Introduction to Probability Theory and Stochastic Processes
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Lecture – 91

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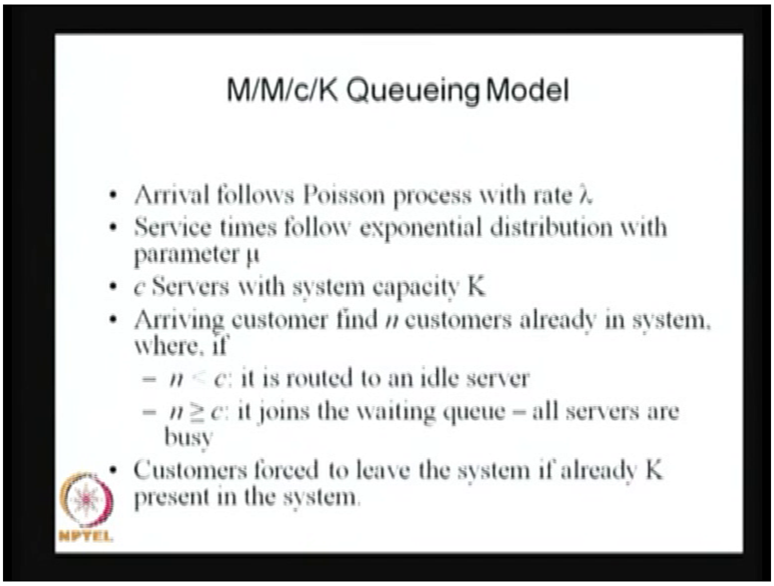

Now, I am moving into MMcK model queuing model. So, here the change is instead of a one server in the MM1 model you have a more than one servers c. And you have a finite capacity that is capital K capacity of the system. So, the arrival follows the Poisson process service is exponential. We have c identical servers, the capacity is a capital K, and this is a scenario in which a whenever the system size is less than c, it will be routed into the ideal server. If it is greater than or equal to c; that means, all the servers are busy; that means, the customer has to wait.

But if the system size is full; that means, a c customers are under service and K minus c customers are waiting in the queue for the service , then whoever comes it will be rejected, forced to leave the system. Therefore, you have a waiting as well as a blocking. Because it is a finite capacity there is a blocking, and since you have a always we choose K, such that it is a K is always greater than or equal to c.

If K is equal to c, then it is a loss system. If a K is greater than c, then K minus c customers a maximum can wait in the system in the queue.

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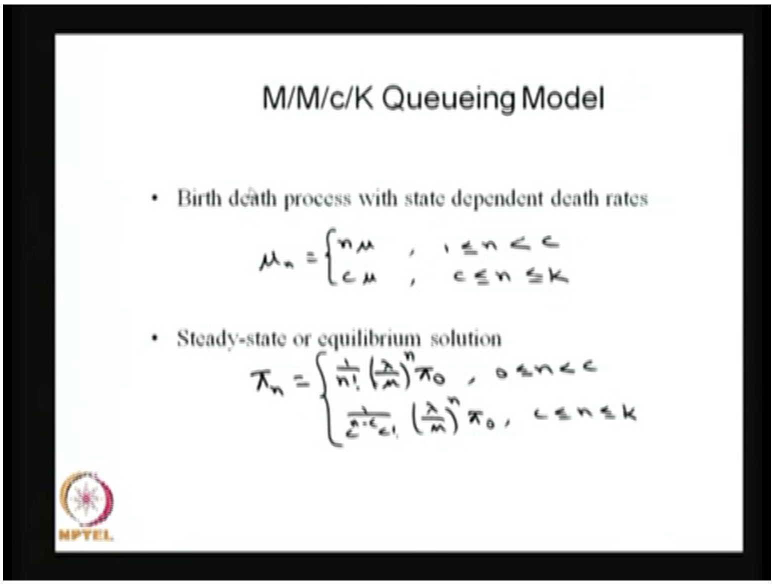

Therefore, the underlying stochastic process here the stochastic process is again number of customers in the system at any time t. Therefore, this stochastic process is also going to be a continuous time Markov chain. Because of these assumptions inter arrivals or exponential distribution service each service is each service by each server is exponentially distributed and all are independent and so on.

So, with these assumptions this stochastic process is a continuous time Markov chain. And at any time only one forward or only one backward can the system can move therefore, it is going to be a birth death process also. And the birth rates are lambda, because it is a infinite source population. So, all the lambda ns are going to be lambda whereas, the death rates are state dependent. That is going to be n times mu lies between one to c from c to K onwards it is going to be c mu.

So, I have not drawn the state transition diagram for MMcK, but you can visualize the way we have a MM 1 n and MMc model. So, with the combination of that that is going to be the state transition diagram. Since it is a finite capacity model, it is easy to get the steady state and the equilibrium solution. So, first you solve pi q is equal to 0; that means, you write a pi n c in terms of pi naught and use a normalizing constant summation of pi I is equal to 1 using that you will get the pi naught.

So, I have not written here. So, you use the normalizing constant the summation of pi I is equal to 1, you get the pi naught then substitute pi naught here therefore, you will get pi n in terms of pi n completely.

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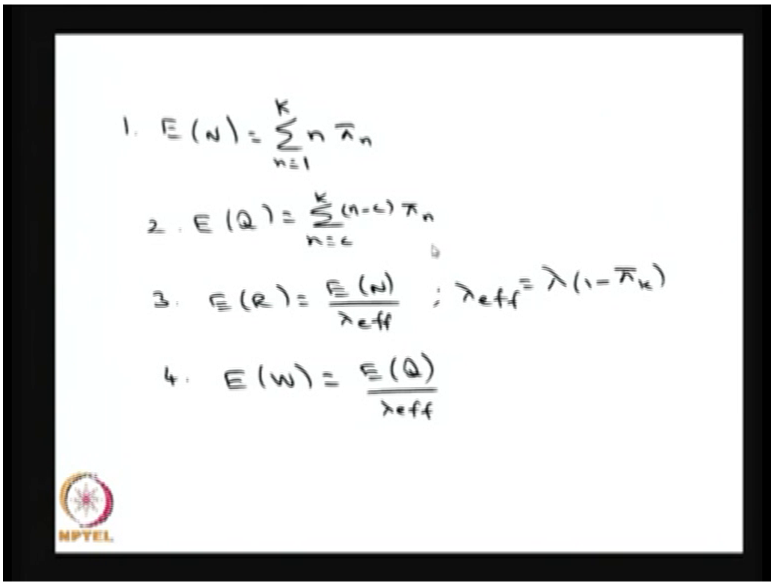

After that you can get the all other average measures the way I have explained the MM1 n, and the MMc infinity the combination of that. You can get the average number of customers in the system, average number of customers in the queue. That is a n minus c times pi n the combination the summation goes from c to K, and the average time spent in the system.

Since it is a finite capacity you have to find out the lambda effective. Effective arrival rate that is a 1 minus it is a capacity is capital K therefore, 1 minus pi K and that is the probability that the system is not full. So, the effective arrival rate is lambda times 1 minus pi K. Substitute here and get the average time spent in the system. And similarly you can find out the average time spent in the queue also using the littles formula.

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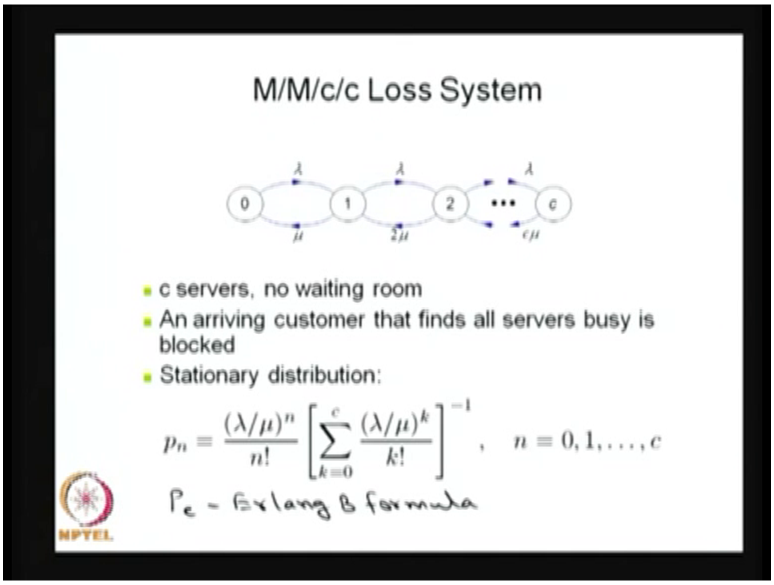

Now I am moving into the 4th simple Markovian queuing model, and first I started with the MMc infinity MM1, n then I did MMcK and now I am going for capital K is equal to c that is a law system. It is not a queuing system, because we have a c servers, and the capacity of the system is also c. Example is you can think of a parking lot, which has the some c parking lots, and the cars coming into the system that is a if you make the assumption is a inter arrival time is exponentially distributed and the car spending time in each parking lot that is exponentially distributed.

Then the parking lot problem can be visualized as the MMc loss system. So, here we have a c identical servers, no waiting room. So, since it is a c capacity and the c waiting room, you can think of a self-service with the capacity c that also you can visualize. So, the, inter here the inter arrival times are exponentially distributed, and the service by each server that is exponentially distributed with the parameter mu. Therefore, the system goes from 2 2 1 1 2 0 and so on, it is going to be a how many customers in the system and completing their service therefore, the time is exponentially distributed with the sum of those parameters accordingly.

Therefore, it is going to be 1 mu, 2 mu till c mu. Since it is a finite capacity and so on it is a irreducible model positive recurrent. Therefore, this listed probability exist limiting probability is also exist. And that is same as the equilibrium probabilities also therefore, by using a p q is equal to 0 and the summation of pi is equal to 1, you can get the steady state or equilibrium probabilities that is p ns. The piece of xc that is nothing but the probability that the system is full. And that is same known as a Erlang B formula.

So, this is also a useful to design the system for a given or, what is the optimal c such a way that you can minimize the probability that the system is full for that you need this formula therefore, to do the optimization problem over the c. And here we denote a piece of X c that is a Erlang B formula. Whereas, a Erlang C formula comes from the MMcK model for the loss system we will get the Erlang B formula.

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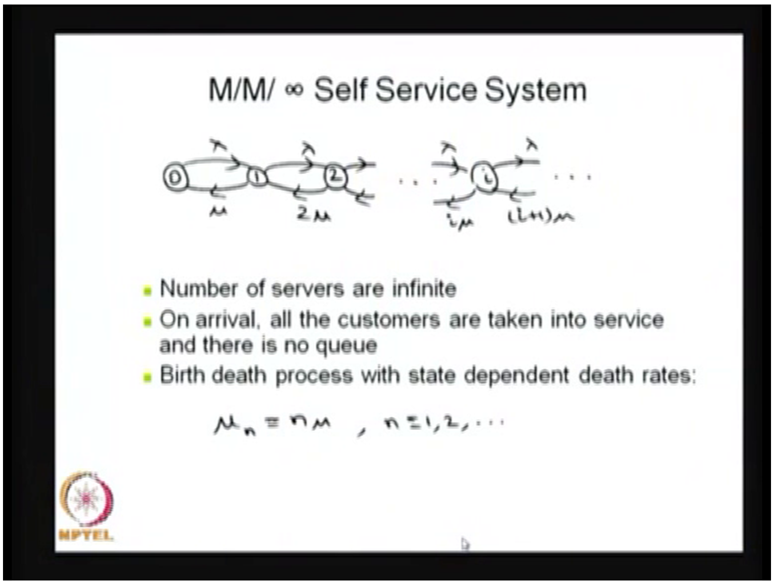

The fifth model that is MM infinity it is not a queueing model. Because in the servers are infinite, unlimited servers in the system therefore, the customer whoever enter he will get the immediately service, the service will be started immediately.

Whereas, the service time is exponentially distributed with the parameter mu by the each server all the servers are identical the number of servers are infinite here. Therefore, you will have a the underlying stochastic process for the system size, that is a birth death process with the birth rates are lambda, because the population is from the infinite source. The death rates are one mu 2 mu and so on, because the number of servers are infinite. So, the model which I have discussed in today's lecture all the 5 models are the underlying stochastic process is the birth death process.

This is a simplest Markovian queuing models.

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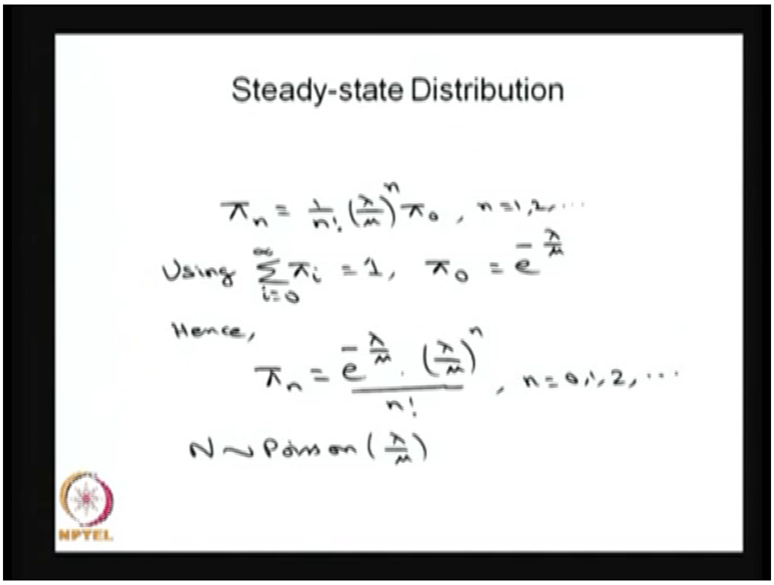

We can get the steady state distribution; you use the same theory of a birth death process. And if you observe these steady state probabilities is of the same Poisson. This of the form that is a probability mass function of a Poisson distribution therefore, you can conclude in a steady state a number of customers in the system that is Poisson distributed with the parameter lambda by mu, because the probability mass function for the pi n is the same as the probability mass function of exponentially distributed random variable with the parameter lambda by mu.