Introduction to Probability Theory and Stochastic Processes
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Lecture – 65

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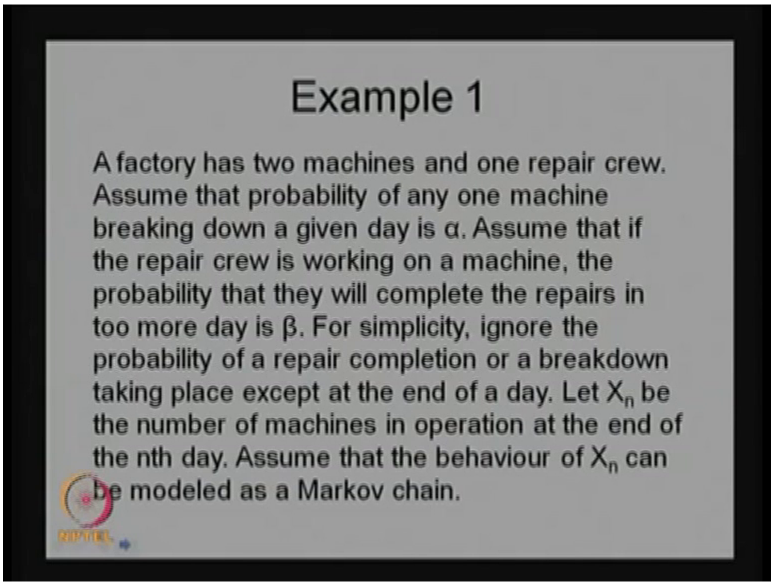

Now, we are moving into simple examples using the n step transition probability matrix and the one step transition probability vector how to find the distribution of Xn for some simple example. The first example which I have discussed in the lecture 1 , and this is a very simple example in which the underlying stochastic process is the time homogeneous discrete time Markov chain with the state space with the state space s is a 0 1 and 2.

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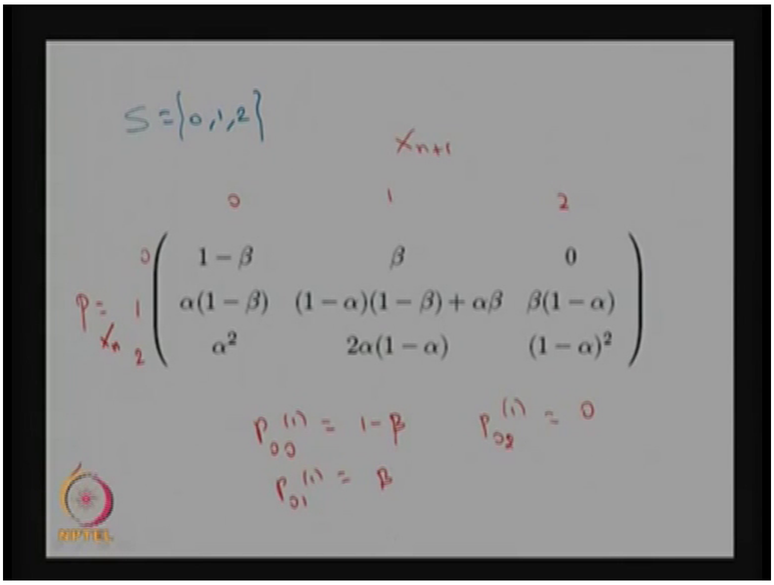

So, this is the state space, and the information which we have based on that we can make a one step transition probability matrix that is nothing but, what is the possible probability in which the system is moving from the state i to j in one step that you can fill it up.

So, this exercise we have done it in the lecture 1, and now our interest is to find out.

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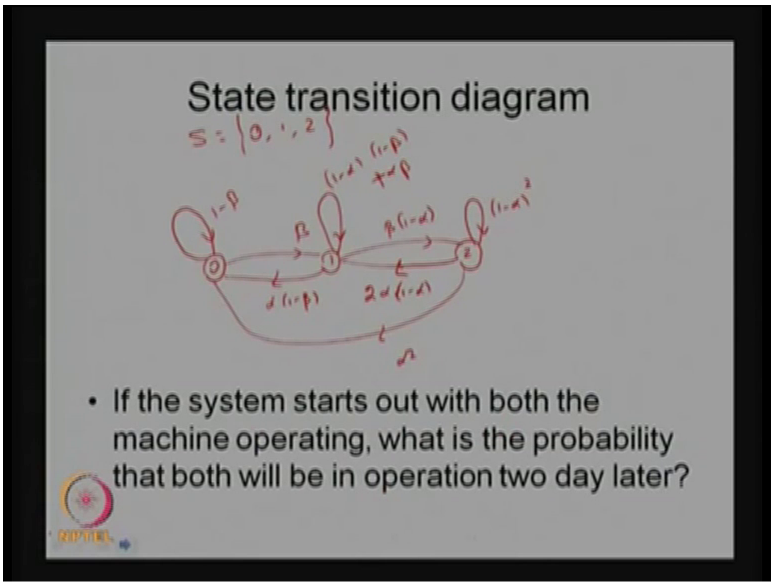

And also we have made the state transition diagram, which is equivalent to the one step transition probability matrix and we have got the state transition diagram

Now, the question is; if the system starts out with both the machines operating, what is the probability that both will be operation 2 days later. So, if you recall what is a random variable, X n be the number of machines in operation at the end of the nth day.

So, the random variable is a how many machines are in the operation at the end of nth day. So, here the clue is, at the time 0 or the 0th step both the machines are operating therefore, X 0 is equal to 2 with the probability 1. So, the given information with the probability 1, the both the machines are working at the zeroth step.

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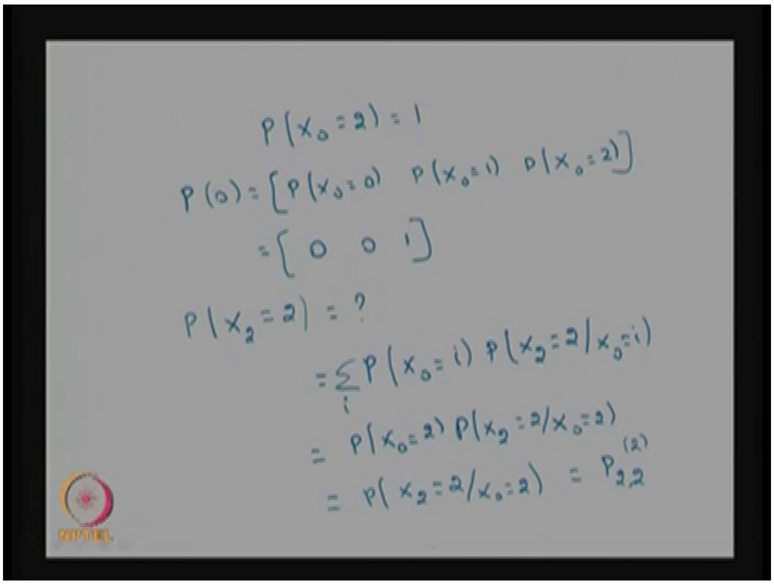

So, this can be converted into the P X 0 takes a value 2 that probability is 1 or we can make it in the initial probability distribution or initial probability vector as what is the probability that at X 0 the system was in the state 0 at the zeroth step the system was in the state 1. So, this is the initial probability vector

So, at time 0 the system was in the state 2 therefore, that probability is 1 and all other probabilities are 0. So, this is the given information about the initial probability vector. Now the question is; what is the probability that, both will be operation 2 days later ; that means, what is the probability that; we can convert this into, what is the probability that X 2 in the second step the system will be in the state 2, given that; the system was in the state 2 at the 0th step.

So, this is what you have to find out what is the conditional probability if the system starts with the both the machines what is the probability that both will be operation in 2 days later. So, not even this is a conditional probability the question is; what is the probability that, what is the probability that , the system will be in the state. So, to find this you can make what is the probability that; with the given information is there X 2 is equal to 2, given that X naught is equal to i for all possible values of i.

And this is same as since the initial probability vector is going to be 0 0 1. So, this is land up, what is the probability that the X naught is equal to 2 multiplied by what is the probability that X 2 is equal to 2 given that X naught is equal to 2 and all other probabilities are 0 that was 0 into anything is going to be 0 therefore, it is same as what is the probability that X 2 X naught is equal to 2 and this is the condition probably. And X naught is equal to 2 is 1 therefore, this is same as what is the probability that X 2 is equal to 2, given that X naught is equal to 2.

So, this is a same as what is the probability that 2 comma 2 in 2 steps. This is nothing but the system was in the state 2 at 0th step and the system will be in the state 2 after the 2 steps. So, this is a 2 step transition probability of system moving from the state 2 to 2.

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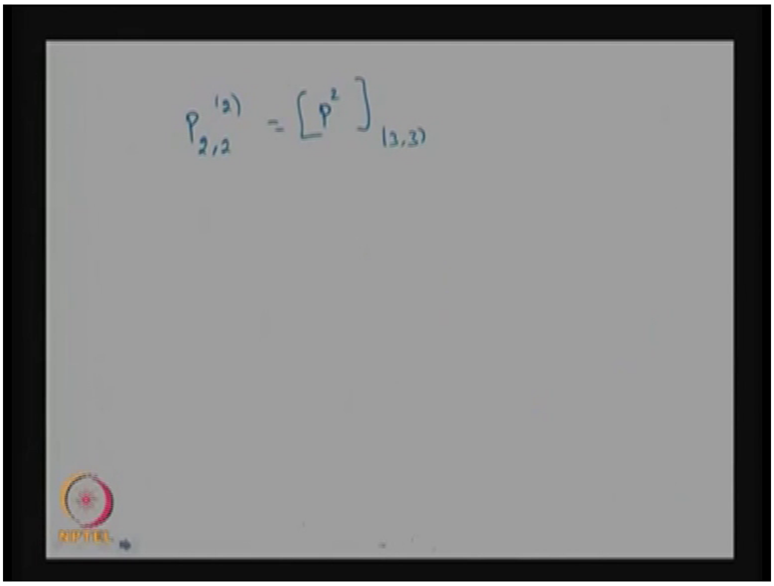

This is same as you find out, you find out the P square matrix and the from the P square matrix this is nothing but, the 2 comma 2 that is going to be the last element , out of that

9 elements the third row third column element that is going to be the row element for the this probability.

So, what do you have to find out is; a find out the P square find out the P square. So, we have provided the P. So, this is the P matrix so from the P matrix you find out the P square. So, the P square is also going to be a 3 cross 3 matrix. So, from the P square 3 cross 3 matrix you take the third row third element third column element and that is going to be the probability for 2 step transition of system moving from the state 2 to 2 that is going to be the answer for the given question, what is the probability that both will be operation in 2 days later. Similar to this we can find out the probability for any day, for any finite day by finding the P power n matrix, then pick the corresponding element and that is going to be the corresponding probability.

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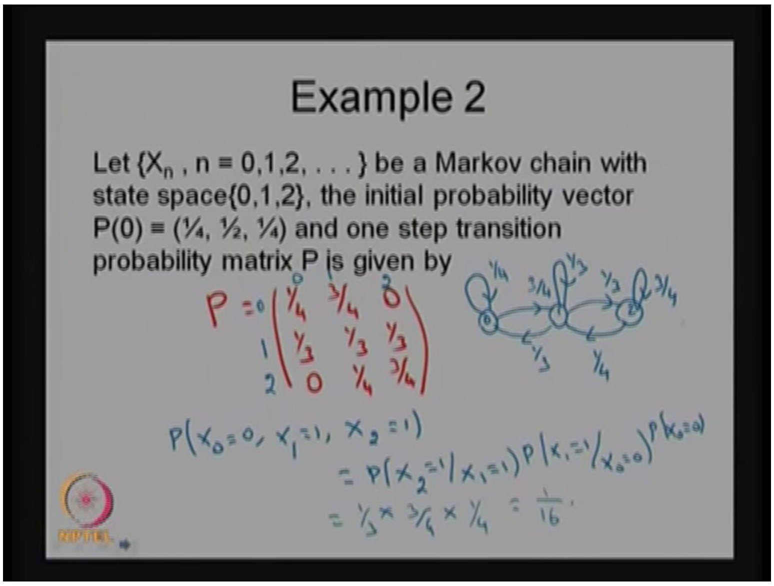

Now, we will move into the next example, this is abstract example in which the Xn be the discrete time Markov chain the default discrete time Markov chain is always it is a time homogeneous. So, this is a time homogeneous discrete time Markov chain with the state space 0 1 and 2.

And also it is provided the initial probability vector that is, a P 0 that is a vector that is 1 4th of one forth so the summation is going to be 1 therefore, this is the initial probability vector; that means, the system can start from nth 0th step with the probability one 4th from the state 0, from the state 1 with the probability of with the probability 1 4th it can start from the state 2.

And also it is provided the one step transition probability matrix from the one step transition probability matrix, you can you can draw the state transition diagram also, because the state space is 0 1 comma 2 therefore, the nodes are 0 1 and 2. And this is the one step transition probability therefore, 0 to 0 that probability in one step up one step, the system is moving from the state 0 to 0 that is 1 4th, and the system is moving from the state 0 to one in one step that is a 3 4th, and there is no probability from thus going from the state 0 to 2 therefore, you should not rather arc.

From 1 the one step transition probability of 1 to 1 is 1 3rd and this is 1 3rd and similarly this is 1 3rd. From the state 2 2 to 2 0 is 0 and the 2 to 1 is 1 4th, and 2 to 2 is a 3 4th. This diagram is very important to study the further properties of the states therefore; we are drawing the state transition diagram for the discrete time Markov chain. So, this is a one step transition probability matrix and this is the state transition diagram.

Our interest is to find out the few quantities that is, what is the probability that X naught is equal to 0, and X 1 is equal to 1 and X 2 is equal to 1. What is the probability that the system was it is a joint distribution of these 3 random variable, X naught is equal to 0 and X 1 is equal to one and X 2 is equal to 1.

So, this is same as the joint distribution, is same as you can write it in the product of the conditional distribution. And the conditional distribution again you can write it using the Markov property. The conditional probability of only one step therefore, this is going to be by using the probability theory you apply the joint distribution is same as the product of conditional distribution by using the Markov property, you reducing into the another conditional distribution.

So, this is same as what is the probability that X 2 is equal to 1, given that X 1 is equal to one multiplied by X 1 is equal to 1, given that X naught is equal to 0 and the probability of X naught is equal to 0.

So, this is the first term is nothing but the one step transition of system is moving from 1 to 1, and this is nothing but the system is moving from the state is 0 to 1 and this is the initial you take the probability from the initial probability vector of X naught is equal to 0.

Yeah now we are going to label the one step transition probability matrix with a 0 1 2 and 0 1 2, from this we can find out this is a one step transition probability of system moving from 1 to 1. So, 1 to 1 is 1 3rd into this is a system probability of system moving from 0 to 1. So, 0 to 1 is a 3 4th, and a system started from the state 0 the 0th step. So, that we can take it from the first element that is 1 4th

So, if you do the simplification you will get a 1 by 16. So, this is the joint distribution of the system was in the state 0 at 0th step. The system was in the state 1 at the first step and the system was in the state 1; at the second step that probabilities 1 by 6 16.

Similarly, you can find out the other probabilities also that is, a suppose our interest is what is the probability that at the end of a second step the system will be in the state 1.

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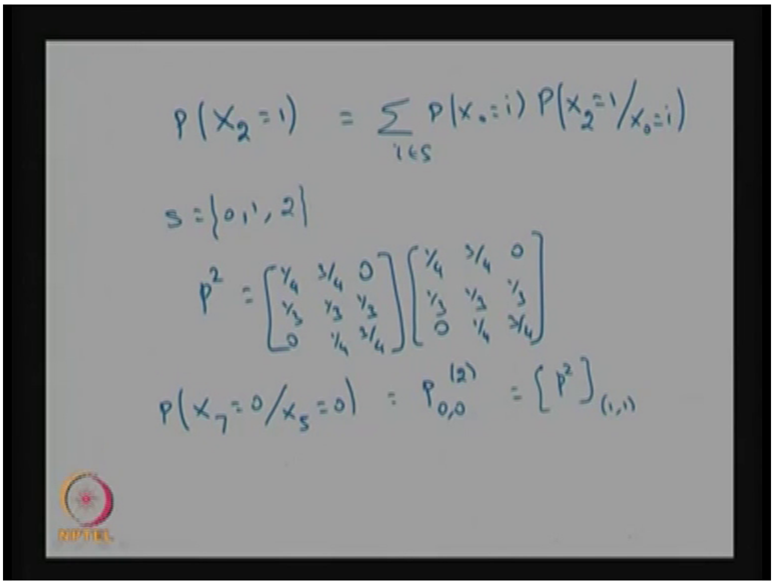

That is nothing but, what are all the possible states in which the system would have been started from the state i and what is the 2 step transition of system is moving from the state i to 1. So, the i is belonging to S so here the S is a 0 1 comma 2. So, already we have given the initial probability vector that is 1 4th of and 1 4th, using this and you need a 2 step transition probability; that means, you need to find out what is the P square. So, the P square will give the 2 step transition probability matrix therefore, the P is provided to you so the P is 1 4th 3 4th and 0 13rd 1 3rd 1 3rd 0 1 4th 3 4th.

So, this is the P so you multiply the same thing 1 3rd 1 3rd 1 3rd 0 1 4th 3 4th, you find out the P square. So, from the P square you pick out the element of X 0 is equal to for all possible i, then multiply this and this that multiplication will give probability of X 2 is equal to 1. So, I am not doing the simplification so once you know the P square you can find out probability of X 2 is equal to 1.

Similarly, one can compute the other conditional probabilities also. Suppose our interest find out what is the probability of X 7 is equal to 0 given that; X 5 was 0. This is same as what is the probability that, the system was in the state 0 with the 5th step given that what is the probability that the system will be in the state 0 in the 7th step.

That is same as what is the probability of 2, what is the probability of sorry, what is the probability of 0 comma 0 in 2 steps; that means, you find out the P square, from the P square the 0 comma 0 is the nothing but you take the first row first column element that is going to be the probability of a probability of X 7 is equal to 0 given that X 5 is equal to 0.

Similarly, you can find out all other different conditional probability and what do you have to do is; always you have to convert because of the given DTMCs a time homogeneous. So, you converted into find out the n step transition probability, and the n step transition probability is same as the P power n. So, you pick the corresponding element to find out the conditional probability.