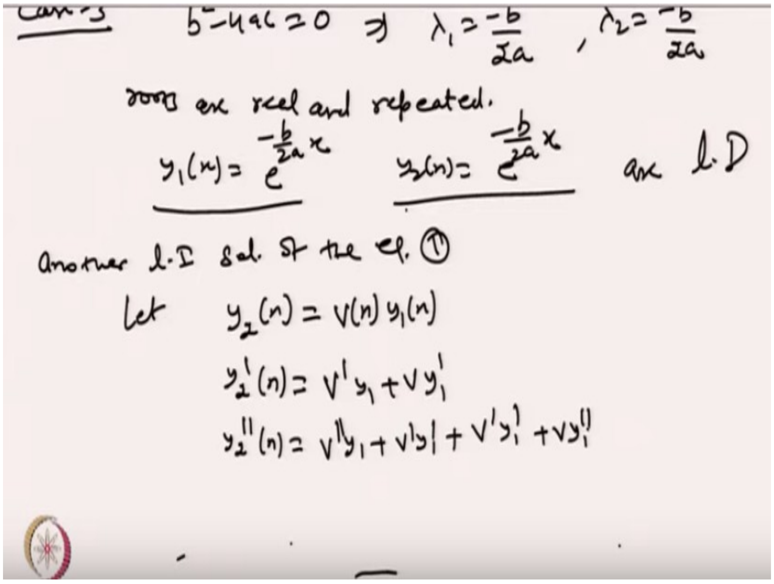

For this one, I will find out the so with the help of this, I want to define so with the help of the one solution because one solution is known to me, I want to find another linearly independent solution of the equation 1 whatever the equation I started with. So I want another linearly independent because in this case I can choose only one solution and the another solution I want to find.

So this another solution we can find with the help of very renowned methods. So that is called the reduction of the order of the method. So this is the reduction of the order of the equation. So how we will do that? So in this case I will write that the let I will take, I will get another solution y 2 x is equal to suppose I have some v x and then y 1 x, I choose this one. So y 1 x I will do like this one and v 1 x.

So v x is a function of x and I do not know that what is the value of the v x. So and this y 2 x is a solution of the equation number 1. Now so before that I just take the derivatives of this one So y 2 dash will be, so taking the product rule, it will become v dash just I skip the writing x and again but it is understood it is the function of x. So I will write v dash y 1 plus v y 1 dash.

So this is the first derivative and the second derivative will be again v dash y 1 + v dash y 1 dash + v dash y 1 dash + v y 1 double dash. So this is the second derivative we have taken.

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another

let $y_2(x) = v(x)\,y_1(x)$

$$y_2'(x) = v'y_1 + vy_1'$$

$$y_2''(x) = v''y_1 + v'y_1' + v'y_1' + vy_1''$$

$$\Rightarrow L(y_2)(x) = a(v''y_1 + 2v'y_1' + vy_1'') + b(v'y_1 + vy_1') + c(vy_1)$$

$$\Rightarrow (av''y_1 + bv'y_1) + v(\underbrace{ay_1'' + by_1' + cy_1}_{=0}) + 2av'y_1' = 0$$

$$\Rightarrow av''y_1 + bv'y_1 + 2ay_1'v' = 0$$

$$\Rightarrow v'' + \frac{b}{a}v' + \frac{2y_1'}{y_1}v' = 0 \Rightarrow \boxed{v'' + \left(\frac{2y_1'}{y_1} + \frac{b}{a}\right)v' = 0}$$

So from here if I substitute this solution in the equation 1 then I will get L of y 2 x will become a, so now I am writing not as a function of x just for the so that the calculation will be easier. So I writing v into y 1. Just this will become because I have to, second derivative. So second derivative will be v double dash y 1 plus now this into this so it will be two times.

So this will be two times v dash y 1 dash plus v y 1 double dash. Now plus b. So b I will so it will be v y 1 plus v y 1 dash plus c and this v y 1 and because this solution is a differential equation. So this is equal to zero. Now from here I will take the, I will separate the, I will separate the all the quantities related to y 1. So if I separate this one and choose the y 1 as a common.

So I will get a v double dash plus b v dash plus now so I will just keep this one here. This one I will because this will be used. So I will now what I do is, I will write a v double dash y 1 plus b v dash y 1 plus, now this is two times I am taking a and this is y 1 dash so from here I can write that v I am taking common so from here I can write a y 1 double dash plus b y 1 dash plus c y 1 plus and then this I have already chosen this I have already taken from here plus two times a v dash y 1 dash equal to zero.

So this will become this. So this is the we have separated the given equation to this form. And now what about this one? So this one I know that this is the equation we have started with and y 1 was the solution of the equation. So its value should be called zero. So from here this reduced to a v 1 dash y 1 plus b v dash y 1 plus 2a y 1 dash v dash equal to 0.

And from here I can write this equation as v double dash plus, because I can divide the whole equation by a y 1. So it will be again b by a v dash plus 2 y 1 dash over y and a will cancel out, v dash equal to zero. So from here I will further write like this one. So it will reduce to two times y 1 dash over y 1 plus b over a y dash equal to zero.

So this equation if you see that this equation is a first order equation for the y dash and this first order equation, I can solve this because we already know that how to solve the first order equation. So the first order equation, this equation I can solve.

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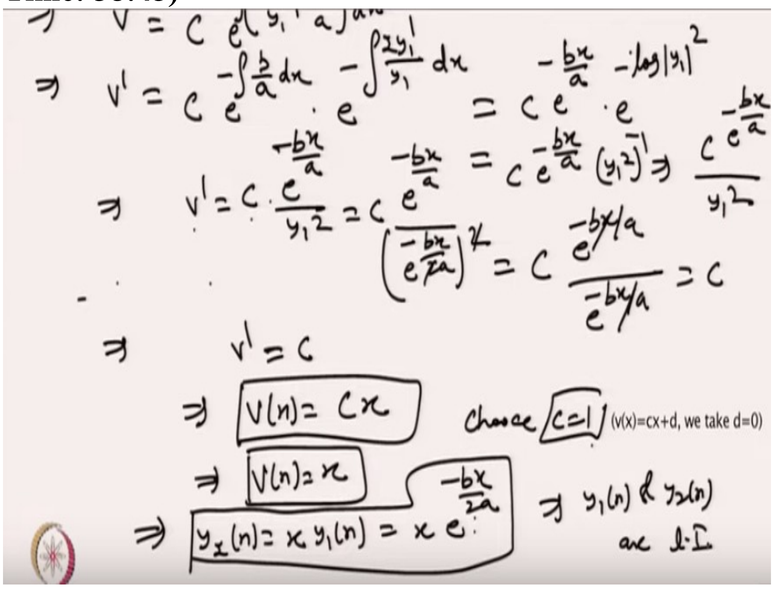

So from here my v dash will become e raised to power. So I will take the constant of integration c, e raised to power minus 2 y dash over y 1 y dash plus b over a x and this is a constant of integration d x. So this is the integration I am taking because just I will take it on the right hand side and then the separating of variables I can find out the solution on… this one. So from here and this c is the constant of integration.

So from here my y dash will be c e raised to power minus integration b over a dx into I just y 1 over 2 y 1 dash dx so separating this one. From here I can solve further. So what will it become? It will become e raised to power minus b x over a. And this will become e raised to power and this is the function and its derivative. So it will be e log y 1 and then the square.

So from here further I can write this one as c e raised to minus bx by a and then it becomes e minus from this one I can write directly that this reduce to because exponential and the minus of log this one so this I can write as y 1 square because this I can take as this one. So from here further I can write this as c e raised to power minus bx by a by y 1 square. So this is my v dash.

Now if I further solve this one from here, I will get v dash is equal to c e minus bx by a by y 1 square. Then what was my y 1 square? My y 1 square was e minus bx by a and what was my y 1, y 1 if you see the y 1, my y 1 was this one e raised to power this. So this is my y 1. So from here, if you see my y 1 was e raised to power minus bx by 2a and the scaling of this one.

So if I take this one it becomes c e raised to power minus bx by a and then taking the square of this, so it again becomes bx by a because 2 will cancel out from this and this will cancel out and you will get the constant of integration that is c. So from here, I will get v dash is equal to c. Now I will get v dash is equal to c and the constant integration is this one.

From here, I will take the integration both side with respect to x and I will get v x will be integration c x. So just for the simplicity I will choose c is equal to 1. So from here because I need only one solution, this is a class of solution and I need only one solution for this one, so I choose c is equal to 1. So from here I can define that my v x will be x.

So if I choose v x equal to v or x, then I can now define from here I can define that my y 2 x will be x into y 1 x and y 1 x. So it will be x e raised to power minus bx by 2. And you can also check that y 1 x and y 2 x are linearly independent. So this will be linearly independent that you can just do yourself, it is very easy to check. So from here now I get y 1 x I know and y 2 x is this one.

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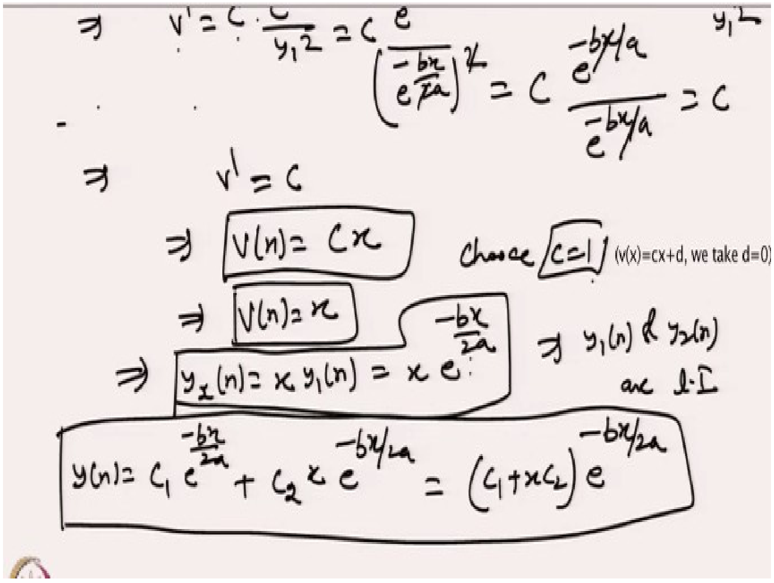

So from here I can define my general solution. So the general solution I can write as y x is equal to c 1 e raised to power minus bx by 2a plus c 2 x into e raised to power minus bx by 2a. And from here I can write this one as c 1 into x c 2 e raised to power minus bx by 2a. So that is my general solution for this differential equation and these are the case when the roots are repeated.

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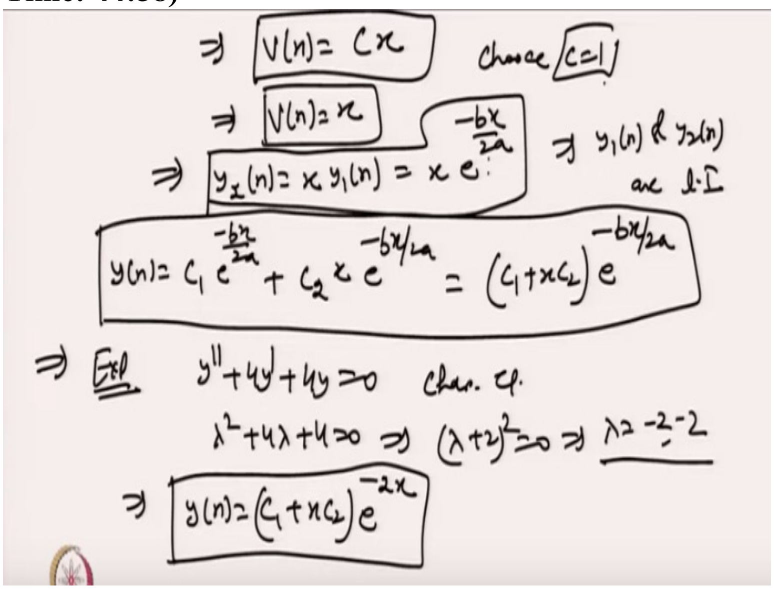

Now, so let us do take one example based on this one. So let us solve this one. I have a differential equation y double dash plus 4 y dash plus 4 y equal to zero. So in this case, I have the my characteristic equation will be lambda square + 4 lambda + 4 = 0 and here we will say that this becomes whole square equal to zer0. So from here my lambda is -2, -2.

So now this is the roots which are real and repeated. So from here if I want to write my solution y x. So y x will be c 1 + x c 2 e raised to power - 2 x. So this is my general solution for this differential equation. So this is the three types of cases that depends on the roots of the quadratic equation and the quad equation is the characteristic equation that is corresponding to the differential equation we are solving.

So these are the three cases and based on this cases we can find the general solution for the homogeneous equation.

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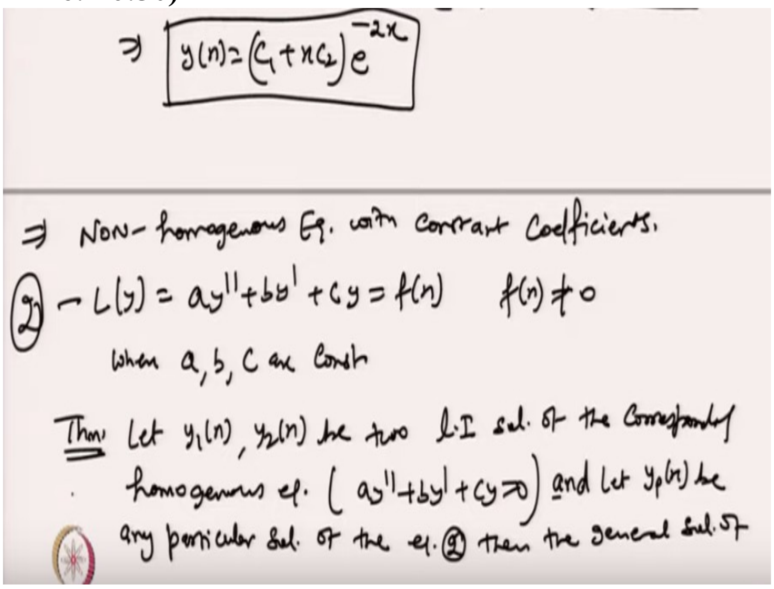

Now so after solving the homogeneous differential equation, now we start with the non-homogeneous, non-homogeneous equation with constant coefficients. So in this case I have, so let us define the equation. So this is my L y. So that is I am writing a y dash + b y dash + c y equal to some function f x. So in this case my F x is not equal to zero. So this function I am defining and this function is not equal to zero.

So this become a non-homogeneous second order linear differential equation where my a, b, and c are constants. Now how we solve this type of differential equation, the non-homogeneous differential equation. Because this is again a second order differential equation. So for this differential equation if I want to write the general solution.

So that general solution should contain two arbitrary constant again to find out the general solution. So for this one I will just define one theorem that let y 1 x and y 2 x be two linearly independent solutions of the corresponding homogeneous equation. So that is I can write as a y dash + b y dash + c y = 0. So this is the corresponding homogeneous equation attached to this non-homogeneous.

So let y 1 and so in this case what do you do? First we solve the homogeneous case and from that homogeneous case I will get the two solution and that we know how to solve this one. So this will be y 1 x and y 2 x. Then and let I will call it y p x. Let y p x be any particular solution of the so let us give it a name. So I call it 2.

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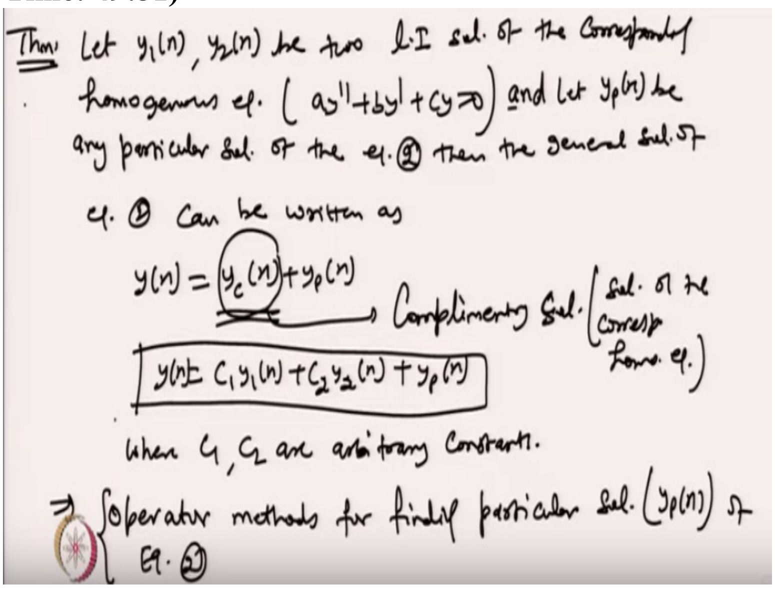

So let y p x be any particular solution of the equation 2 then the general solution of equation 2 can be written as so I can write the general solution that is y x. So we call it actually we separate this one into two parts. So I will call it first one is y c x and then y p x. So what is the y c x? This is the solution and it is called the complimentary solution.

So this solutions, I will call it complimentary solution or I can say that the solution of the corresponding homogeneous equation. So the now put right hand side equal to zero and whatever the equation we get if we solve that one. So this is the complimentary solution and the solution corresponding to the homogeneous equation. So from here I can write that my y x will be, so this my y x will be c 1 y 1 x + c 2 y 2 x + y p x. And that will be my general solution.

So in this case, I have two arbitrary constant that should be there in the solution of the second order differential equation and this is the particular equation. So this is the same way we used to solve the system of linear equations, the algebraic equation. So in the system for non-homogeneous algebraic equation, we always first find the solution corresponding to the homogeneous linear system.

And then we find the solution for the corresponding non-homogeneous system and then we add together. So this is the general solution of the equation 2 where same as c 1 and c 2 are arbitrary constants. So now I want to find out because this c 1 y 1 x + c 2 y 2 x the complimentary solution is very easy to find now because we already know the methods to find out. But how to find the v p x that is the main challenge.

So and the v p x is also based on that what is the value of my function whether my function on the right hand side is a quadratic, is a polynomial or whether my function on the right side is a exponential function or a trigonometric function or any function. So based on that which type of the function f x on the right hand side we have we will go for the different (different) methods to solve the differential equation to find out my particular solution that is y p x.

So this will be we start I go for different methods. So in this case we choose for this method that is called operator methods for finding particular solution and that is y p x of equation 2. So this is I am going to use operator method. There is another method also. But in this course we are going to do with the operator methods for finding particular solution y p x of the equation number 2.

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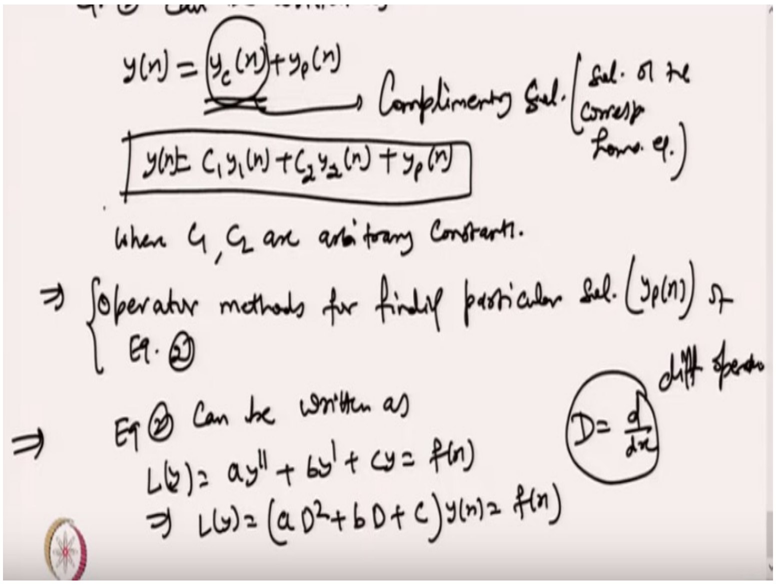

Now we already know that the equation 2 can be written as so this is L y I have, so this I can write as a y double dash + b y dash + c y = f x which can also be written as L y = (a D square + bD + c) y, y means the function of x. So this is y x is equal to f x where my D is a derivative. So this is called differential operator. So I will make use of this differential equation.

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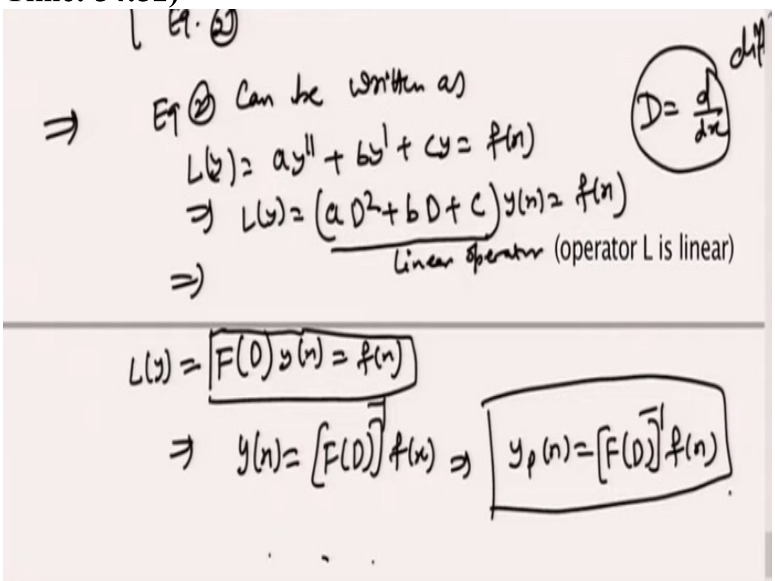

I will I can write this differential equation further and this becomes L y is equal to so this if you see that it is a D square plus bD plus c. So it is a some function of D. So I can write this as F(D) y (x) and that is equal to f(x). So this is my differential equation where F D is the differential operator. From here now if I want to solve this one, I define y x is equal to F D inverse function f x.

So from here I can solve this one. So that becomes my y p x will be F D inverse f x. So this is my particular solution. Now the problem is that how to find this inverse of my F D. So this we will discuss in the next lecture. Thank you very much.