Introduction to Fuzzy Set Theory, Arithmetic and Logic
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Lecture 02
Fuzzy Sets Arithmetic and Logic

Welcome students to the second lecture on Fuzzy sets, arithmetic and logic. If you remember, in the last lecture I have introduced the concept of fuzzy sets, which tries to represent the inherent vagueness with respect to certain concepts.

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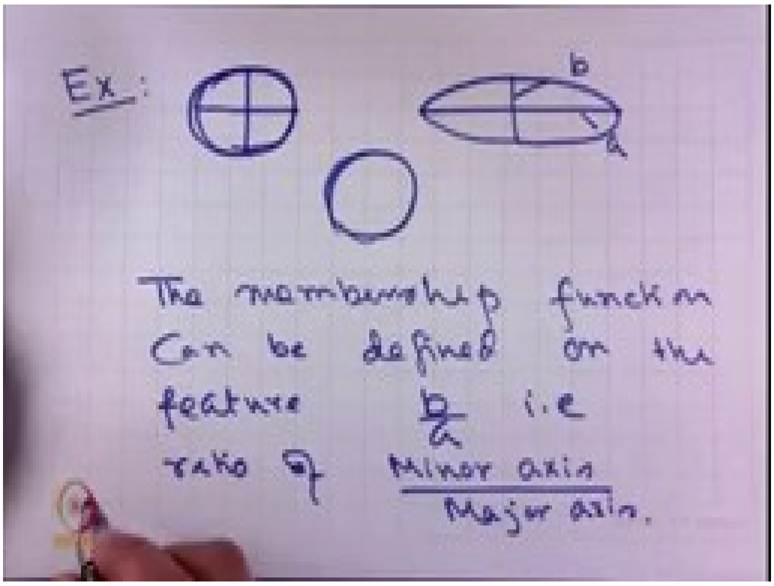

For example, I have given difference between a circle and an ellipse. The basic notion whether it is a circle or ellipse essentially depends upon the ratio of the major axis and minor axis. The closer it is to , the more it appears to be a circle and if that ratio say, . If I call it and if I call it , the smaller it is the more it resembles towards an ellipse. So, our shape which need not be a circle but appears somewhat elongated, so some of us may feel it is closer to a circle some may feel it is closer to an ellipse.

So, how do we distinguish between them? We look at membership function which is a function of; can be defined on the feature that is, ratio of minor axis by major axis. As it is minor upon major it can never exceed . So, perhaps one can think that up to the value of say

0.8 say to it may look like a circle. Its membership to the class circle will be higher than the membership to the class ellipse and below slowly the membership for ellipse will increase.

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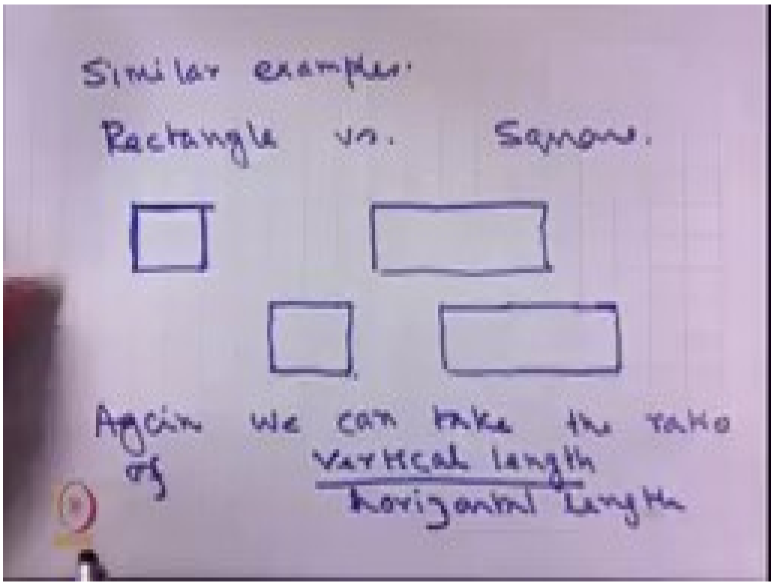

Similar examples rectangle versus square. This is a square, on the other hand, this is clearly a rectangle. What about this? Although it is horizontal sides are slightly bigger than the vertical side it still resembles more of a square than a rectangle. On the other hand, if this length is significantly more than the vertical length then it resembles more of a rectangle. Therefore, again we can take the ratio of vertical length upon horizontal length.

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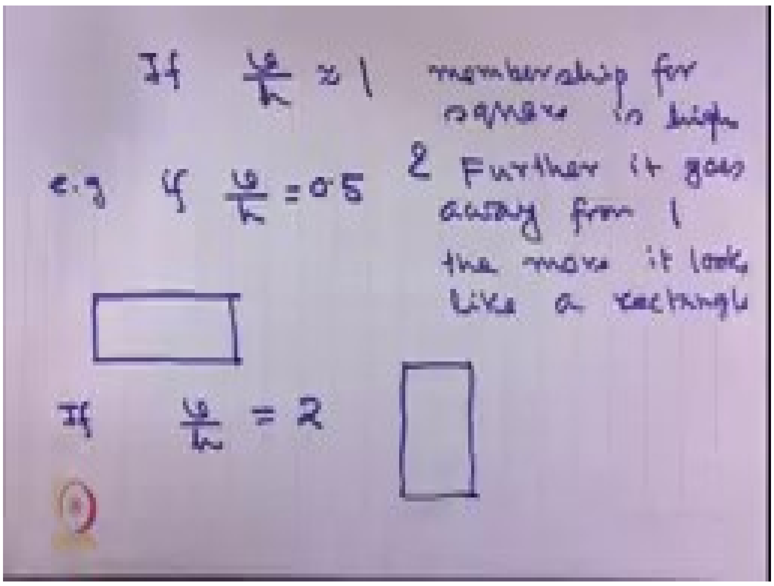

If we call them and . So, if is close to 1, membership for square is high. And further it goes away from , the more it looks like a rectangle. Example, if   it will look like this, if   then it will look like this. Clearly, both of them look like rectangles.

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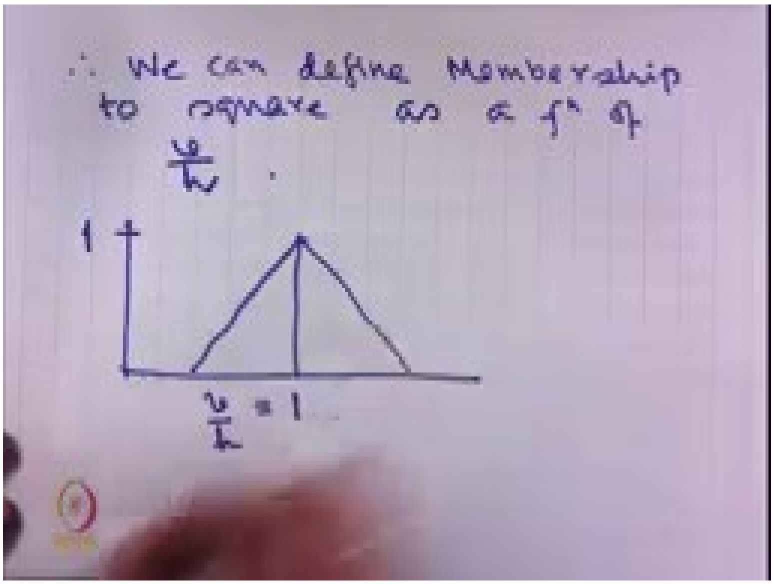

Therefore, we can define membership to square as a function of and if we plot it, we expect a graph like this. When then, membership is . But, as we go away from the membership to square decreases.

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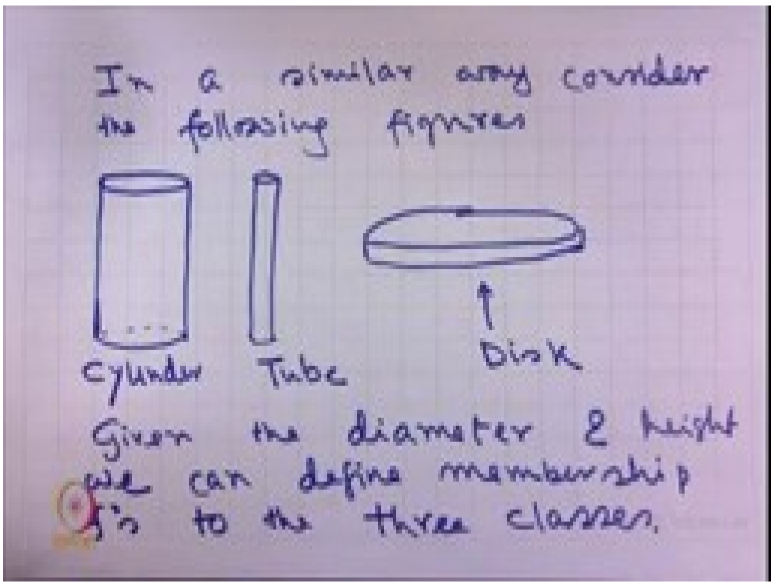

In a similar way consider the following figures. So this looks like a cylinder. On the other hand, if the diameter is very small in comparison with the height then, we call it a tube. On the other hand, if diameter is very big in comparison with the height we may call it a disk. Therefore, given a figure with two parameters diameter and height as before we can define membership functions to the three classes.

Question is how to define a membership function? One I have shown that has a triangular shape say for example with respect to square which is a triangular shape this is the vertex and this is the base of that triangle. There may be several other shapes that I have shown in my last class.

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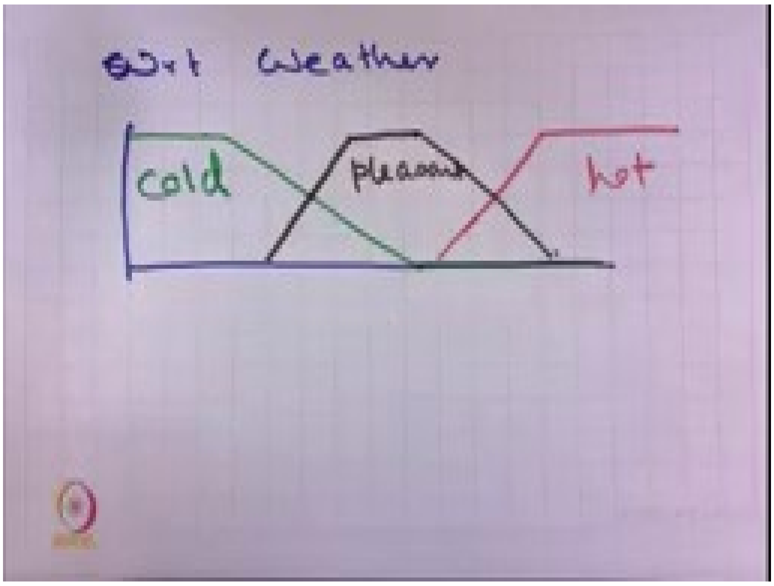

With respect to weather, if you look at the different shapes you might have noticed that, if it is a cold then it may have this shape. If it is pleasant it may have this type of shape and if it is hot it may have shape like this. So, this is hot, this is pleasant, and this is cold. So, depending upon the shape, this has a pie shape, this is called S and this is reverse of S you can say. Once the concept of fuzzy set is clear the question comes how to perform set operations on Fuzzy sets.

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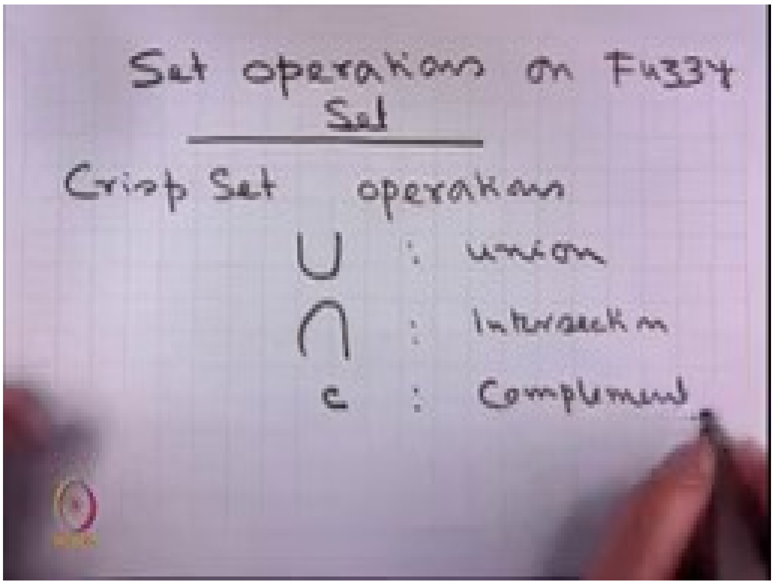

When we talk about a crisp set, the standard operations that we look at

- : Union,
- Intersection and
- : Complement.

Given 2 sets and we know that contains all the elements of and . Similarly, intersection contains all the elements that belong to both and And complement of a set in terms of a universal set is all the elements which belong to but do not belong to .

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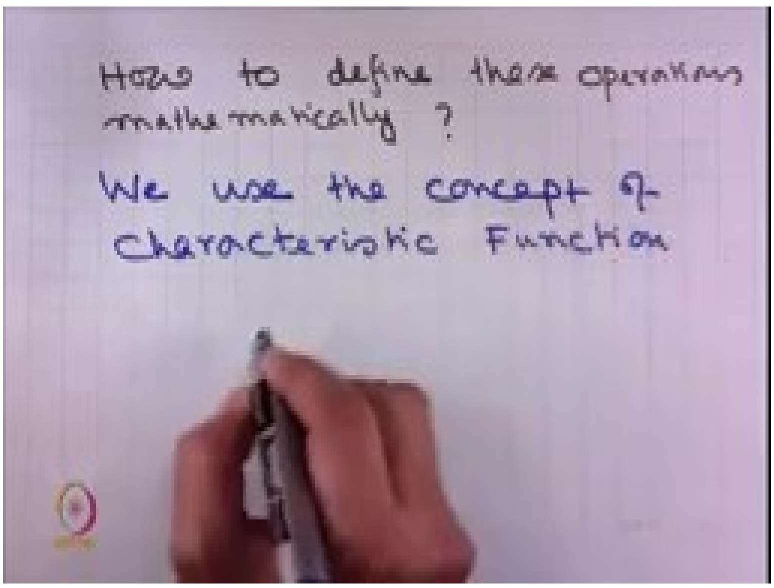

How to define these operations mathematically?

We use the concept of Characteristic Function.

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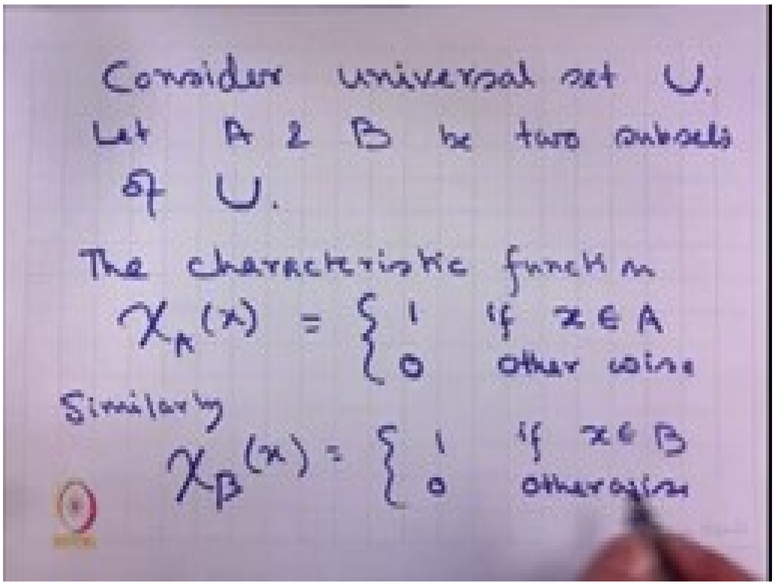

Consider universal set . Let and be two subsets of . The Characteristic Function . Similarly, .

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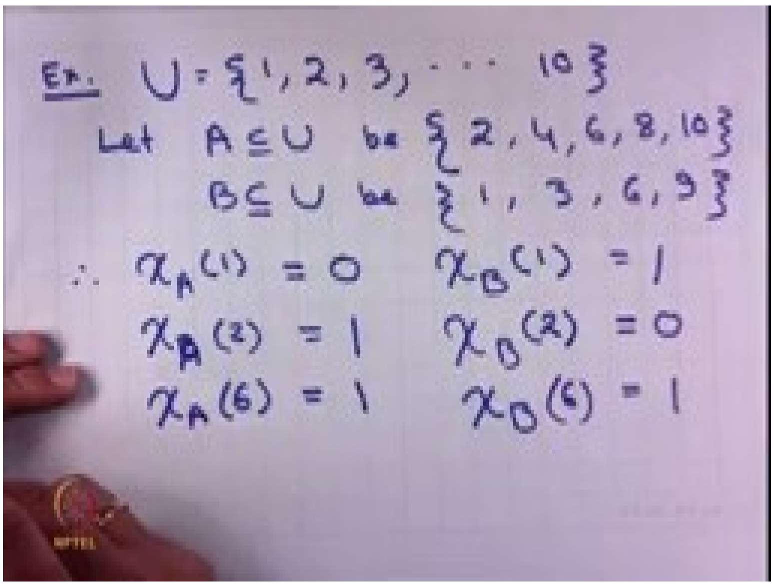

Example, . Let be and be . Therefore,

As it belongs to both of them.

In a similar way, we can compute the characteristic function with respect to all the elements of namely

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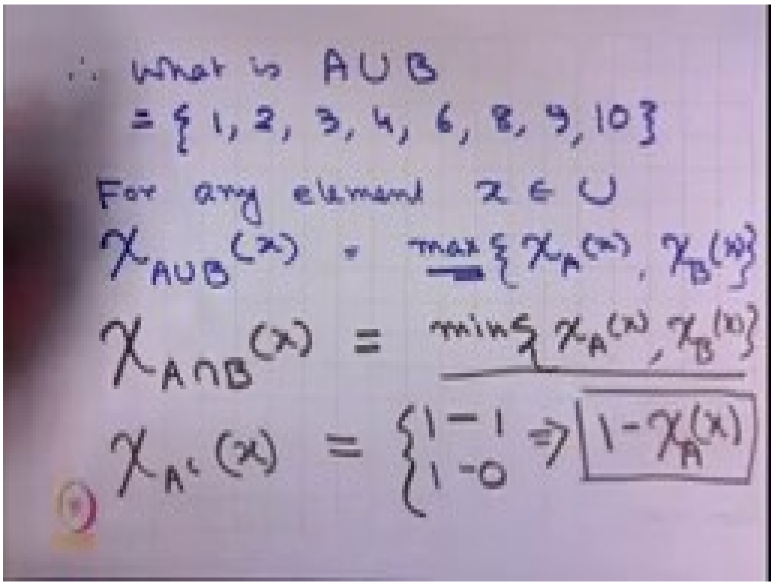

Therefore, what is ? This is equal to . How do you compute it? We can easily see that for any element ,

Say for example,

- with respect to 1 it is 0 here and 1 here. Since the maximum of these two is 1 it belongs to .
- Similarly, with respect to 2 it is 1 and 0 therefore the maximum is 1 therefore it belongs to .
- With respect to 6 it is 1 here and 1 here therefore, it also belongs to .

In fact, we can check that all these belong to and we can get therefore, the membership function to by taking the maximum of these two values.

What about ? if it belongs to both of them.

Therefore, we can think of . If we look at these two sets the only common of them is 6 and we see that and 1. Therefore, the minimum is 1 and therefore, defined by this function. For all other elements you will see either or or both of them 0. Say something like which neither belongs to nor to and therefore it cannot belong to .

What about If its characteristic value is therefore its value to should be which is equal to . If it is if it does not belong to then it will belong to and therefore its value should be 1 – 0. Therefore, together we can write . Okay. Zadeh extended the similar concept with respect to membership functions to describe union, intersection and complement.

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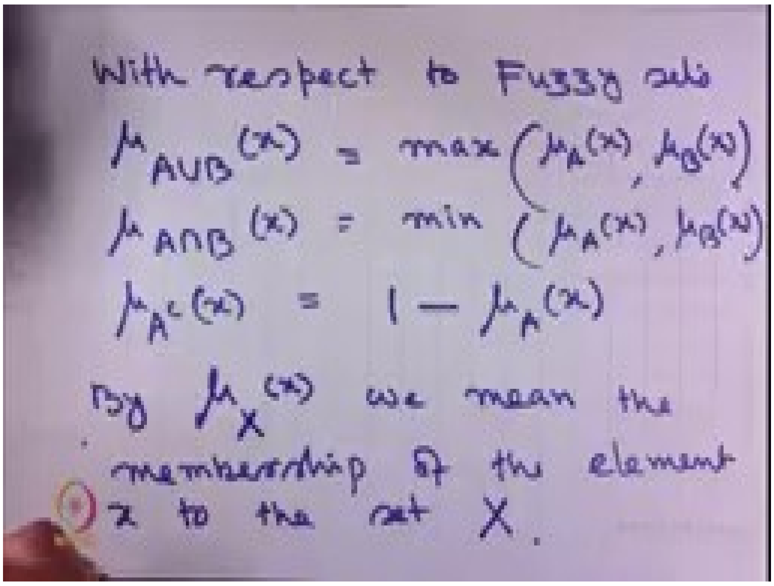

Therefore, with respect to fuzzy sets is defined as . and . If you are confused let me just say that by we mean the membership of the element to the set .

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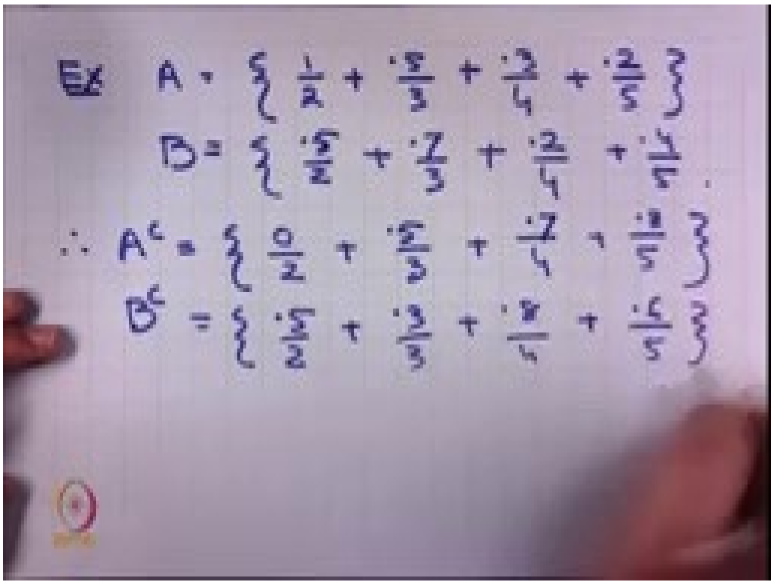

Example,

What does it mean?

It means that the membership of 2 to the set is 1 membership of 3 to the set is 0.5 for 4 it is 0.3 and for 5 it is 0.2,

Therefore, is equal to as you know it is 1 minus the membership.

Similarly,

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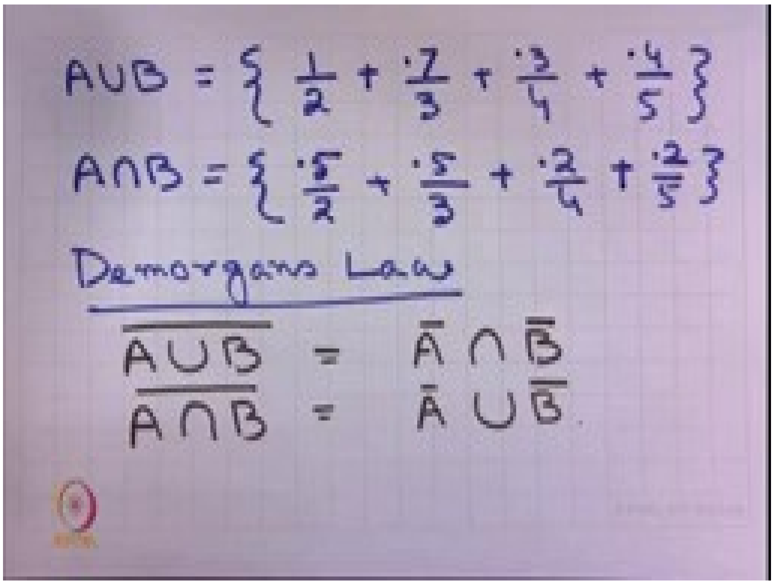

Therefore, what is ?

We can define the membership as the maximum of the two memberships of and . Therefore, .

What about ?

It is the minimum of the memberships to and

Therefore,

What about de Morgan's law?

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Question comes whether these two properties hold for a fuzzy set.

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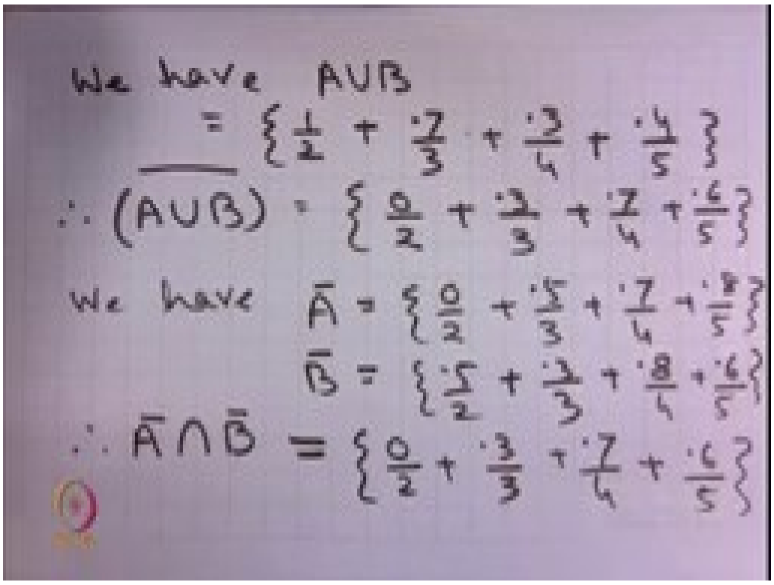

Let us see. We have

We have and

and we can see that .

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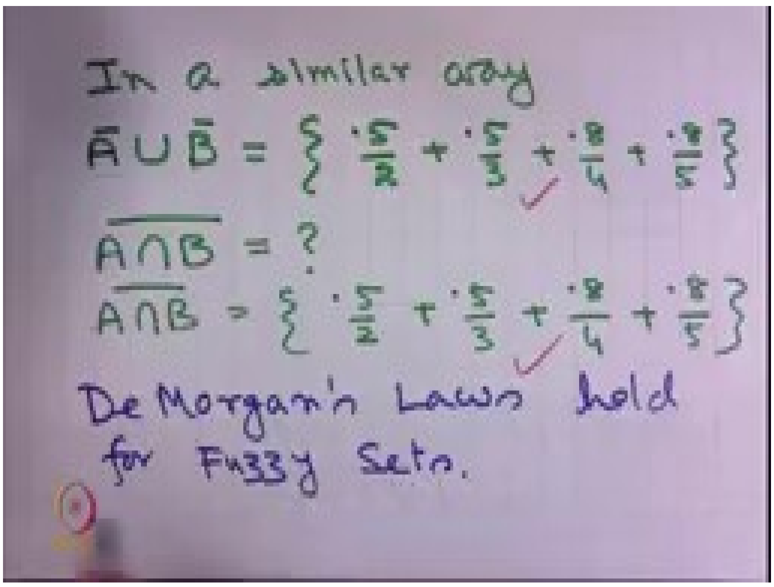

In a similar way, we have to take the maximum of these memberships, so it is coming out to be . Since and if we take the union of these we will get the maximum of individual memberships. Therefore, we can see that .

What is ?

We have computed therefore, . For each of the elements we are taking one minus the membership values so it is coming out to be and if we compare these two we find that they are same. Therefore, we can say that de Morgan's law holds for fuzzy sets.

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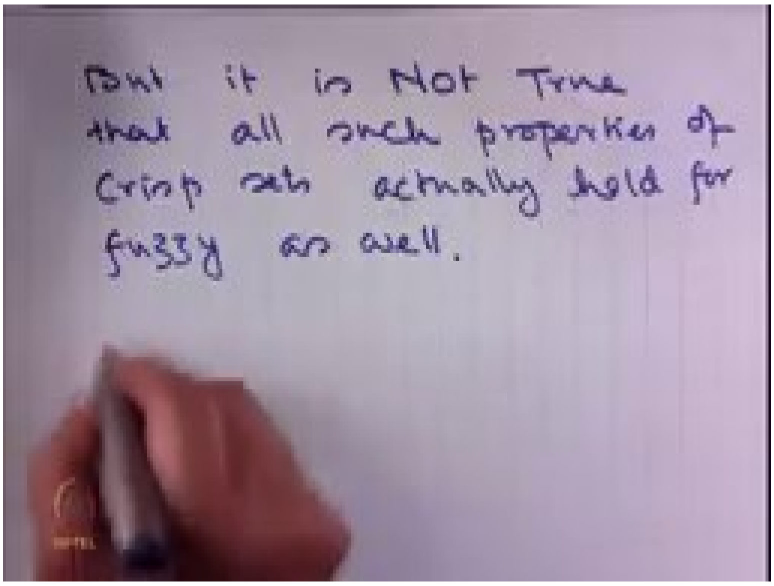

But it is not true that all such properties of crisp sets actually hold for fuzzy as well.

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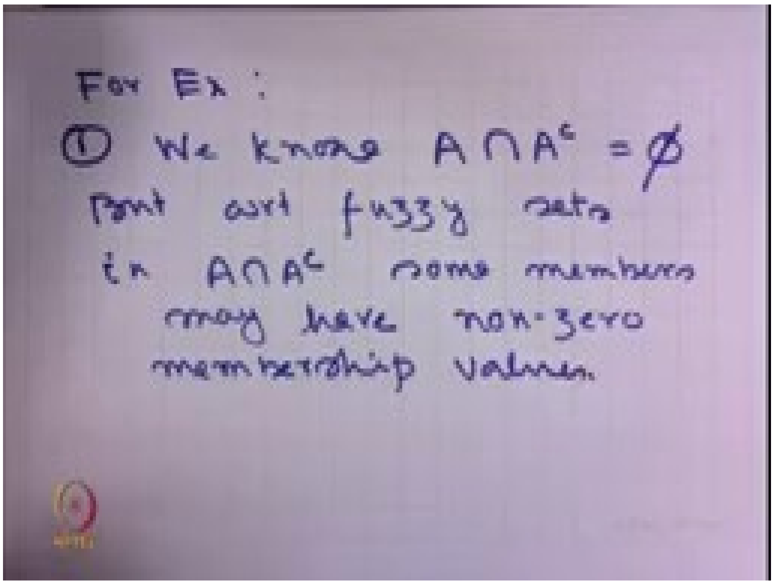

For example, we know but with respect to fuzzy sets, in some members may have non-zero membership values.

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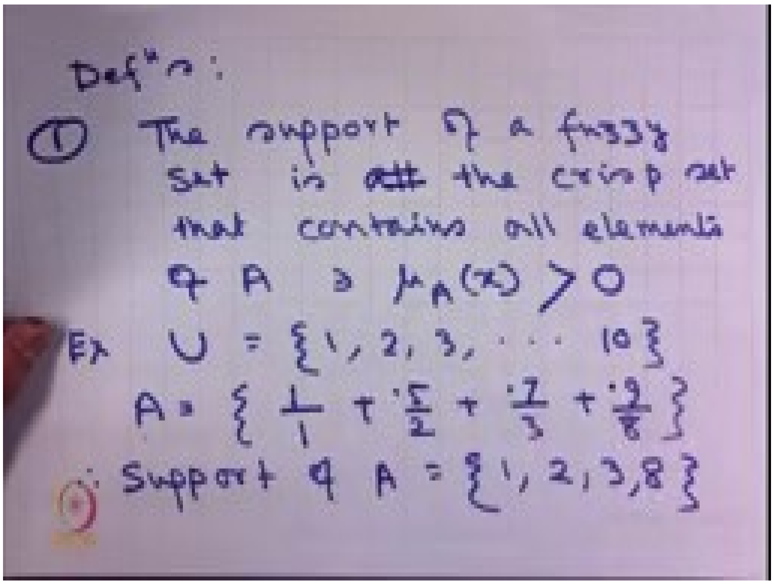

Let me give some definitions.

Support:

The support of a fuzzy set is the crisp set that contains all elements of such that.

Say for example .

Suppose

If we take this example so, we have defined a fuzzy set on . Therefore, support of is equal to the crisp set . For all other elements of , their membership to is .

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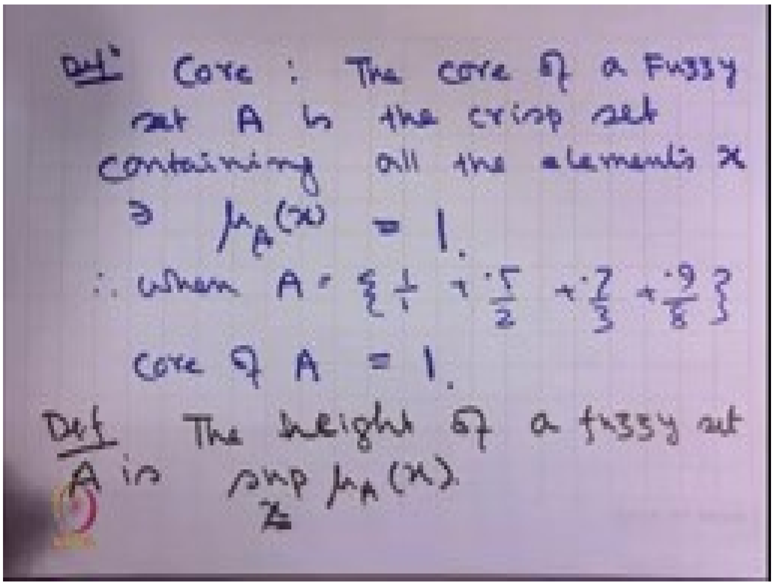

Core:

The core of a fuzzy set is the crisp set containing all the elements such that Therefore, in this case we can see that the Core is 1.

Therefore, when . Core of is equal to .

Height:

The height of a fuzzy set is .

Again the set is called A therefore, supremum over all the membership of supremum over the membership of all the elements of is called the height of the fuzzy set.

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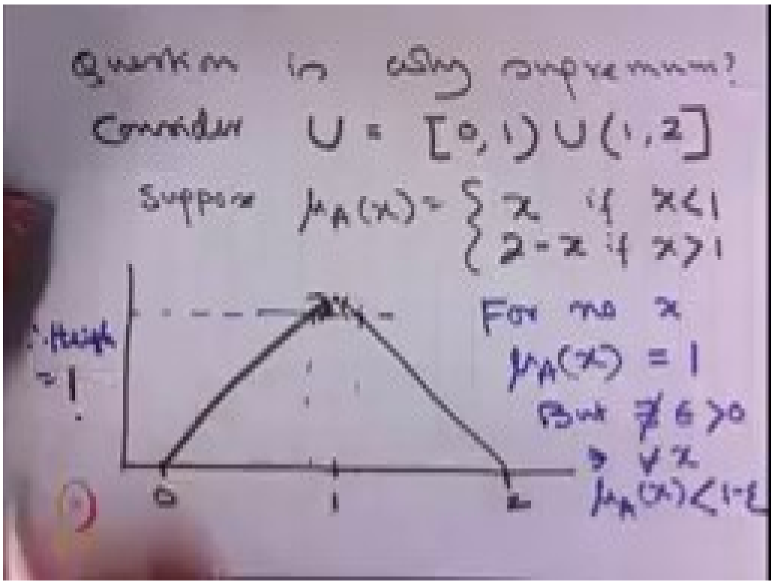

Question is why supremum?

If it is a discrete then, of course the maximum of the different membership values is considered to be the height. But consider So, I am looking at a set defined over the interval but I exclude the point 1. Suppose . Then we get this membership. So, between to it is this line, but it does not touch . Therefore, it will not be included there.

Similarly, between to . So, if you look at it for no , .

At the same time, such that for all , .

If you take epsilon here, you can see that still there will be some for which the membership is greater than .

Therefore, actually if you want to look at the height we need to go to the supremum of these things which is going to be in this case.

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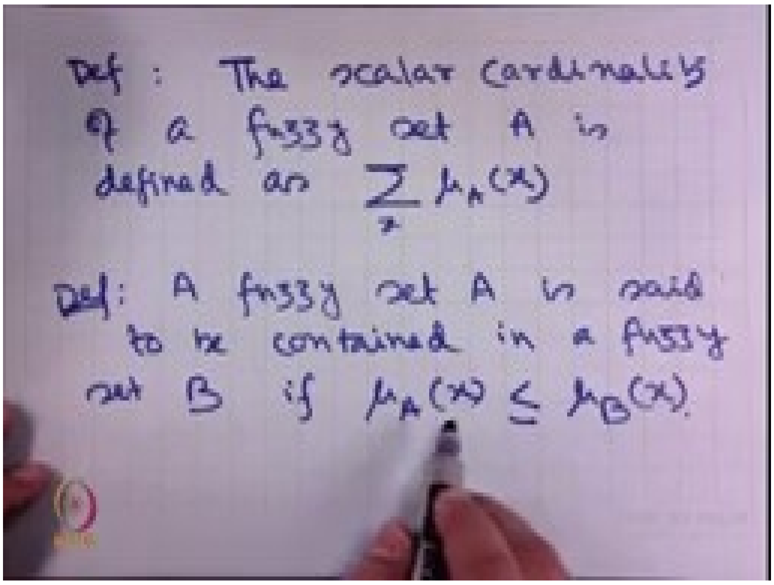

The scalar cardinality of a fuzzy set is defined as .

That is, we are adding up the membership values of all the elements belonging to the set .

A fuzzy set is said to be contained in a fuzzy set if .

That means for all the elements of the membership to is less than equal to the membership to .

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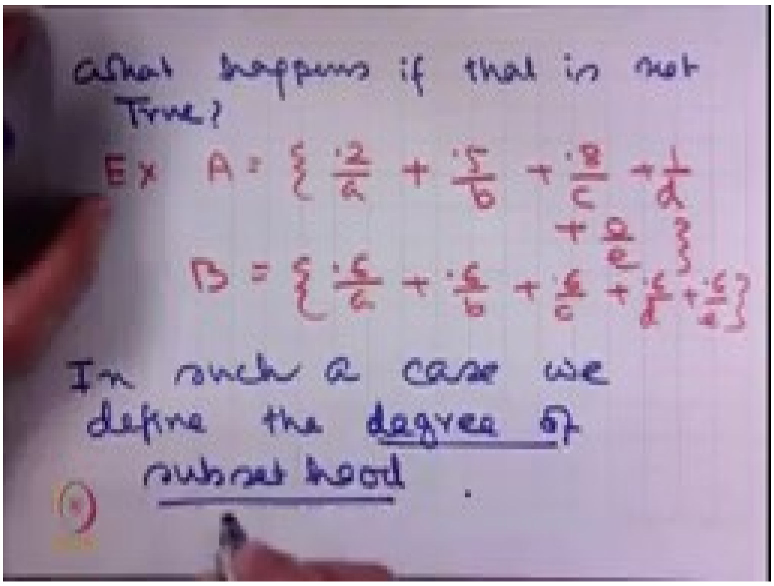

What happens if that is not True?

Example:

and

Therefore, if we look at it we can see that, for all the elements it is neither the membership values of is smaller than the membership value of nor it is true that (for all values) for all the elements of the membership is less than the membership of.

Therefore, neither is fully contained in nor is fully contained in . In such a case we define the degree of subsethood.

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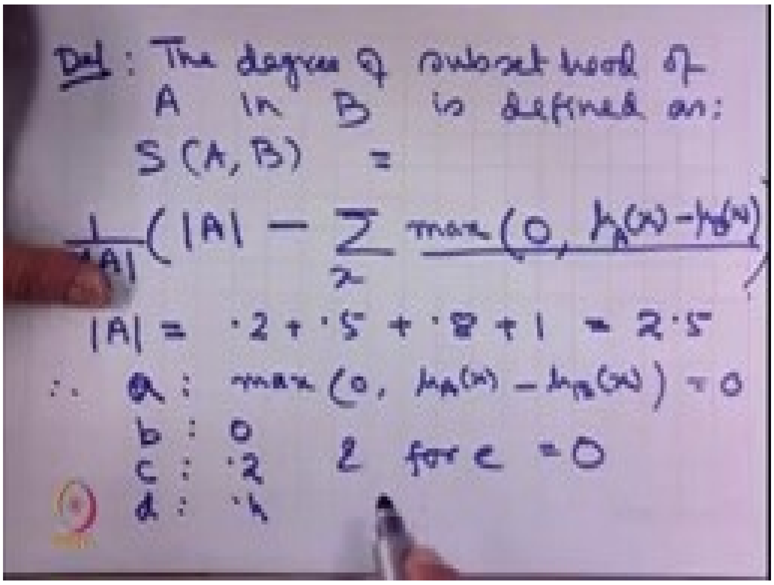

The degree of subsethood of in is defined as

So, what is happening?

If then, is but, if then becomes negative. Therefore, we are getting 0. Therefore, let us understand what is the subsethood of in .

So, according to the definition, therefore, . Therefore, what is with respect to the 5 elements?

For ,

For ,

For ,

For ,

For ,

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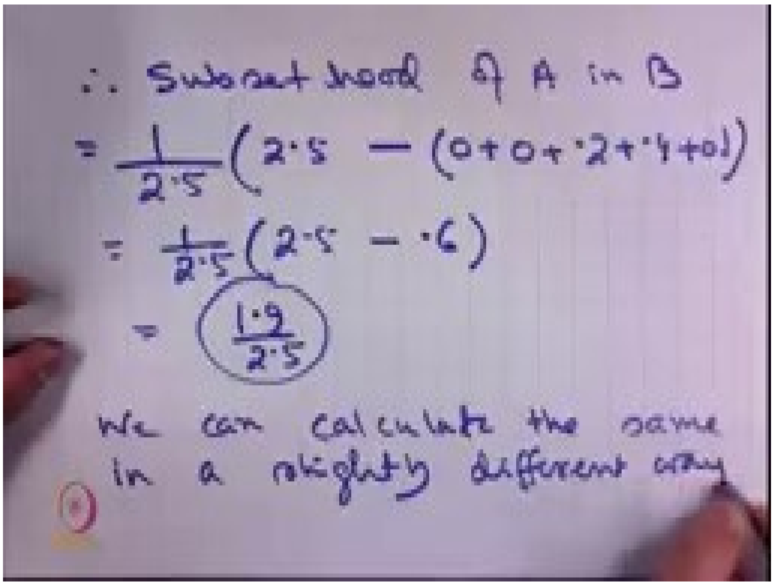

Therefore, subsethood of in

 .

We can calculate it in a slightly different way.

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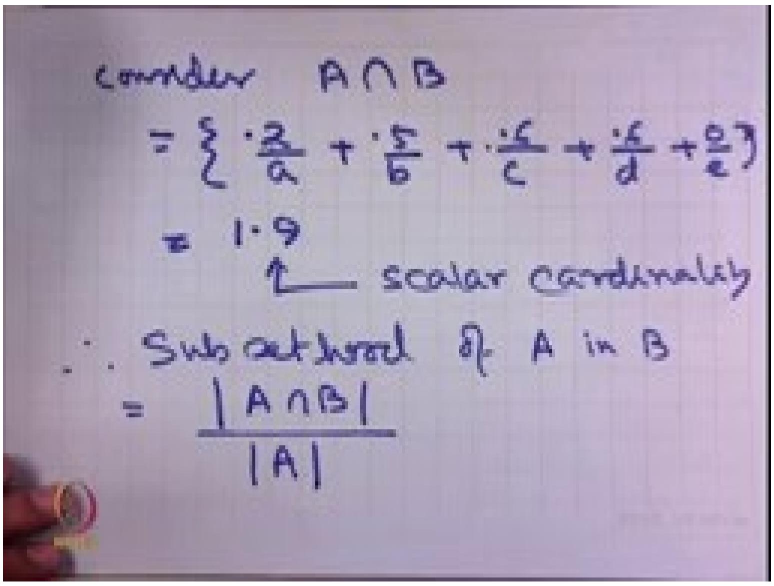

Consider . We know that with respect to standard intersection which is the minimum, (the intersection) the membership values come as the minimum of these corresponding memberships. So, therefore the is going to be .

Therefore, subsethood of A in B

So, this is another way of defining subsethood with respect to fuzzy sets.

Okay students I stop here today. In the next class I shall start with different types of membership functions and also I will introduce you to the interesting property and very important property of alpha-cut with respect to fuzzy sets. Thank you.