Introduction to Probability Theory and Stochastic Processes
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Lecture – 68

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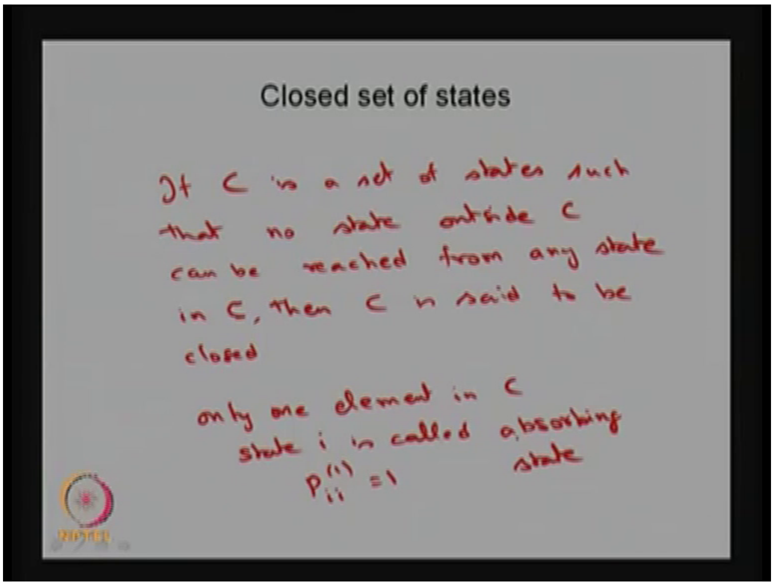

Now, i am going for the next concept called a closed set of states closed set of state. If C is a set of states such that, no state outside C can be reached from any state in capital C, then we say then this collection or the set C is said to be closed.

So, whenever you create a collection of states and that is it, we call it as a C if it satisfies this property, then we say that a set is called a closed set. So, we can combine the class property with the closed to set property. If both the properties are satisfied, the communicates with each other as well as the closed property satisfied then, we can say that a closed communicating class.

So, the any subset in the state space s if it satisfies each element within the set is communicating each other and satisfies this property, then we say that collection is going to be a closed communicating class.

There is a possibility in a set you can have a more than one elements, more than one states. In that collection, the class may have only one element or it may be more than one element if any closed to set or the closed communicating class has only one element; that means, you cannot include one more state and to make it as the closed or communicating class, then that closed set is called or that state is called only one element in capital C, then the state i is called absorbing states.

Here, state i is said to be absorbing state, then it is going to form a closed communicating class which has only one element in that class, there is a possibility more than one element is also possible in the closed communicating class.

So, we can define the observing state with the through the closed communicating class or we can make it in the same observing state. Using the definition, P ie or i in steps 1 that is going to be 1; that means, if you see the one step transition probability matrix, the diagonal element of that corresponding state, that corresponding row the element is going to be 1.

That means, the system starting from the state i and in one step the system moving to the same state i that probability is 1. If this probability is 1, then we say that state is going to be observing state. In the other way round, you can go for defining the absorbing state via close communicating class has only one element also.

So, there are 2 ways you can say the observing state. Using these concepts, I am going to develop the next concept called a irreducible Markov chain. We are discussing a time homogeneous discrete time Markov chain where is this concept called irreducible that is valid for the discrete time Markov chain as well as the continuous time Markov chain. So, that we are going to discuss later.

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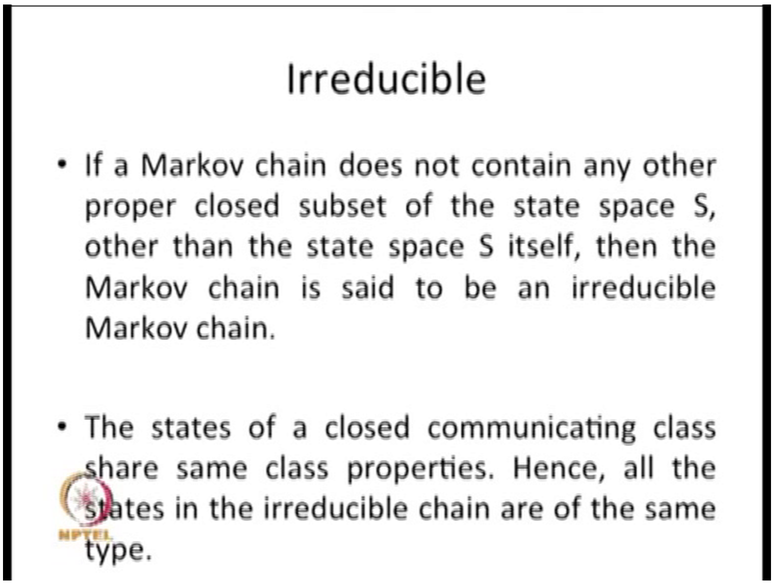

Now, I am defining the irreducibility for a time homogeneous discrete time Markov chain, if the Markov chain since the irreducible concept comes further discrete time Markov chain and the continuous time Markov chain, we use the word called a Markov chain that is valid for both.

If the Markov chain does not contain any other proper closed subset other than the state space S, then the Markov chain, in short we can use the word MC for Markov chain.

Then, the Markov chain is called irreducible Markov chain. Whenever the state space cannot be partition into more than one closed set, the proper set; that means, you can have only one closed set and that is same as the capital S all the elements in the state space is going to be form a only one closed set. In that case, that Markov chain is going to be call it as a irreducible. Irreducible means, you cannot partition, you cannot partition the state space.

If more than one closed proper closed subsets are possible from the state space, then that Markov chain is going to be call it as a reducible Markov chain. If more than one or we can able to make the partition of the state space into more than one closed set as well as a few transient states and so on that I am going to discuss later.

So, whenever you are not able, if you are able to partition the closed partition the state space, then that is going to be a reducible Markov chain. If you are not able to partition the state space and the whole state space is going to be of only one closed proper closed to subset then, that is that Markov chain is going to be call it as a irreducible Markov chain.

In this case, all the states belonging to the that class is going to be form one class and since it is going to have only one class, all the states going to have if one state has the period something, then all the other states also going to have the same period because, you are not able to partition so you have only one class. Therefore, if one state has the period some number some integer then that same period will be for the all other states also.

So, the Markov chain which are not irreducible are said to be reducible or non-irreducible Markov chain fine. Now, I am going to give the next concept called the first visit. We did not come to the classification of a state before that, we are developing a few concepts. Using these concepts, we are going to we are going to classify the states.

The next concept is called first visit. What is the meaning of first visit?

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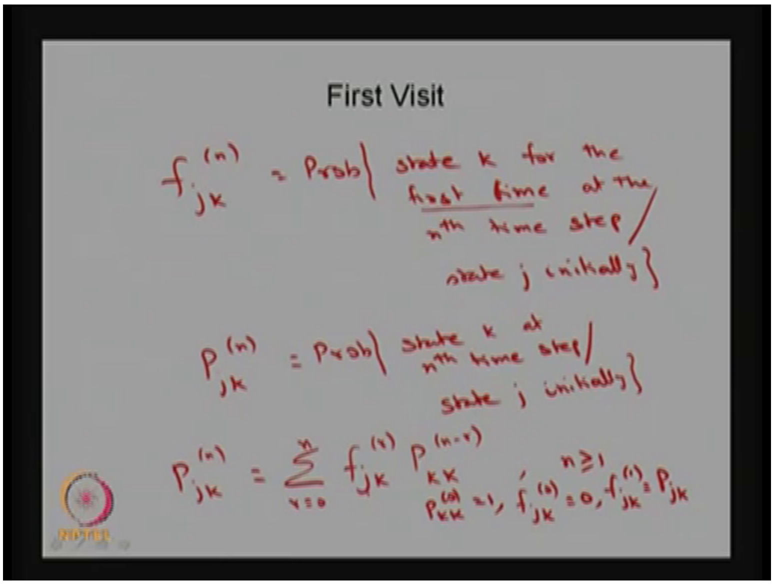

I am going to define the probability mass function as the f suffix j k with superscript n; that means, what is the probability that the system reaches the state k for the first time, that is important for the first time at the nth step, nth time step given that the system starts the state j initial.

This is a conditional probability mass function of a system moving from the state j to k and system reaching the state k are the nth time step, for the first time that is important.

So, this is the first time the system reaches the state k are the nth step exactly at the nth step. And this conditional probability mass function that I am going to write it as the f suffix j k of n. This is different from the j k of n. This is also conditional probability whereas, this probability is defined what is the probability that in the system reaches the state k at the nth time step given that, it was in the state j initially.

This is also conditional probability the only difference is the first time; that means, there is a possibility the system here the P suffix j k of n means there is a possibility the system would have come to the state k before nth step also. So, that probability is included.

Whereas the f suffix j comma k at the nth step means a this is the only the nth step it reaches the state k therefore, the way I have a given the first time conditional this probability and this is not necessarily the first time this is also conditional probability. I can relate the f suffix j comma k with the P suffix j comma k both are in the n step transition probability, but one is for the first time the other one is not necessarily.

I can relate both in the form of P j suffix k superscript n, that is, a n step that is same as f suffix j k of r steps and P suffix k k of n minus r steps and r can be vary from 0 to small n for n is greater than or equal to 1.

This means, if the system is moving from the state j to k in n step not necessarily the first time that can be written as the union of mutually exclusive events for different r, in which the system moves from the state j to k in r steps for the first time. And the remaining n minus r steps there is a possibility the system would move from the state k to k not necessarily the first time and the possible or can be 0 to small n and this n can vary from 1 to infinity.

Obviously we can make out the; I can give the P k k of 0 step that is going to be 1. And similarly, you can make out a f suffix j of k that is 0 steps also 0 and f j k of 1 step that is nothing but the P j k. The first time the system is moving from the state j to k in 1 step that is same as the 1 step transition probability the first time and once 1 step transition probability same.

Whereas, for n is greater than or equal to 1, then it is going to be the combination of the first time with the not necessarily the first time n minus r step transition probability that all possible events that will give the altogether final probability. So, here we have used the total probability rule as well as the Chapman Kolmogorov equation for the time homogeneous discrete time Markov chain to land up giving the relation between the P j k with the f j k.