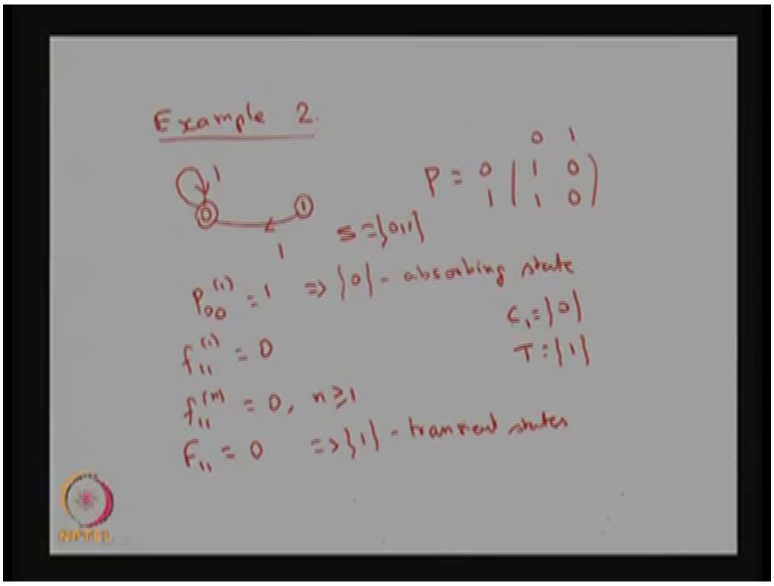
Introduction to Probability Theory and Stochastic Processes
Prof. S. Dharmaraja
Department of Mathematics
Indian Institute of Technology, Delhi

Lecture – 72

(Refer Slide Time: 00:02)



Example 2, here I am going to discuss a reducible Markov chain. Here also we have a only 2 states. The probability of system is moving from state 0 to 0 in the next step, it is the probability is 1. And the system is coming from the state 1 to 0 in one step that probability is 1. So, this is the state transition diagram of a time homogeneous a discrete time Markov chain. So, I am going to write what is the one step transition probability matrix for this state transition diagram or for this discrete time Markov chain.

So, 0 to 0 1 step that probability is 1, 0 to 1 is 0, 1 to 0 is 1 to 0 is 1, 1 to 1 is 0. You can verify whether this is going to be a stochastic matrix, because each elements are lies between 0 to 1, and the row sum is one therefore, this is a stochastic matrix. So, both are equivalent the state transition diagram and the one step transition probability matrix is one of the same.

Now, we will try to find out what is the classification of the states. Go for the state 0, the p 0 0 of 1 that is 1. That is one step transition of system is moving from the state 0 to 0 that is going to be 1. This implies the state 0 is a absorbing state.

Now, we will try to find out what is the classification of the state 1. So, if you find out f 1 1 of 1 what is the probability that the system will come to the state 1 given that it was in the state 1, and the first time we see to the state 1 exactly in the first step. So, that is going to be not possible because with the probability 1, it moved to the state 0 therefore, this is going to be 0. And if you find out f 1 1 of all the subsequent steps also, that is also going to be 0. Because if the system starts from the state 1 in the next step itself it goes to the state 0 with the probability 1 and it is not coming back.

Therefore, now you try to find out what is capital F 1 1. That is nothing but the summation of all the f i's summation of all the f i's and that is going to be 0. If you recall the way we classify the state is going to be a recurrent or transient we said f, ia is going to be one or f ia is going to be less than 1. So, that less than one includes f ia is equal to 0.

So, basic ally our interest is to classify whether with the proper distribution, the system is a coming back to the same state with the probability one that is f ia is equal to 1, and all other things we say that is a transient state, it includes f ia equal to 0. So, here with the probability 0 the system is not coming back to the state 1, if the system start from the state 1. This is always a conditional probability and this conditional probability f 1 1 is equal to 0 implies the state 1 is going to be a transient state. So, whenever any for any state i f ia is equal to 1 that concludes the state is going to be a recurrent state, and whenever the f ia is lies between including 0 excluding 1, that is less than 1 then that state is going to be call it as a transient state.

Since, the close since you have only 2 states that is a state space is 0 and 1 and you land up having a one absorbing state and one transient state. Therefore, the state space is a partition into one closed communicating class which has only one element and the transient state is 1. Therefore, I can say the state space s is a partition into closed communicating class c 1 which consists of only one element. And the collection of all the transient states that is only one element.

So, this is a notation for capital T collecting all the transient states in this state space in the DTMC, and the c 1 is the first closed communicating class, and which has only one element. If any closed communicating class has only one element, then it is going to be call it as absorbing state. Therefore, 0 is absorbing state and 1 is a transient state. Since, you have a c 1 union T becomes the state space s therefore, this Markov chain is not a reducible irreducible Markov chain. Therefore, this is called a reducible Markov chain.