Introduction to Fuzzy Sets, Arithmetic and Logic
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Module - 3
Lecture - 9
Fuzzy Sets, Arithmetic and Logic

Welcome students, to ninth lecture of fuzzy sets, arithmetic and logic. In the last class, we have discussed fuzzy numbers and we have seen different types of fuzzy numbers. In today's class, we shall study arithmetic operations on fuzzy numbers.

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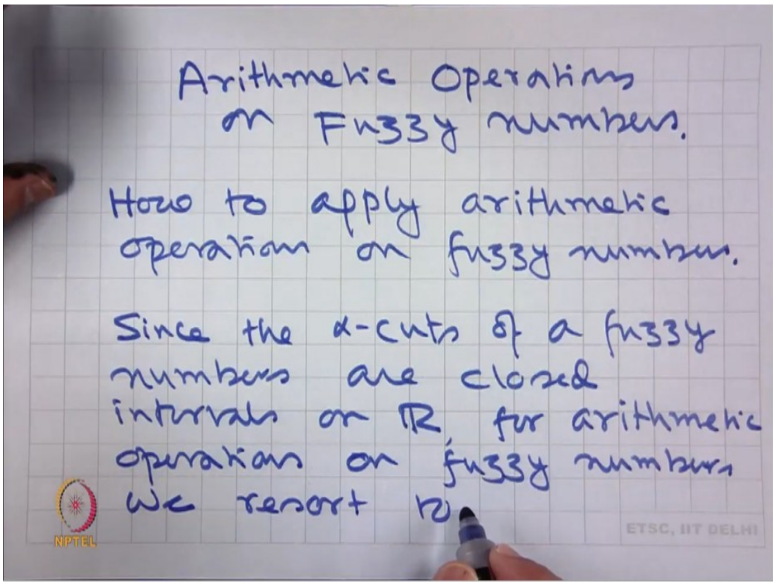

Question is: How to apply arithmetic operations on fuzzy numbers?

Because a fuzzy number is not a unique number, it is a set characterized by its different alpha-cuts, which are closed intervals on the real line.

Therefore, since the -cuts of a fuzzy number are closed intervals on , for arithmetic operations on fuzzy numbers, we resort to what is called interval-based arithmetic.

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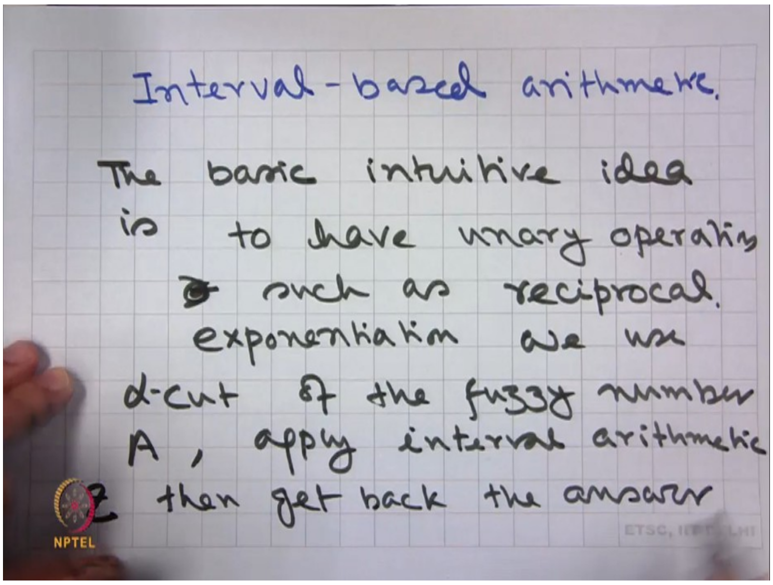

The basic theory is as follows:

To have unary operations such as, say reciprocal, exponentiation, etc., we use -cut of the fuzzy number , apply interval arithmetic and then get back the answer;

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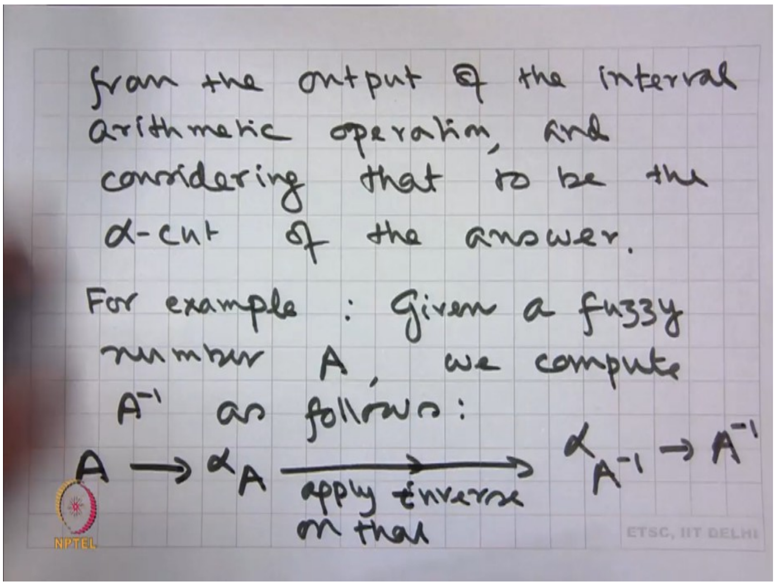

from the output of the interval arithmetic operation, and consider that to be the -cut of the answer.

For example: Given a fuzzy number , we compute as follows.

From , compute .

Apply inverse on that interval to get . And from there, get ; that is reciprocal.

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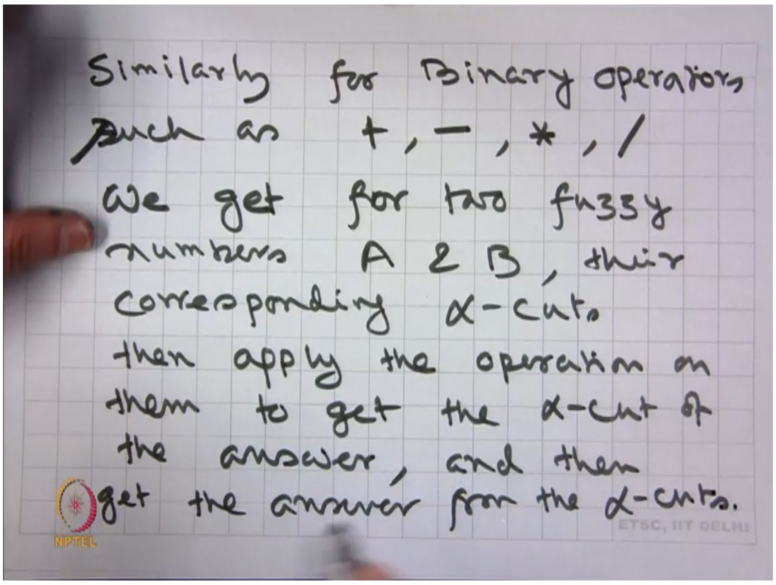

Similarly, for binary operators such as plus, minus, multiplication, division; we get for two fuzzy numbers and , their corresponding -cuts. Then, apply the operation on them to get the -cut of the answer. And then, get the answer from the -cuts.

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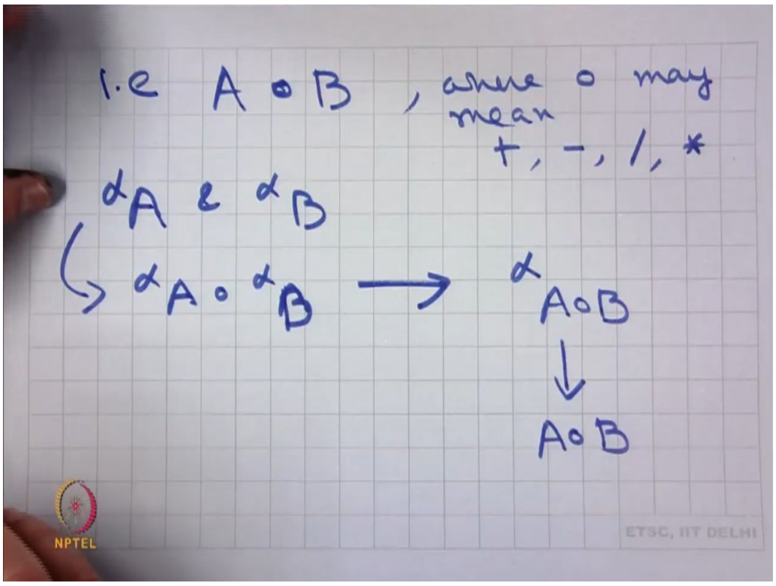

That is,

, where may mean ; such binary operators.

What we do, we compute and . Then we apply interval operation on this and , that is, . And from there, we get. And from there, we recover . That is the intuitive idea for computing fuzzy arithmetic.

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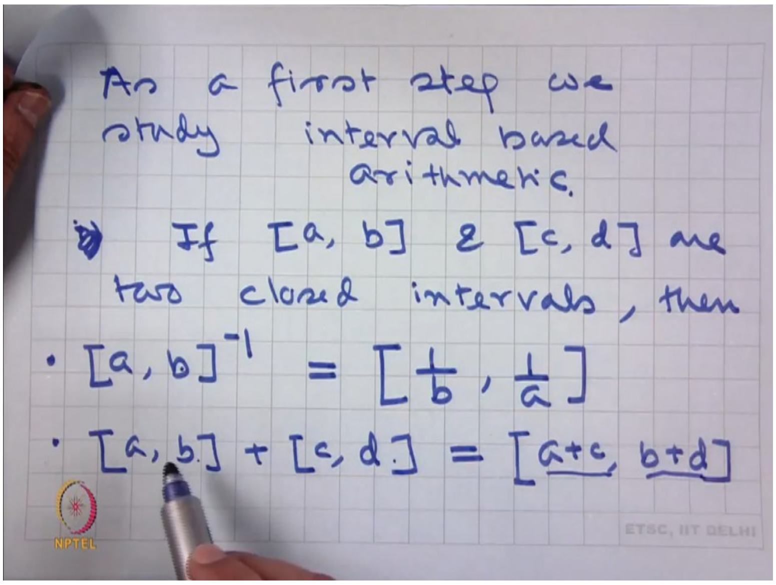

Therefore, as a first step, we study interval arithmetic or interval-based arithmetic.

So, if and are two closed intervals, then

 only if

Since in an interval, , therefore, when we take their reciprocals, has to be smaller than . And therefore, this is going to be the corresponding interval for reciprocal.

What is ?

This is the interval .

So, for addition, we get the answer by adding the lower bounds of the two intervals as the lower bound of the output. And the upper bound of the two intervals, their sum is going to be the upper bound of the output.

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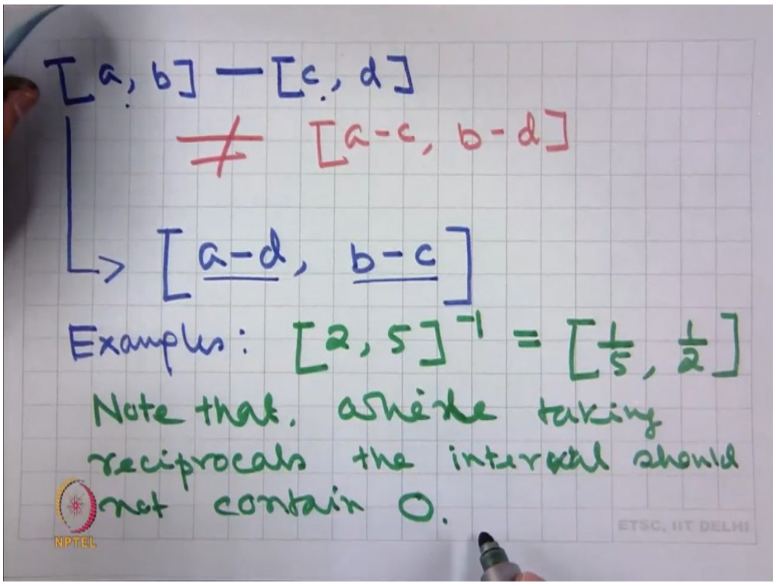

What is ?

It is not . So, people often make this mistake. This is not correct.

In fact, .Why?

Because, in forming the interval of the output, we look at what is the smallest value that it can attain and what is the largest value that it can attain when we apply the subtraction on these two intervals.

The difference will be maximum. But rather than the lowermost value of the difference is going to be . When is the smallest and is the largest, the difference is going to be the least one. And when is the highest and is the smallest, their difference is going to be the largest one in the intervals.

So, let me give you examples.

What is ?

This is actually is equal to . And we know that . One has to note that, when we were taking reciprocals, the interval should not contain Because, if is inside, then is not defined.

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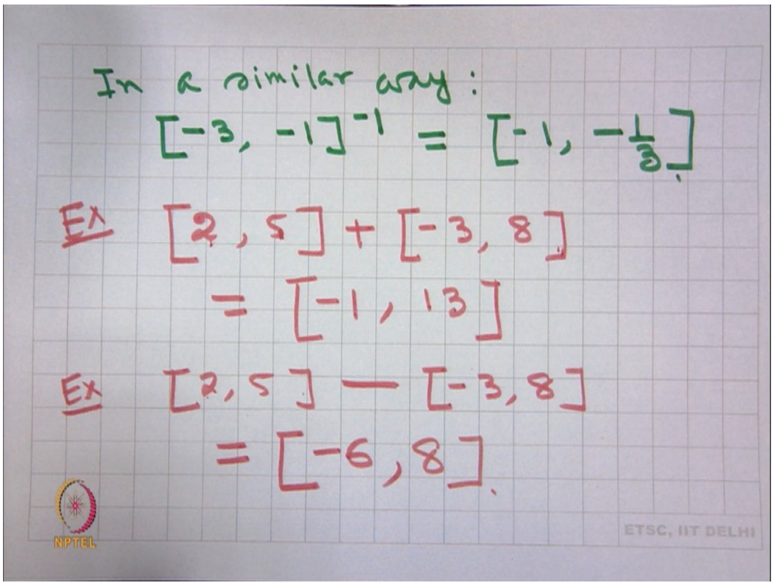

In a similar way,

Example for addition:

To understand that, suppose we take any arbitrary number from and one arbitrary number from , what is going to be the smallest value for that one?

The smallest value comes when I take the smallest one here and the smallest one here; and their addition gives me . In a similar way, the largest value of the sum is going to be the sum of the largest value of and the largest value of .

What is ?

This is going to be the smallest value that we can attain by subtracting any arbitrary number chosen from from any arbitrary number chosen from . That comes when this number is smallest, that is and this number is the largest, that is Therefore, the smallest possible value that we can get is . On the other hand, the largest possible value we can get is If we understand this, then we go to the next step.

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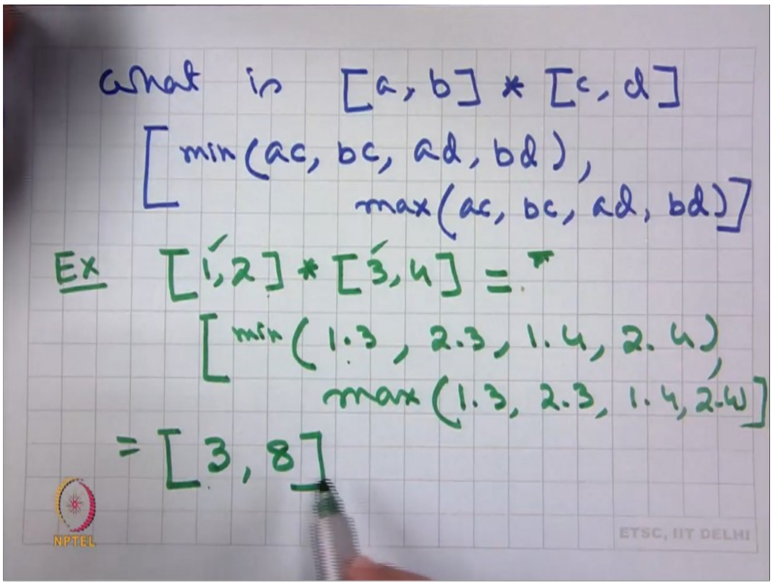

What is?

Again, as before, we want to get the interval that should contain all possible values which come as a product of any arbitrary number chosen from this interval and any arbitrary number chosen from the second interval. Since the minimum of that will depend upon the respective signs of these.

We will get the interval to be .

So, let me illustrate. What is ?

This is equal to.

That is minimum of and . Therefore, this is going to be . And maximum of and is going to be .

Therefore, what we are getting is on lower side and on the upper side.

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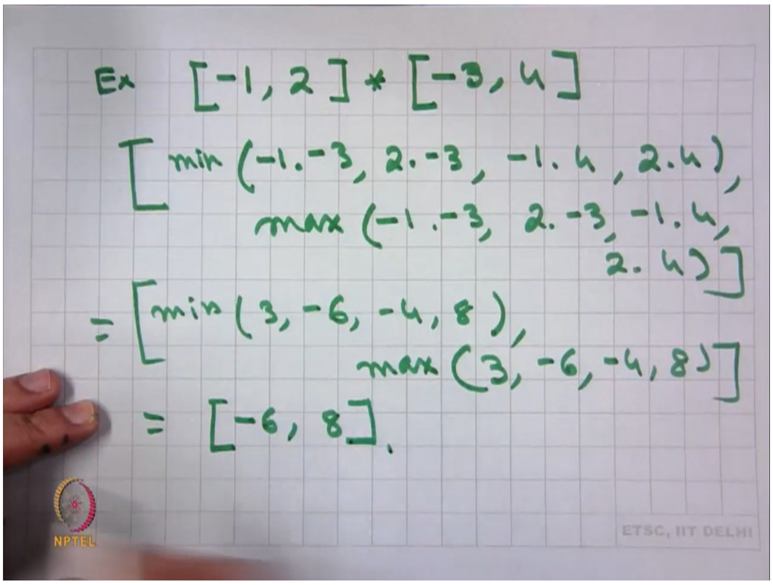

But consider

Therefore, this is going to be the interval,

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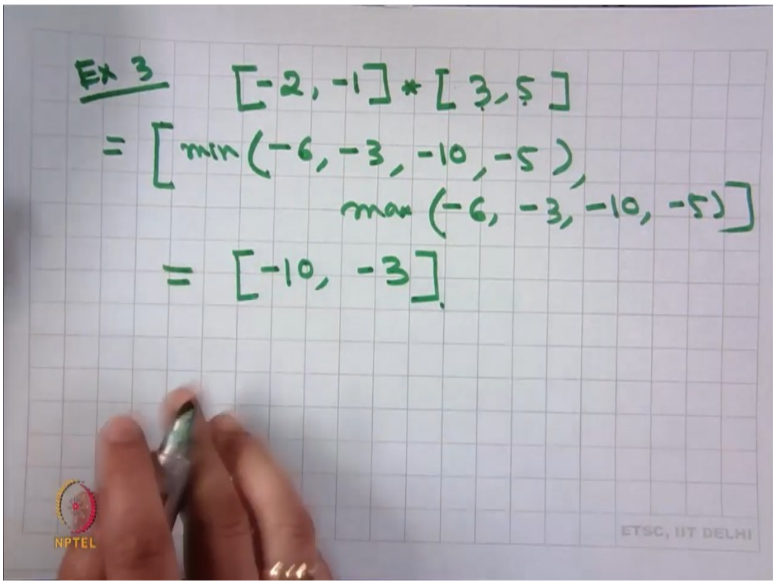

A third example:

What is ? ?

This is is equal to

So, I feel that the concept of multiplication of intervals is clear. With that, let us look at division.

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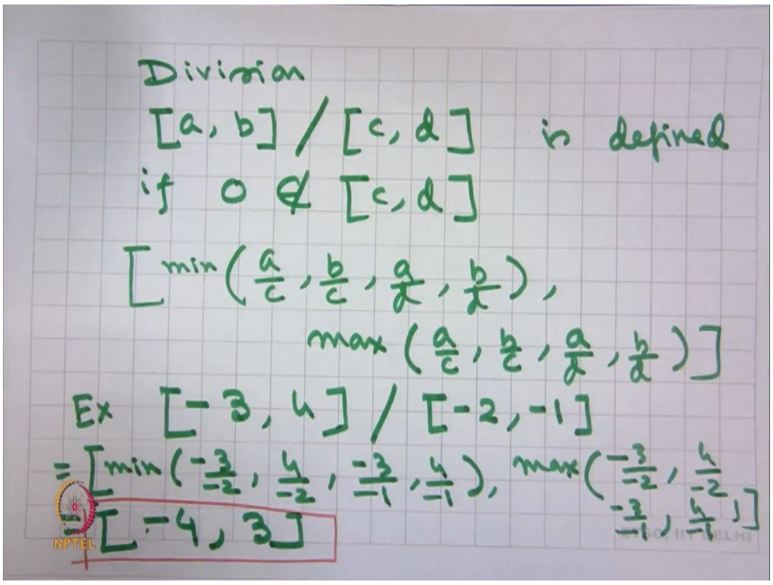

Division

is defined, if .

And, in this case, the output is

Example:

This is going to be the

The output is this interval .

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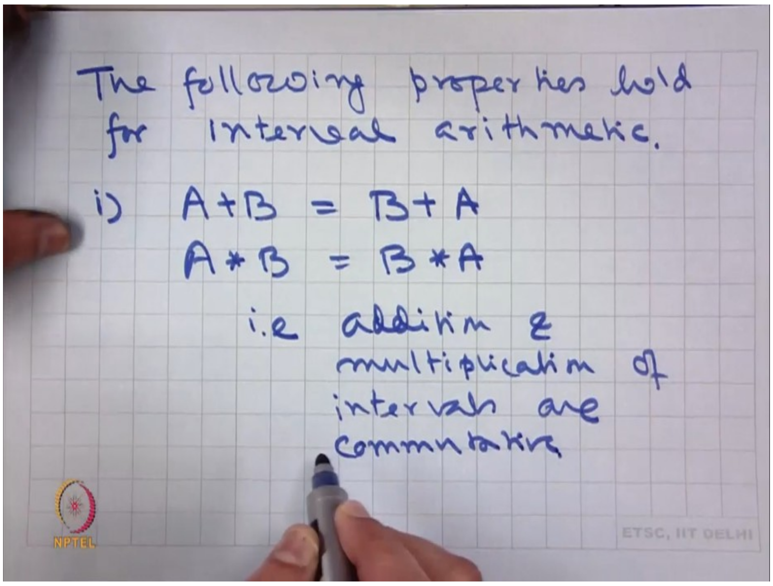

The following properties hold for interval arithmetic.

i.

That is, addition and multiplication of intervals are commutative.

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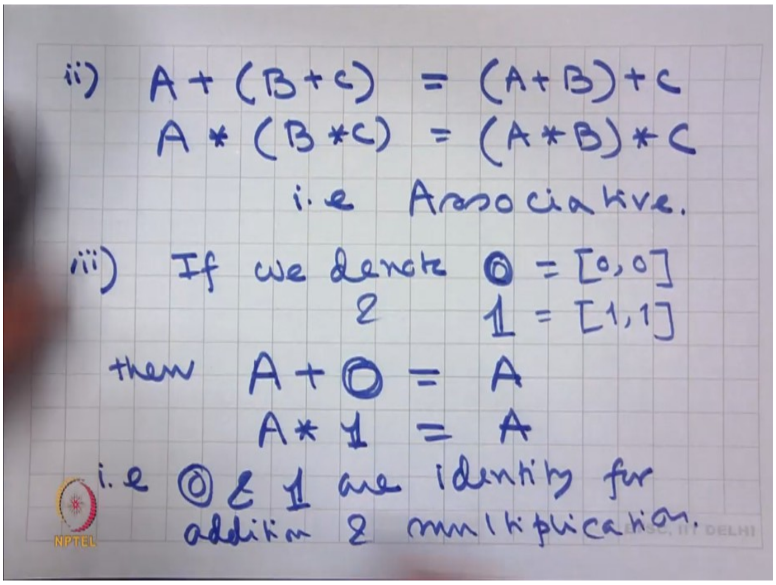

ii.

That is, associative.

iii. If we denote by and then,

and That is, and are identity for addition and multiplication.

The above properties are very similar to what we have seen with respect to real numbers. But the following is slightly different.

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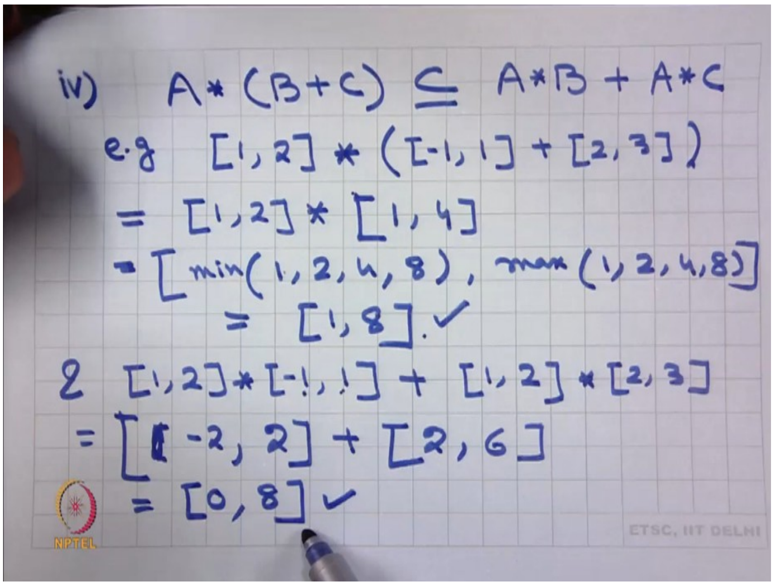

What is ?

In reals, we know that it is

But with respect to interval arithmetic,

iv.

Example:

Consider

So, we can see that, .

And they are not same.

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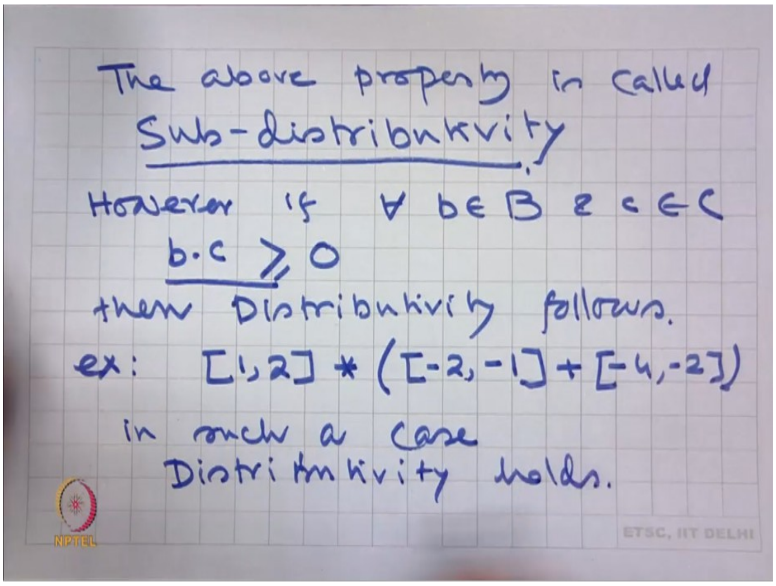

The above property is called sub-distributivity.

However, if for all and , then distributivity follows.

Say, for example,

Since all the members of is negative and all the members of is negative, therefore these conditions hold.

In such a case, distributivity holds.

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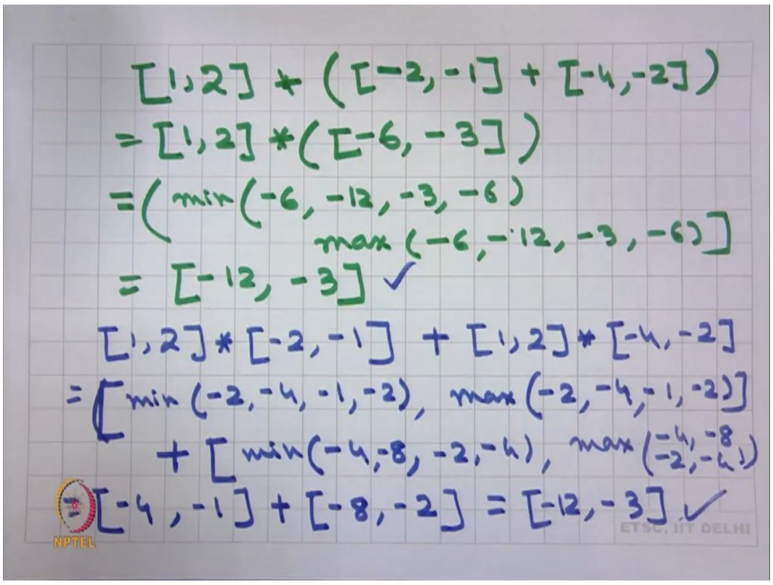

And

Therefore, distributivity holds if all the elements of and have the same sign. Another property:

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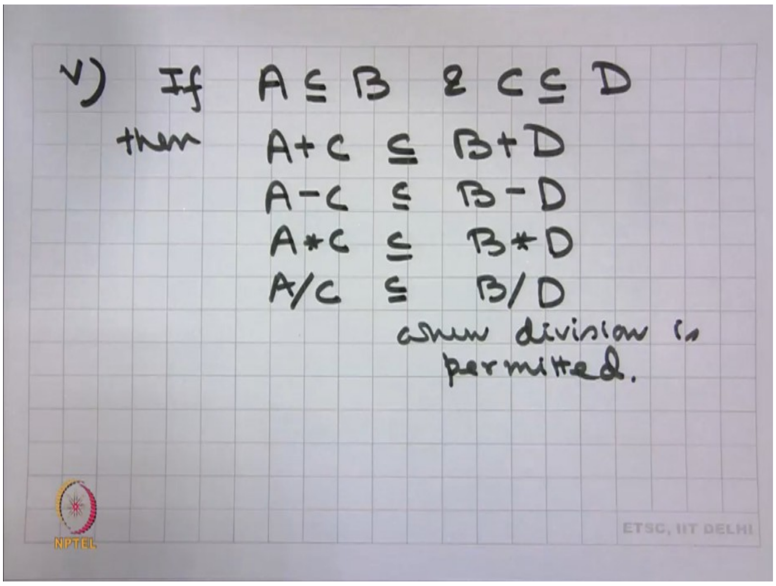

v. If and then,

I like you to verify this results. Once the basics of interval arithmetic is clear, we now focus on arithmetic operations on fuzzy numbers.

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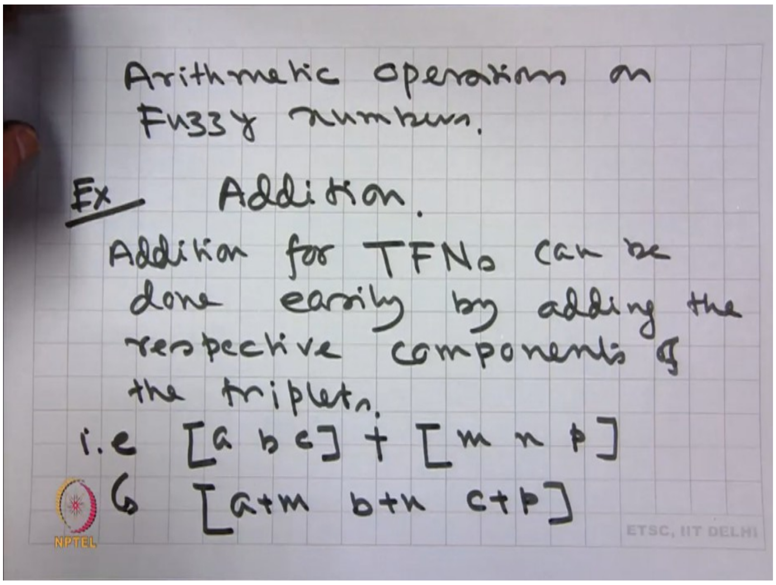

As I said, we will look at the -cuts. Then, we will operate on the -cuts. And from there, we will try to recover the output.

So first, for example, Addition.

Addition for triangular fuzzy numbers can be done easily by adding the respective components of the triplets.

That is, if is a triangular fuzzy number and is another triangular fuzzy number; then their addition is, the triangular fuzzy number . I hope you remember the definition of a triangular fuzzy number.

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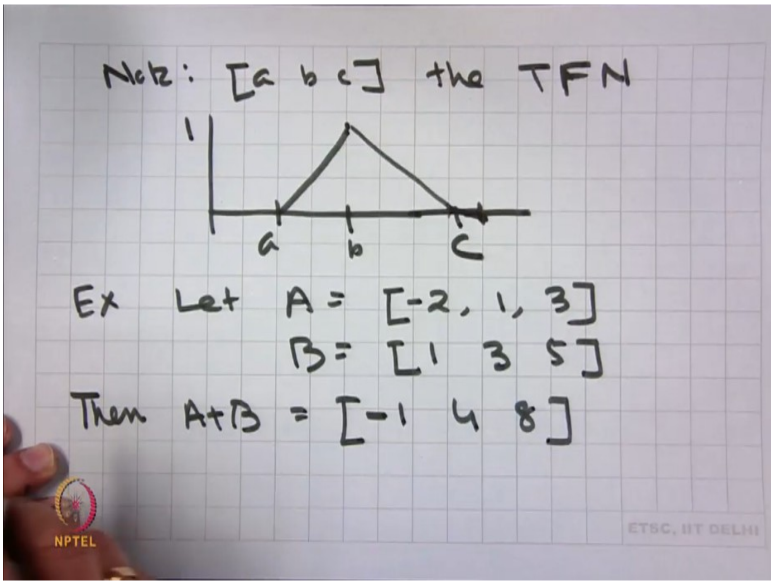

denotes the , which is having full membership at .

And between to , it is rising linearly. And between to , it is decreasing linearly.

So, for example, let and ;

then, . Now, we show it using -cut.

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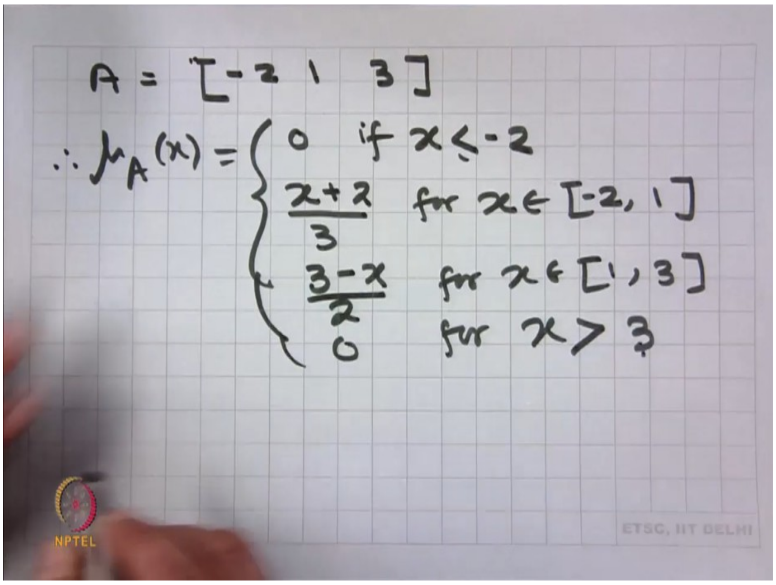

Therefore,

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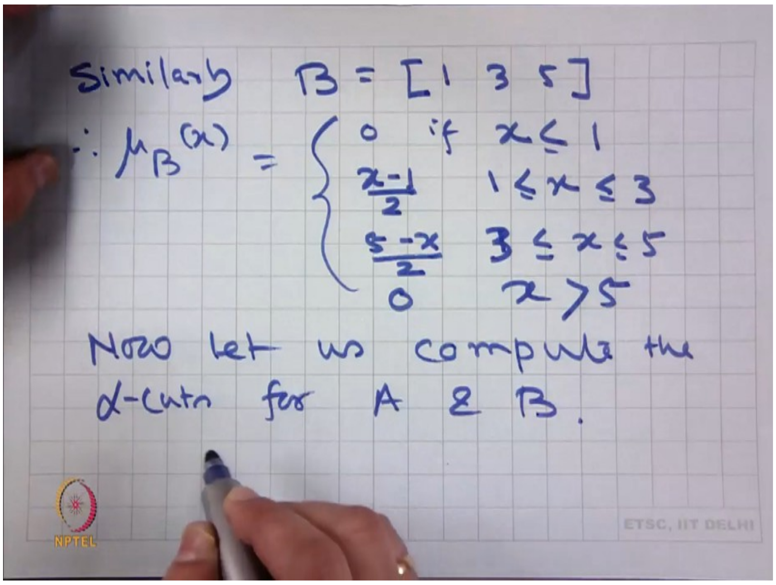

Similarly,

Therefore,

Now, let us find out the alpha-cuts for and .

We have already obtained the membership values for different corresponding to the set .

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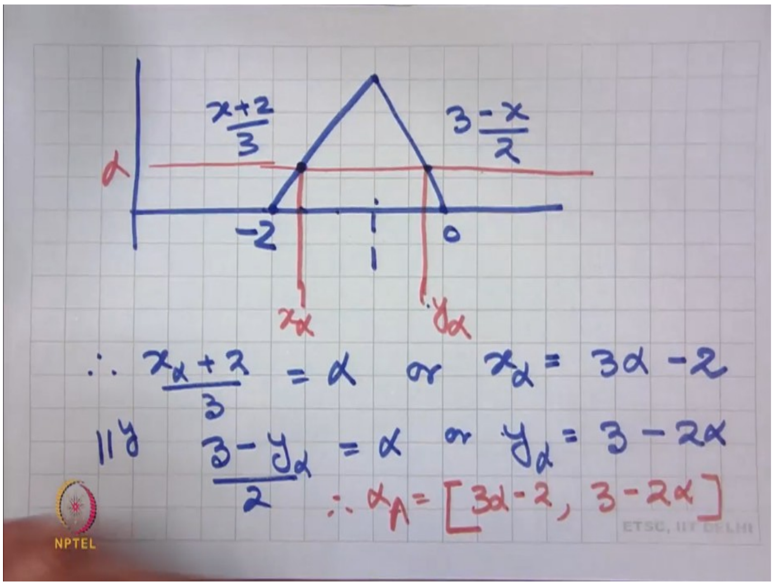

Let us draw the membership function for which is;

at 1, this is ; at , it is ; and at , it is .

So, we get this triangular fuzzy number.

And we know that the and

So, how do you get the alpha-cut?

Corresponding to any arbitrary alpha, we draw the line.

Therefore, this interval is going to give us the -cut.

So, let us call it . So, at the point , the membership value is .

Therefore, .

Similarly, at the membership value is . Therefore,

Therefore,

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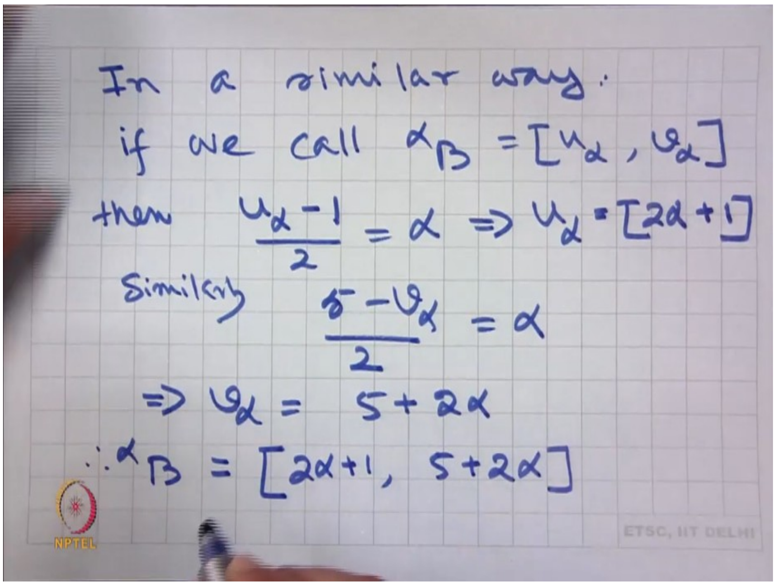

In a similar way, if we call then,

Similarly,

.

Therefore, on one hand we have .

On the other hand, for .

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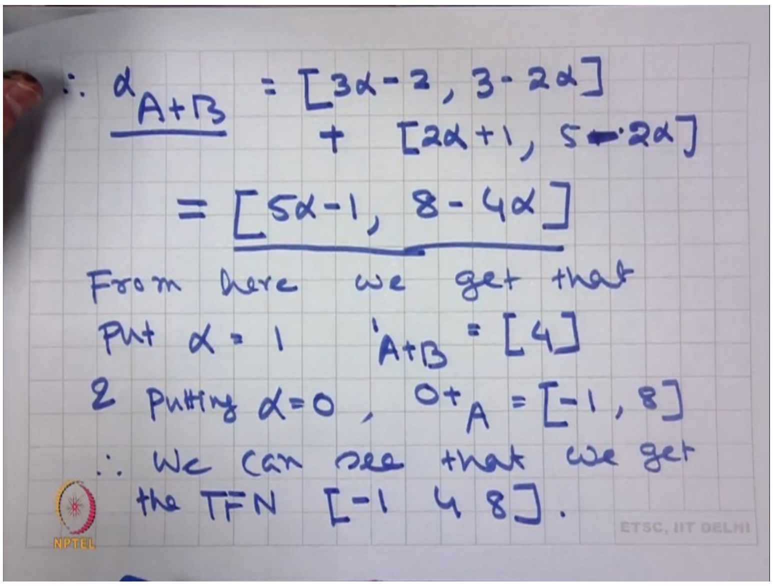

Therefore, .

From here, we get that, putting , .

And putting ,

Therefore, we can see that we get the .

And if we look at; we have got this same TFN when we have added the respective components.

This shows that, if we are given triangular fuzzy numbers, then addition is straightforward. But, we can arrive at the same result by going through the -cuts.

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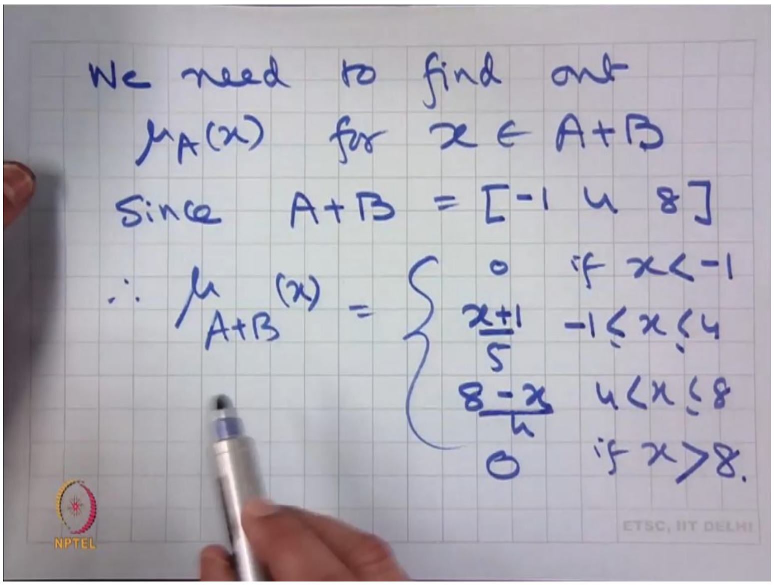

Now, we need to find out for . Since

Once we know the triangular fuzzy numbers the 3-tuples, we can easily compute the membership values.

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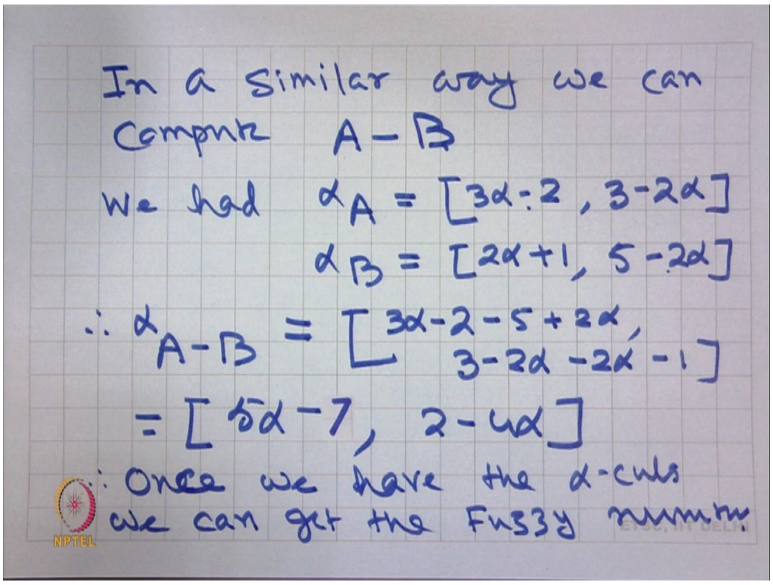

In a similar way, we can compute .

We had and .

Therefore,

Therefore, once we have the -cuts, we can get the fuzzy number.

I leave that as an exercise for you to compute the membership functions corresponding to A minus B.

Okay students, I stop here today. In the next class, I shall give you examples of summation for Trapezoidal Fuzzy Numbers. Since addition and subtraction are linear operations, it will be easy to compute the corresponding membership functions by establishing the linear equations. But, if we go for multiplication and division, we shall see that the corresponding numbers are not going to be Triangular Fuzzy Number at all, because we will get equations in, quadratic equations in

Therefore, we have to be more judicious in computing the multiplication and division for fuzzy numbers. We shall investigate that in our next class. Thank you.