Introduction to Probability Theory and Stochastic Processes
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Lecture – 88

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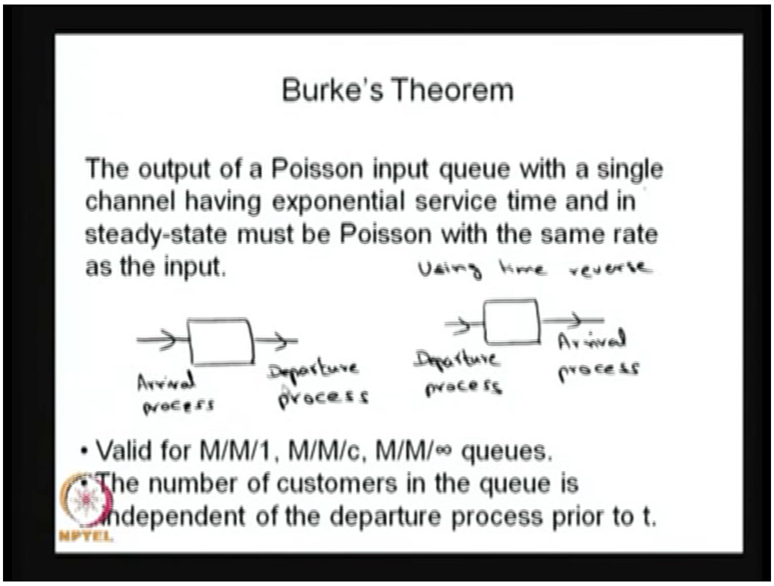

Here I am giving the concept of output process. The arrival follows the Poisson process for the M 1 queue and the service is exponentially distributed it is independent of the arrival process. And the customers leave the system. Now the question is what is the distribution of the departure process; that means, what is the inter departure time. After first customer leaves how much time it takes for the second customer leave the system? Then the third customer how much how much time it takes a for the inter departure time?

And therefore, what is the distribution of the departure process. And that is given by the burkes theorem, the output of a Poisson input queue with a single channel having exponential service time, and in steady state must be a Poisson with the same rate as the input. So, whenever you have a system in which the arrival follows a Poisson. And whenever the system has a single channel and the service time is exponentially distributed in a longer run, the departure process is also going to be a Poisson process. And the rate will be the same rate as the arrival process.

So, this can be proved, this can be proved, but here I am giving the interpretation using the time reverse process because when a steady state this model is going to satisfies the time reverse model therefore, the stationary distribution exists. And if you make a this M M 1 queuing model the underlying birth death process satisfies a time reversibility equation. Therefore, using the time reverse you can conclude the departure process you can reverse it, and that is going to be a independent of the arrival process.

And this is also going to be a again Poisson process. So, using the time reverse concept one can prove the departure process is independent of the arrival process. And departure process is also Poisson process with their same rate as the arrival rate. And even though I said it is a single channel having a exponential service time, and this is valid for M M 1 queue, the multiserver Markovian queue, as well as a infinite server Markovian queue also. So, all those thing all those models can be combined with the single channel having a exponential service time, whether it is a single server or multi server or infinite server this result holds good.

And the next result is the number of customers in the queue is a independent of the departure process prior to it that is also satisfies.

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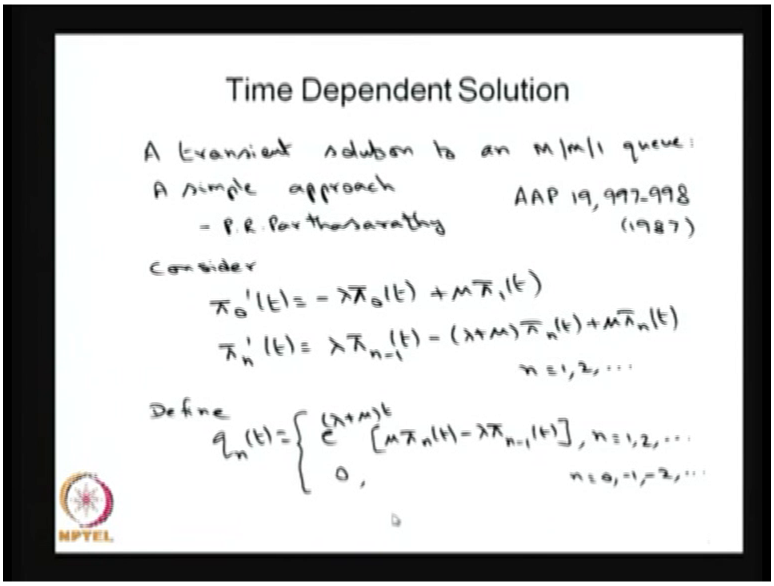

Now, we are giving the time dependent solution of a M M 1 queue. There are many more methods to find out the time dependent solution for a M M 1 queue it started with the spectral method and the combinatorial method. And also their difference equation method, like that there are many more methods in the literature to find out the time dependent solution.

And here I am presenting the time dependent solution by PR Parthasarathy, and this work is appeared in the advance applied probability volume number 1987. So, in this paper they have he has considered the system of difference differential equation that is nothing but the forward Kolmogorov equation and making a simple function q n of t that is a difference of a pi n's with the multiplication e power lambda plus mu of t. So, once you use this definition, once you convert this system of difference equation with the q n of t by making a proper generating function that is of the form n is equal to minus infinity to infinity q n of t times s power n.

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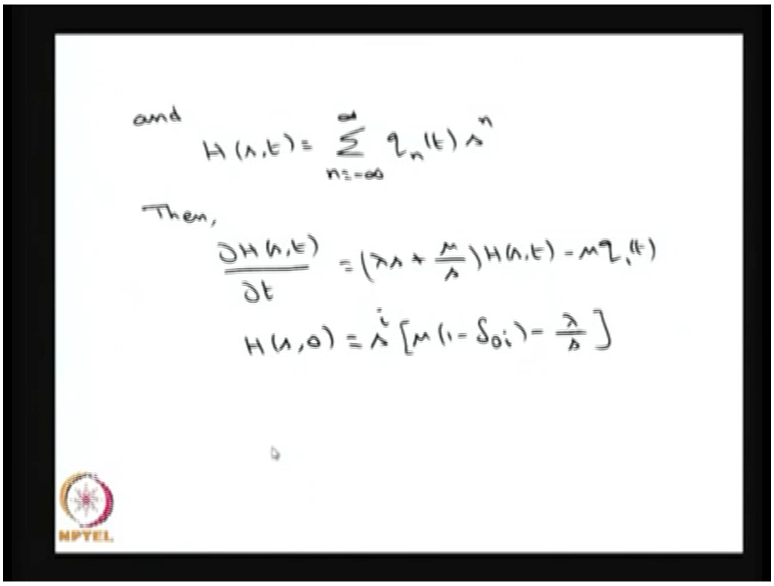

Therefore, this is sort of a generating function in terms of q n of t, where q n of t is for n is equal to 1 to infinity, this is of difference of a mu times pi n minus lambda times pi n minus 1 multiplication e power this function. And for n is equal to 0 minus 1 2 and so on 0 therefore, you have a generating function. So, you can convert the whole difference differential equation in terms of pi n into the one partial differential equation. With the initial condition also changes because if you assume that i customers in the system at time 0, and this is going to be a initial condition for a function h of s comma t at a t equal to 0.

So now the question is you have to solve this equation with this initial condition this pde using this initial condition.

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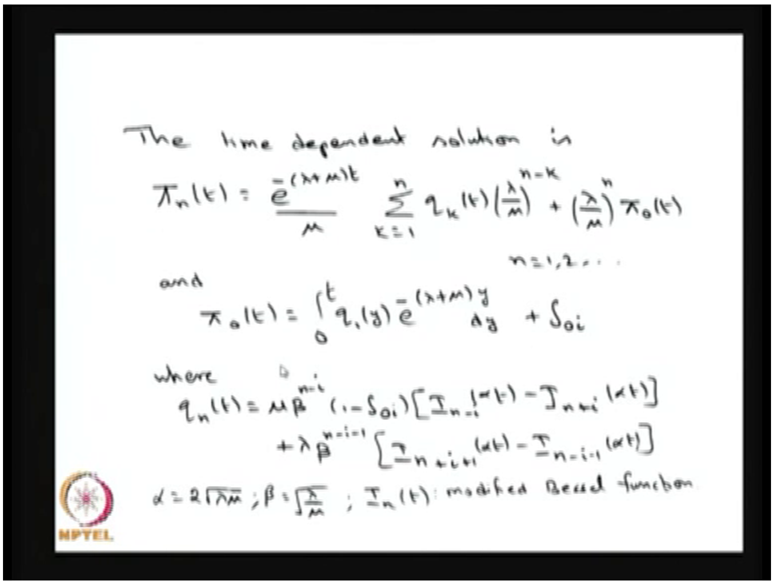

So, use the some identity of a modified Bessel function, one can get the solution pi n of t in terms of pi naught where pi naught you can get it in terms of q 1 where all the q n's satisfies this equation. That is in terms of the modified Bessel function. So, one can see the complete solution in this paper. But here I am giving the very simple approach of getting the time dependent solution for the M M 1 queue by changing this system of differential equation into 1 pdi with the initial condition and solve that pd and obtaining the pi n’s and pi naught in terms of a modified Bessel function.

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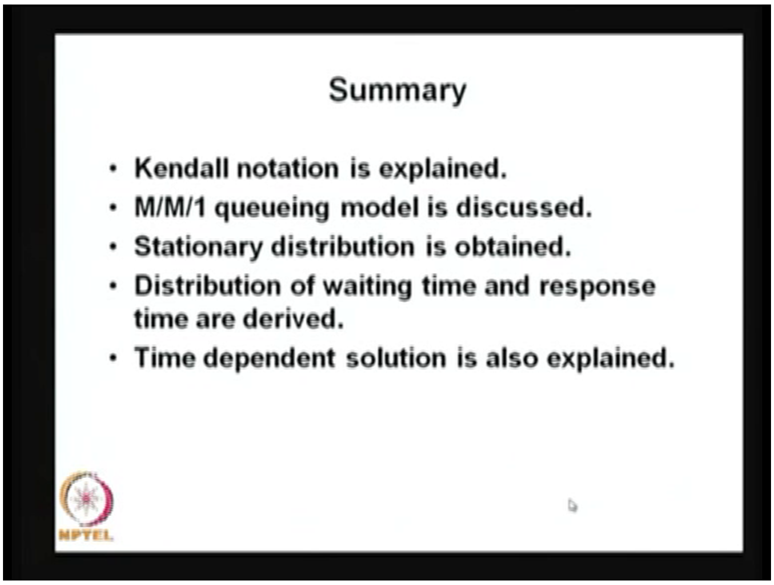

Before I go to the summary let me give the simulation of a M M 1 queue.

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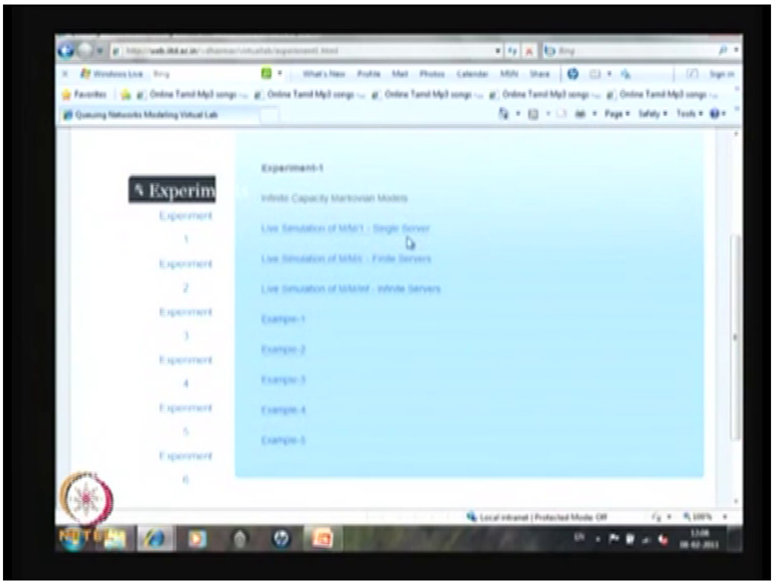

So, this is a queuing network modeling lab. So, from in this queuing network modeling lab one can simulate the queuing network models.

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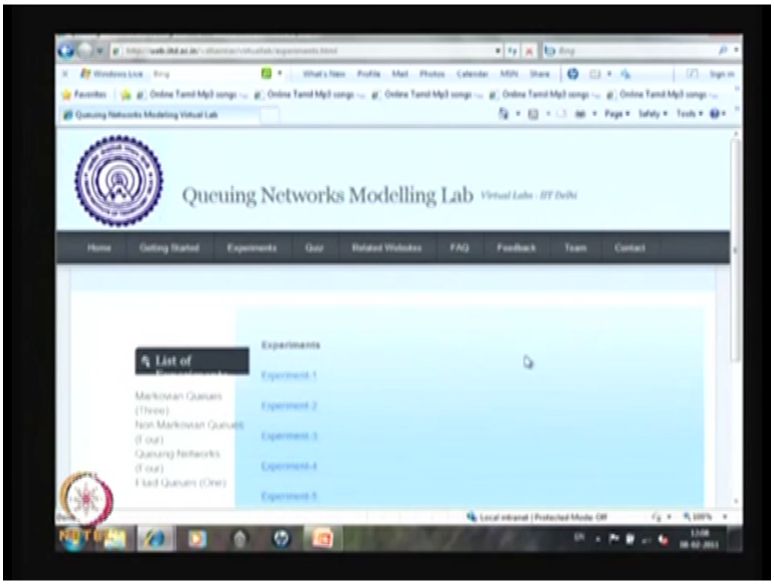

So, for in this I am going to explain how to simulate the M M 1 queue, and the first experiment. That is nothing but live simulation of a M M 1 queue single server as well as you can simulate a multi-server queue model and you can go for the infinite server model also.

So, here I am simulating the M M 1 queuing model.

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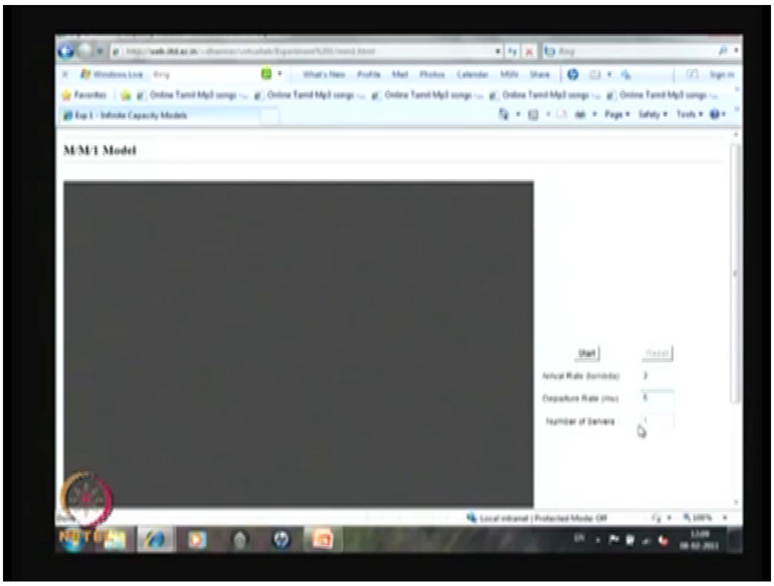

So, to simulate the M M 1 queuing model you need the information about the inter arrival time that is exponential distribution you need a parameter lambda. The value of lambda as well as you need a value of mu that mu is nothing but the service rate. So, suppose you supply the arrival rate suppose the arrival rate is 2, and the departure rate is 5 the number of servers is it is a M M 1 queue therefore, it is already one is placed it is a number of servers.

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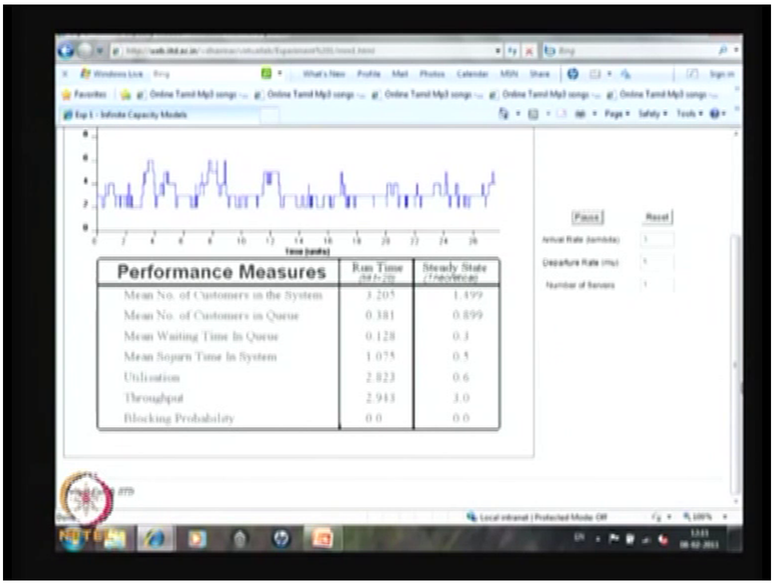

So, you can start so, this is the way the system increases. So, this is the actual simulation goes with the is a time x axis, and a y is the number of customers in the system. And here the information is how many customers entered till this time that is 15 customers entered, and nobody is blocked because it is a M M 1 giving system, therefore, all the customers who are entering it will be queued, and a how many customers are served during this time, and a number of customers in the orbit this is nothing to do with the M M 1 queue this is for the retrial queues.

And now how many customers are in the system at this time. And here this table gives the performance measures, the one we have calculated the average number of customers in the system e of r, and the average number of customers in the queue e of, this is mean number of customers in the system that is e of n. The mean number of customers in the queue e of queue mean waiting time in the queue that is a, mean waiting time that is a e w mean sojourn time in the system sojourn time spending time or response time all are the same. The mean sojourn time in the system is nothing but e of r.

So, this is nothing but the e of r, this is nothing but e of w, this is nothing but e of q, and this is nothing but the e of n. And the iteration is nothing but a what is the probability that the so, here I am giving the run time, what is the average values till this time, and what is the result is going to be in a longer run in a steady state. And the blocking probability is here 0, because the system is a infinite capacity model therefore, there is no one blocked therefore, the blocking probability is 0.

So, this is the way we can reset, and you can give some other values and you can, you can start again and you get the another simulation also.

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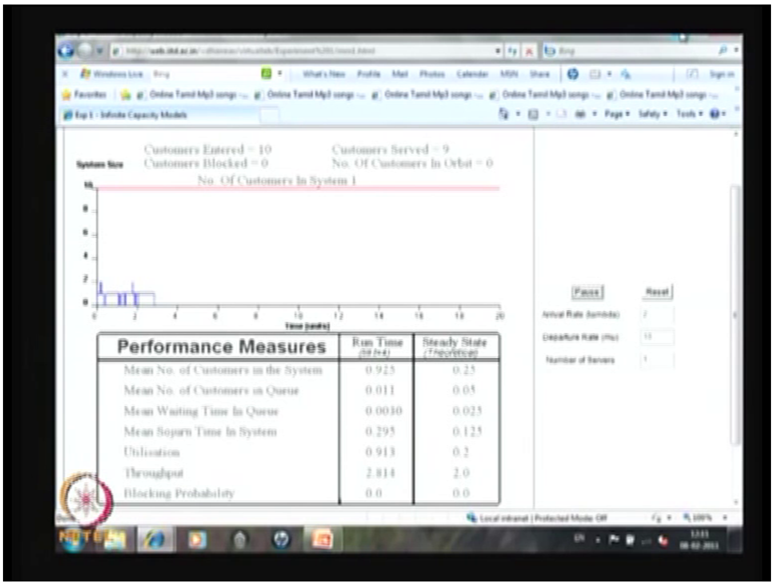

And initially it gives the fixed steady state results in the steady state theoretical result, and the runtime is nothing but what is the result for the over the time. With this let me complete the simulation. So, in the summary we have started with the Kendall notation and a M M 1 queue is a discussed stationary distribution, waiting time distribution, response time distribution is discussed for the M M 1 queue and also the time dependent solution, and I have given the simulation of a M M 1 queue also.