Introduction to Probability Theory and Stochastic Processes

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Lecture – 63

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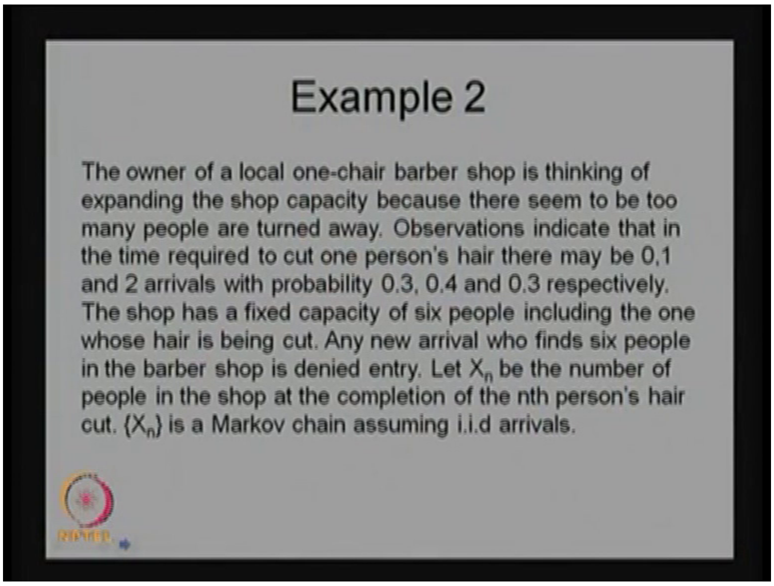

Now, I am moving into the second example. In this example, I have taken the barbershop example which I have discussed in the model 1 also the same example. So, the owner of a local one chair barber shop is thinking of expanding the shop capacity. Because there seems to be too many people are turned away. Observation indicates that in the time required to cut a 1 person's hair there may be 0 1 and 2 arrivals with the probability 0.3, 0.4 and 0.3 respectively.

So, this information is very important; that means, during one persons a hair cut, what is the probability that no people turned up with the probability 0.3 and a 1 people may turned up with the probability 0.4. And there is a possibility 2 arrivals is possible during the one person's hair cut with the probability 0.3; therefore, the summation of probability is going to be 1. So, during the one person's hair cut these are all the only 3 possibilities are possible with the 0 arrival or 1 arrival or 2 arrival.

The shop has a fixed capacity of a 6 people including the one whose hair is being cut; that means, maximum 6 people can be allowed in the system. So, 5 people can wait maximum and a 1 people under the service any new arrival who finds 6 people in the barber shop is denied entry that is the meaning of a capacity of the system is finite with the size 6. Now I am going to define the random variable let, X n be the number of people in the shop at the completion of the n th person's hair cut.

This is very different random variable or this is a very different stochastic process. Usually the parameter space is a time, but here the parameter space is the number of people in the shop. The n is the parameter space is the person who leaves after the hair cut. So, it is a n th person who leaves the system that becomes the parameter space. Whereas, the random variable is how many people in the system when the n th person leave the system; that means, you should not count that person when you are finding the values of X n; that means, this number is countered at the departure time point.

So, when the n th person leaves, how many people in the system? And the system is a maximum 6 people allowed therefore, he cannot see more than 5 people in the system when he leaves. So, because of this constraint because of a during the one person's arrival either 0 or 1 or 2 arrivals can takes place and so on. Based on this information the stochastic process X n is going to be a discrete time, discrete state stochastic process as well as the Markov property satisfied.

That means the probability of X n plus 1 takes some value given that all the previous values are known, that is same as the conditional probability distribution of X and plus 1 takes some value given that X n was some value. So, all so, the future distribution given that a present as well as the past information is same as the future distribution given the present not the whole past information. So, these Markov properties will be satisfied by this stochastic process therefore, this X n will form a discrete time Markov chain.

Obviously it is the time homogeneous discrete time Markov chain also. So, in this example our interest is to find out what is the one step transition probability matrix.

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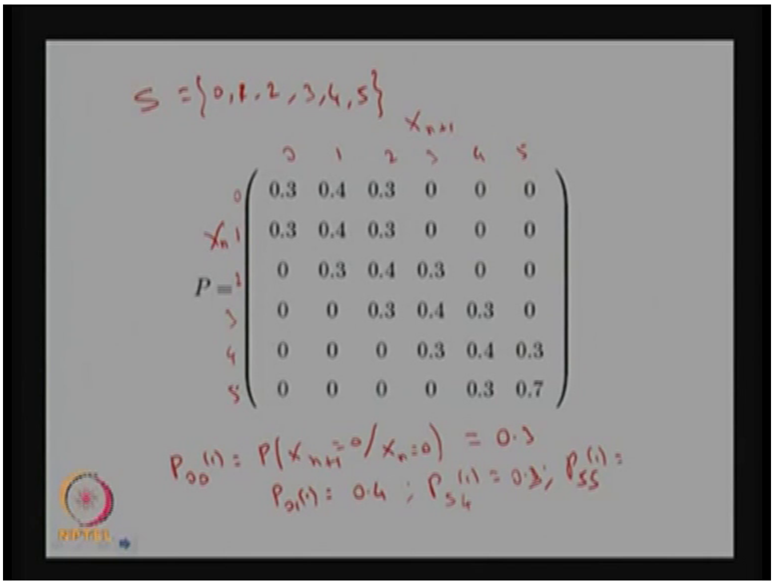

This is going to be the one step transition probability matrix, and the possible states S S is going to be 0, 1, 2, 3, 4 or 5. Because the capacity of the system is 6 and whenever he whenever the n th person leaves either first person second person third person leaves, how many people are in the system. Therefore, the maximum will be 5 and there is a possibility when he leaves a no one in the in the system also.

And this is the one step transition probability matrix, and this is a also going to be a square matrix, because it is going to be a countably finite number of elements and this is a 0, 1, 2, 3, 4, 5 now we can discuss what is the probability that 0 comma 0 in one step. That is nothing but in the n th when the n th person leaves no one in the system, when the n plus 1 th person leaves no one in the system. What is the probability for that? That is a X n plus 1 is equal to 0 given that X n was 0. It is a 1 step transition probability it is a independent of n, because it is a time homogeneous it is a 1 step transition probability matrix.

So, this is possible at some person leaves whatever; be the n nobody in the system. When the next person leaves nobody in the system so, that is possible by when some person leaves the system was empty for some time, you do not know how much time it was empty, then the n plus 1 th person enter into the system. And a during his hair cut no one turned up or no arrival takes place during his or a n plus 1 th hair cut is going on therefore, when he leaves no one in the system.

So, we are not bothering when he enter into the system and so on, our interest is how many numbers of people in the system when the n plus 1 th person leaves and this probability is a n plus 1 th person leaves a 0 people in the system. And given that when the n th n th person leaves also 0 person in the system. So, that is possible with the explanation I have given no one enter into the system during the n th n plus 1 th persons hair cut.

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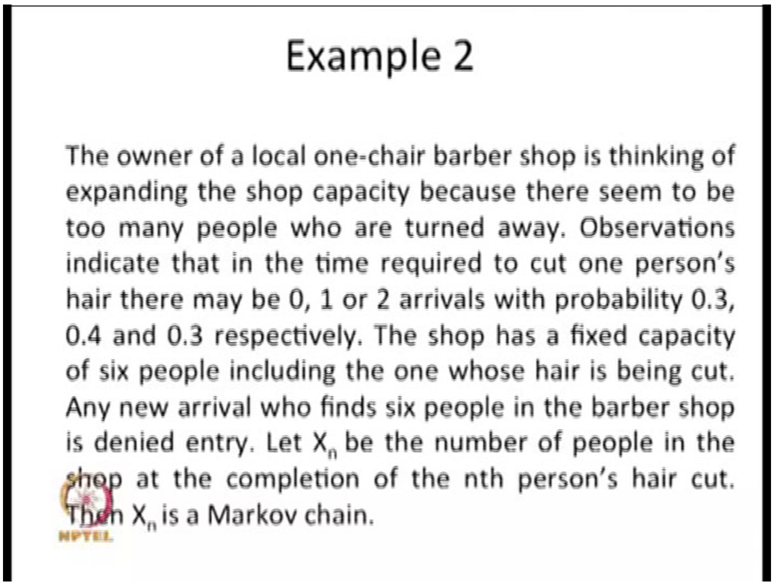

And the information is provided indicate that that time required to the hair cut one-person hair cut there may be a 0 1 or 2 arrivals with the probability 0.3. So, no arrival takes place during the one person's hair cut is a 0.3. Therefore, this probability is possible with the probability 0.3. Whereas, a P 0 1 of one step in the same way we can write probability that X n plus 1 is equal to 1, given that X n is equal to 0. That is possible when the n th person leaves no one in the system, and the n plus 1 th person leaves, one person in the system; that means, during his hair cut one person enter into the system that is possible with the probability 0.4.

Similarly, from 0 to 2 in one step that is going to be 0.3 with the probability 2 arrival takes place during the n plus 1 th persons hair cut. Now the second row; second row what is the probability that when n th person leaves one person in the system? When n n plus 1 th person leaves 0 person in the system. That is possible during the n plus 1 th person hair cut there is no one in the system, no, no arrival takes place therefore, that probability is 0.3.

And from one to one that is possible with a 1 person arrived during the n plus 1 th person hair cut therefore, that probability is 0.4. And going from the state 1 to 2 that is possible 2 person's arrived at during the n plus 1 th person hair cut. Whereas, from 2 to 0, that is not possible because when n th person leaves the 2 person in the system therefore, n n plus 1 th person in the leaves definitely he will see one person in the system. Because of no arrival and one arrival and 2 arrival therefore, it will be shifted by one column, and it will be keep continuing till the end. Whereas, the last one what is the probability that the 5 people in the system when the n th person leave? And 4 people in the system when the n plus 1 th person leave, that is same as no arrival takes place during the n plus 1 th arrival , n plus 1 th hair cut moving on.

So, therefore this is going to be 0.3 whereas, a P 5 to 5 in one step that is possible with the combination of one person arrive the system, or 2 person arrive the system this system size is going to be maximum 6. Therefore, when n plus 1 th person hair cut is going on, if one person arrives then he will be entered if 2 person arrives then he cannot be accommodated. Therefore, he will he would not join the system therefore, the system the number of customers in the system in the n X n that is going to take the value 5.

And the combination of a 0.4 as well as 0.5 therefore, this probability of system is moving from 5 to 5 is 0.7 because of 0.4 plus 0.3. Now I can give the state transition diagram for this example.

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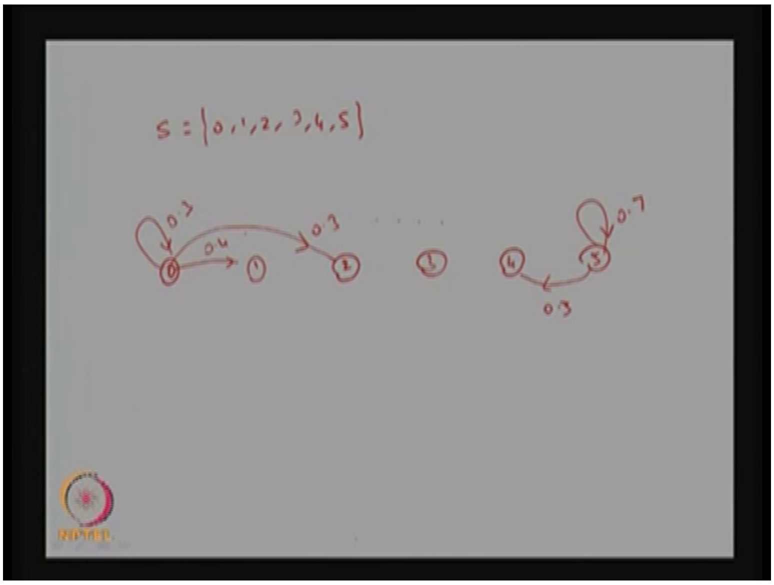

Because S is going to be 0, 1, 2, 3, 4, 5 therefore, the nodes are going to be 0, 1, 2, 3, 4 and 5. And the possible values from the state trans from the one step transition probability matrix I can make out. So, 0 to 0 that probability is 0.3 and 0 to 1 is 0.4, and 0 to 2 is 0.3. Similarly, I can fill up the all other things, and a 5 to 5 that is very important. And 5 to 4 that is possible with the probability 0.3, and 5 to 5 is possible with the probability 0.7.

So, this is the state transition diagram. I didn't complete the state transition diagram you have to fill up all the aux with the weights going from 1 or 2 other arc with the positive probability. Wherever there is a probability 0 you should not rather arc for it. So, in this lecture we have discussed the discrete time Markov chain.

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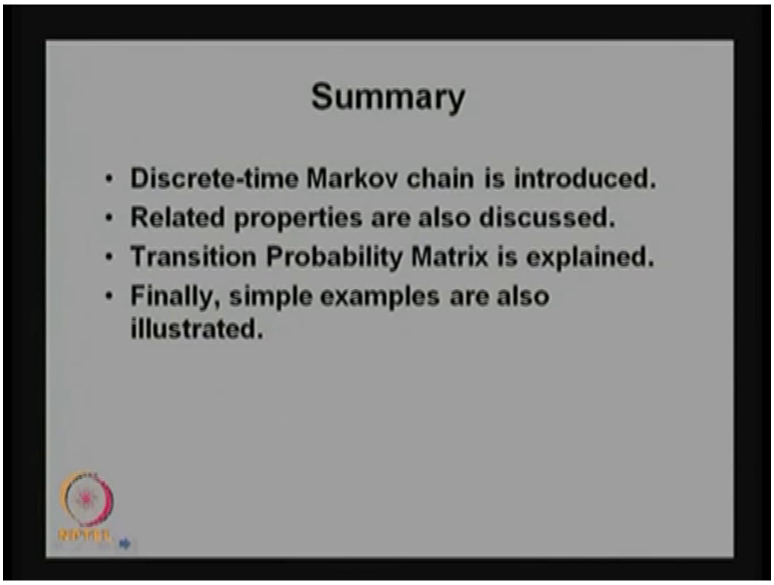

Then we have given the few important properties, followed by we have explained the one step transition probability matrix, and also we have given 2 simple examples. With this the lecture one is over for the module 4.

Thanks.