Introduction to Probability Theory and Stochastic Processes

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Lecture - 08

So, now we have explained how to get the cumulative distribution function of a random variable, through the concept of distribution function. That is what we explained the distribution function first that is a real valued function satisfying the four conditions. Then we are connecting through the probability x is less than or equal to x therefore, it is going to be the CDF of the random variable.

First we will go for the very easy example, in which how to get the cumulative distribution function for the random variable. So, for that we will take a very easy example start from the scratch.

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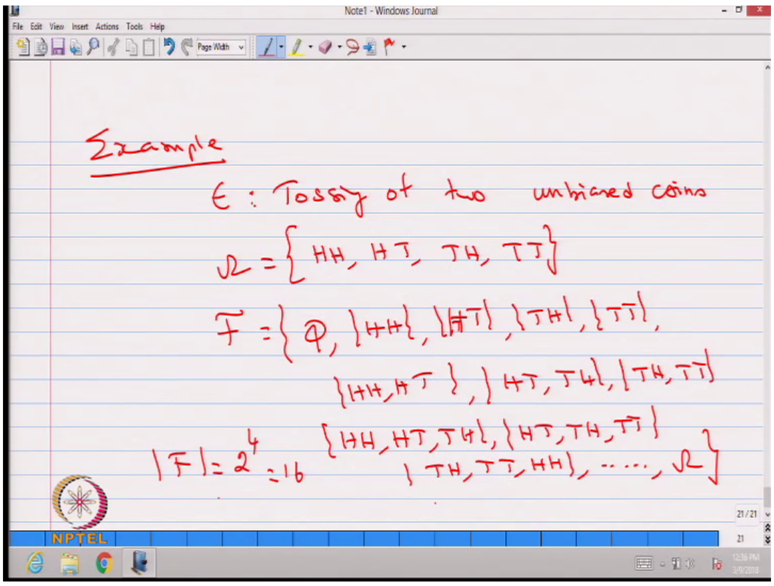

Start from the scratch means you start with the random experiment. So, the random experiment is denoted as a tossing. Yes the random experiment is nothing but tossing of tossing of two unbiased coin. Random experiment is nothing but tossing of two unbiased coins; either you can use unbiased coins or fair coin; that means, there is equiprobable of getting tail or head that is the meaning of unbiased coin or fair coin. So, the random experiment is the tossing of two unbiased coins.

Therefore, you will get collection of all possible outcomes is going to be I use a notation capital H for getting head capital T for tail. So, HH means both we got head and head, head and tail, tail head, or tail tail. When you tossing two unbiased coin therefore, you will get either head head or head tail or tail head or tail tail. Therefore, these are all the 4 possibilities

We have taken the largest sigma field F further consideration for this problem that is empty set then singleton element, any 2 elements oh I have written wrongly that is HH, H tail any 2 elements then any 3 elements then; so 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, so the largest sigma field consisting of a 16 elements. So, instead of writing other elements I am just putting dot dot dot dot, you can fill up all the other elements so, that this is going to be the largest sigma field.

So, for this problem the F is taken as a largest sigma field, which has number of elements is 2 power 4 that is 16 elements. So, one can fill it up all the elements now we are going to define the random variable that is nothing but the real valued function where x denotes number of heads obtained.

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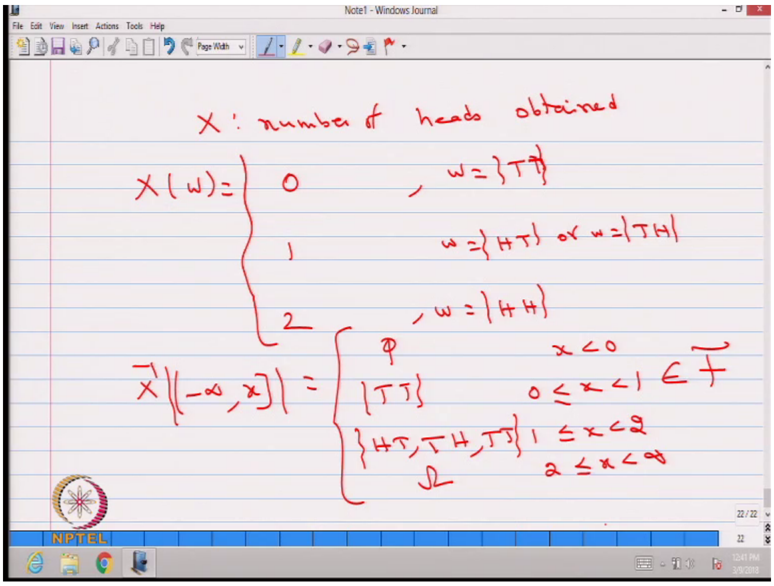

X is a real valued function which denotes number of heads obtains, when you toss a two unbiased coins.

So, x of w the possible values are going to be since it is a number of heads therefore, there is a possibility no heads or there is a possibility you may land up with only one head or 2 heads therefore, the possible values are 0 1 or 2. You can list out what are all the ws will give 0 as well as 1 and 2. So, for 0 it is both are going to be tail then it is going to be the both tail will give x of w is equal to 0 when w is equal to h tail o r or w is equal to tail head, you are going to get h of w is equal to 0.

When w becomes both head you are going to get the value. So, the possible values of the random variable x or 0 1 and 2. So, this is a real valued function. One can verify whether this real valued function is going to be a random variable or not. So, if you go for finding x inverse of minus infinity to x, since the possible values are 0 1 and 2. Therefore, when x is less than 0 you do not have any possible outcomes. Therefore, it is a empty set. When x lies between 0 to 1 when x takes a value from 0 to 1 excluding 1, the x inverse of minus infinity to x becomes tail t when x takes the value from one to 2 excluding 2, it is going to be head tail, tail head, tail tail when x lies between 2 to infinity, all possible outcomes will be included therefore, this is going to be the omega.

Now, we have to verify whether these 4 elements is belonging to F or not. Since we have taken F as the largest sigma field therefore, empty set tail tail head tail tail head tail tail and the omega all are belonging to F. Since for all x, x inverse of minus infinity to x belonging to F, X is a random variable X is a random variable, because it satisfies the condition.

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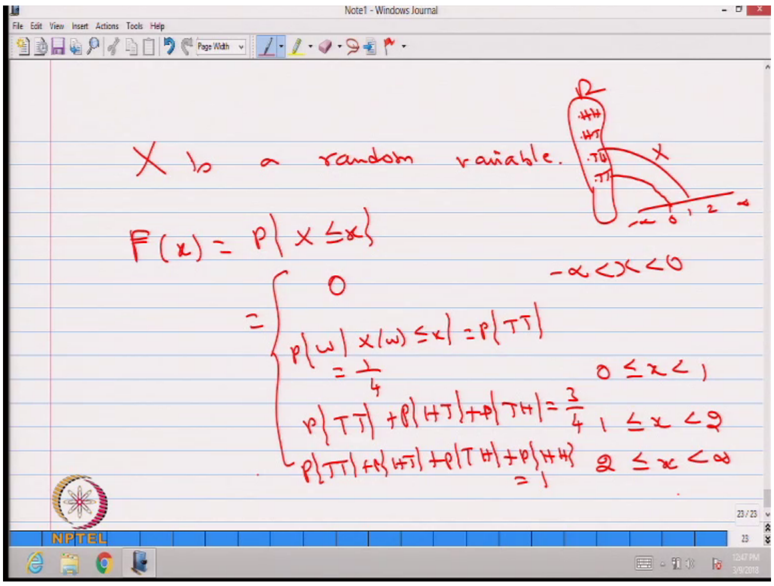

Now, we will go for finding the cumulative distribution function of the random variable x, that is defined as capital F of x, that is P of x is less than or equal to x. For all possible values of x F of x is defined p of x is less than or equal to x. Since, the possible values of x is going to be 0 1 and 2 therefore, we can define the F of x when it is before 0 between 0 to 1, 1 to 2 then 2 to infinity.

So, based on the problem you can always split the ranges. So, since here the x is the number of heads obtained and the possible values of heads are going to be 0 1 and 2 therefore, we are going to define F of x is a value, when x is less than 0 then 0 to 1 then 1 to 2, then 2 to infinity. So, this is minus infinity to 0. So, what is the value when x lies between minus infinity to 0, excluding 0 when x is lies between 0 to 1, then x lies between 1 to 2 then 2 to infinity. When x lies between minus infinity to 0 the P of x less than or equal to x is nothing but collection of possible outcomes, which is going to give the value from minus infinity to till x where x lies between 0 to infinity.

Since the possible values of x I can just draw one small diagram also. Omega consist of head head head tail tail head tail tail and this is mapped with 0 1 2 under the operation x, when P of x is less than or equal to x when x is lies between minus infinity to 0 it is nothing no possible outcomes therefore, p of empty set. So, the p of empty set is going to be 0.

When x lies between 0 to 1, since the tail tail is mapped with 0 collection of possible outcomes this is nothing but w such that x of w is less than or equal to x, here it is P of the w is here it is a tail tail and since it is the fair coin or unbiased coin, it is equiprobable of getting tail as well as head and the probability is 1. Therefore, it is 1 by 2 probability of heading head 1 by 2 probability of heading tail, and probability of getting head tail in both the tosses is going to be 1 by 2 into 1 by 2. Therefore, it is going to be 1 by 4. The probability of obtaining head in one toss is 1 by 2 and probability of getting another toss is 1 by 2. Therefore, it is a probability of a 1 by 4 getting head tail tail.

When x is lies between 1 to 2 that is nothing but collection of possible outcomes gives the values x of w is less than or equal to x, here it is going to be P of tail tail plus probability of a head tail plus probability of tail head. So, all these possibility gives the value of a probability of a x is less than or equal to x, when x is lies between 1 to 2. So, we know that the probability of tail tail is 1 by 4 and a head tail is another 1 by 4, tail head is another 1 by 4 therefore, the probability is going to be 3 by 4. When x lies between 2 to infinity this is going to be probability of tail tail plus probability of head tail plus probability of tail head plus probability of head head because that gives the value 2.

So, 1 by 4 plus 1 by 4 plus 1 by 4 plus 1 by 4 therefore, this is going to be 1. So, the CDF is 0 between minus infinity to 0, the CDF value is 1 by 4 between 0 to 1 and 3 by 4 between the interval 1 to 2, and 1 between the interval 2 to infinity.

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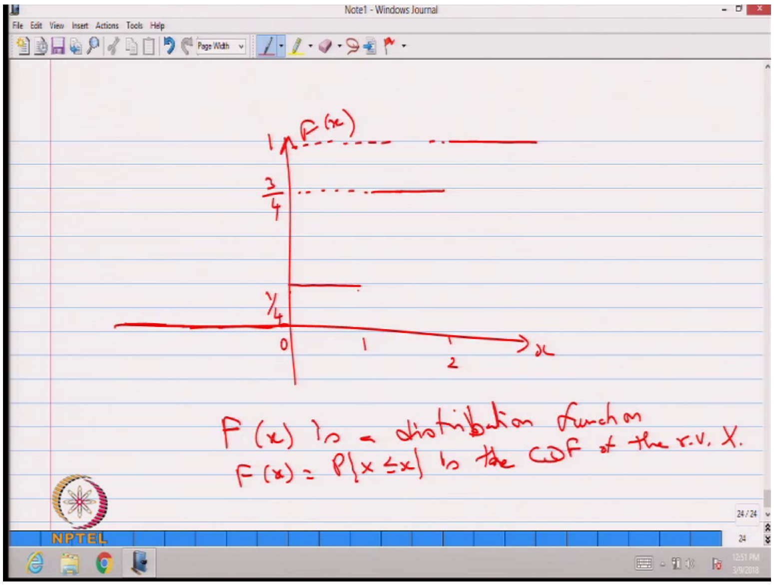

we can draw the diagram for this nicely when x takes the value minus infinity to 0 it is 0 0 there is a jump of height 1 by 4, I am not scaling the graph. At the 0.1 it has a next jump of height 3 by 4. So, suppose this is 1 by 4 another 1 by 4 another 1 by 4, so this much height at the point 2. So, this values is basically 3 by 4. At the point 2 it has a next jump. So, that jump values one; that means, it is right continuous function because the left limit at the value at 0 that is 0 value at the 0 is 1 by 4 right limit at the point 0 is 1 by 4 therefore, its a right continuous at the point 0.

Similarly, the value of CDF at the left limit of one that is 1 by 4, the value at the point one that is 3 by 4 and the right limit at the point 1 that is again 3 by 4 therefore, it is a right continuous at the point 1 also. Similarly the function value at the point 2 that is a right continuous therefore, this is the example in which the CDF is satisfying all the four conditions of the distribution function starting from values lies between 0 to 1 monotonically increasing and left limit is 0, right limit is 1 the fourth condition here it is the right continuous function.

Therefore, this function capital F that is the first it is distribution function the way we defined the distribution function through P of x is less than or equal to x. Therefore, this is a CDF is the CDF of the random variable capital X. It satisfies all the four conditions including it is a right continuous function. Therefore, this distribution function is the cumulative distribution function of the random variable x.

Similarly, we can create a examples for CDF, which is continuous function, similarly one can give the example for CDF which is continuous in some interval it has jumps in some other interval we can create. So, the one example which I have created which has only jumps; we can create a example of no jumps we can create a example of jumps as well as increasing in some interval, so this easiest example.

So, in the next class we will classify the random variables based on the CDF of the random variable.