Introduction to Probability Theory and Stochastic Processes
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Lecture – 62

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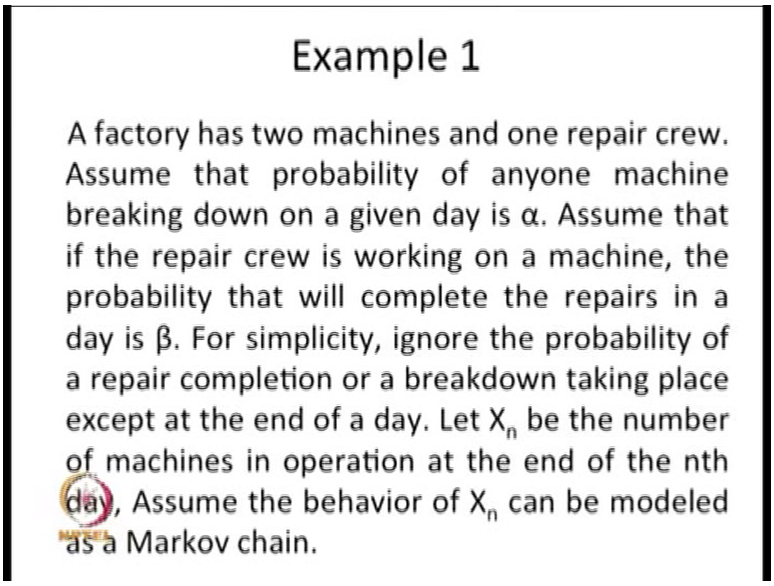

I am going to explain the discrete time Markov chain with the 3 simple examples, the first example is as follows. A factory has a 2 machines and one repair crew, assume that probability of any one machine breaking down a given day is alpha. So, the alpha is the probability. Assume that if the repair crew is working on a machine the probability that they will complete the repairs in 2 more day is beta. For simplicity, ignore the probability of a repair completion or a breakdown taking place except at the end of a day; that means, we observe the system at the end of the day how many working machines in the system.

Let Xn be the number of machines in operation at the end of the nth day. Assume that the behavior of Xn can be modeled as a Markov chain. So, based on the information available here the machine can be break down and we have only one repair person, and the probability of a he can do the repair in a day that probability is beta. And 1 minus beta is the probability that he cannot be able to complete the repair of a machine in a day. And the random variable Xn is a it is denotes; how many machines are in the operation at the end of the day.

Therefore, the possible values of Xn since we have a 2 machines the possible values of Xn will be 0 1 or 2. So, this will form a state space capital S. So, the S consists of the element 0 1 and 2 and the Xn over the n it is going to form a discrete time Markov chain because it is a discrete time a discrete state stochastic process and also the based on the clue the number of machines are working in any day depends on how many machines are working on the previous day, and how many things are under repair and so on.

So, the dynamics of the number of machines in operation depends only on the number of machines working in the previous day, not all the previous earlier days therefore, the memory less property is satisfied by the stochastic process Xn therefore, this is called a discrete time Markov chain. Our interest is to find; what is the one step transition probability matrix with the assumption that the Xn is the time homogeneous also since it is a time homogeneous or DTMC.

Therefore, we are trying to find out what is the one step transition probability matrix for the given time homogeneous DTMCs.

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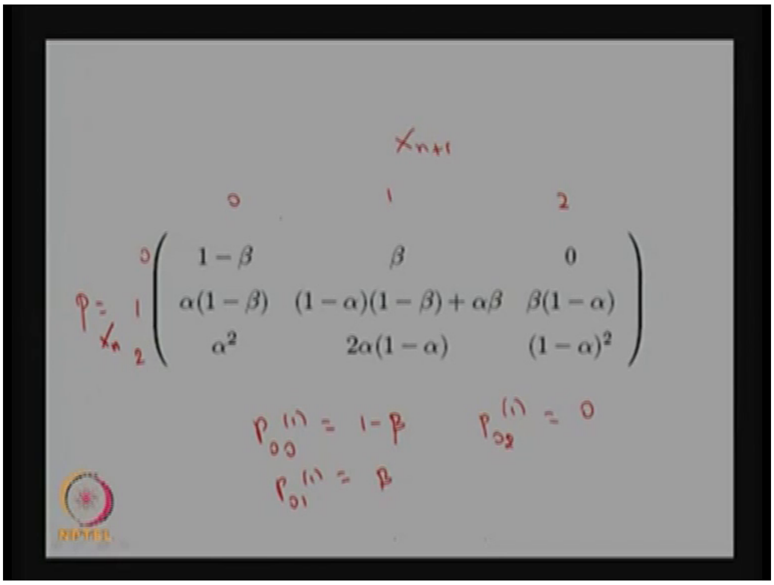

So, this is the one step transition probability matrix Pn and the possible states are 0 1 and 2. And suppose the system was in the state 0 1 or 2 in the nth step, where the system will be in the n plus 1th step. Therefore, this is the possible values of Xn plus 1 and this is the possible values of Xn, and this one step transition probability matrix will give suppose the system was in the state in the nth step, what is the probability that it will be in these states in the n plus 1th step.

So, the first index will give what is the probability that 0 comma 0 in one step; that means, in the nth step, number of a working machines are 0 and what is the probability that in the n plus 1th step also 0 machines are in the working condition; that means, all are under repair all 2 machines are under repair and the probability of a crew is going to be not repair that is going to be 1 minus beta. Therefore, the probability is 1 minus beta.

In one step what is the probability that; the number of working machines going from 0 to 1 that is because of the crew is finishing the repair in a day and that probability is beta. And since he can do the only one repair in a day therefore, the possibility of a repairing a more than one machine, in a day it is not possible it is a rare event and the probability is going to be 0, therefore P 0 2 is going to be 0.

Similarly, now we can visualize; what is the probability that; number of working machines is 1 in the nth step and what is the probability that in the 0 machines will be working in the n plus 1th step. That is possible with the 2 independent things the one machine can be failed and the other machine cannot be finishing the repair. Therefore, the crew is not finishing the repair that probability is 1 minus beta multiplied by one machine is going to be repair therefore, the total number of machines working will be 0 in the n plus 1th step that is alpha times 1 minus beta.

And similarly you can evaluate the other element also and for example, the system is going from the state 2 to 0, that is nothing but at nth step 2 machines are in the working condition. And the n plus 1th step 0 machines are the working condition; that means, both the machines got a repair got failed in the same day therefore, that probability is a alpha times alpha that is a probability both the machines got failed in the same day.

Therefore, in the next day the number of working machine is going to be from 0 2 to 0; like that you can visualize the other elements also. The same one step transition probability matrix can be visualized with the state transition diagram.

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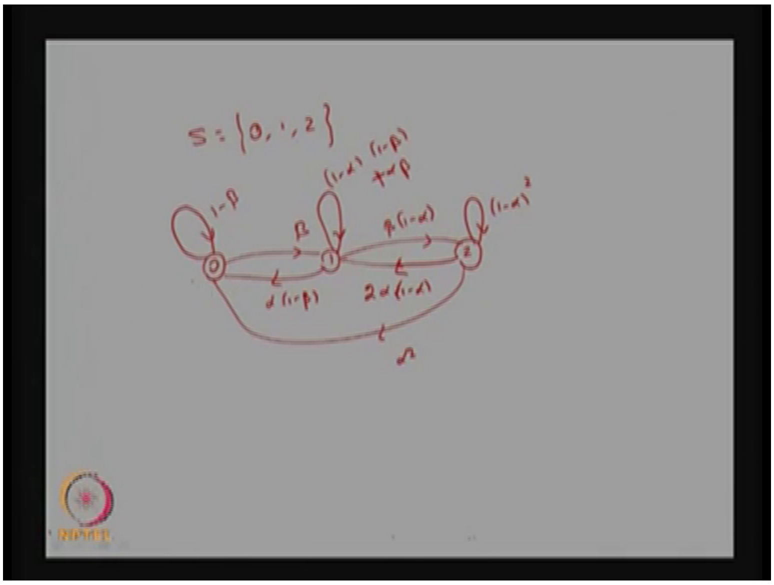

And state transition diagram you have to make with this state space as a vertices or the nodes. And the weights of the director dogs are nothing but the one step transition probability of system is moving from one state to other states; those are going to be the weights. If the probabilities are zeroes then no need to draw the directed arc from that particular node to the destination node.

So, first you start with the nodes as the possible values of the state space. So, you list out all the state space as a node, now by seeing the one step transition probability matrix you should make the arc from 0 to 0, self-loop is allowed if the probability is going to be greater than 0. So, you should draw the self-loop from 0 to 0 with the arc value 1 minus beta, and you should draw the arc from 0 to 1 with the arc weight beta. And you should not draw any arc from 0 to 2 because that probability is going to be 0.

Therefore, 0 to 0 that probability is 1 minus beta and 0 to 1 it is going to be beta and there is no arc from 0 to 2 because that probability is 0. And similarly now we can go for filling the second row. So, 1 to 0 is alpha times 1 minus beta, 1 to 1 is 1 minus alpha times 1 minus beta plus alpha beta and 1 to 2 so you have all 3 probabilities are greater than 0.

Therefore 1 to 0 that arc is the alpha times 1 minus beta, and 1 to 1 is 1 minus alpha times 1 minus beta plus alpha beta. 1 to 2 is beta times 1 minus alpha. Similarly, you can draw the arc for the 2 to 2 to 0 that is alpha square and 2 to 1 and so on. Therefore, 2 to 0 that is alpha square and 2 to 1 2 to 1 is 2 alpha times 2 2 alpha times 1 minus alpha and 2 to 2 that is; 1 minus alpha whole square.

So, the state transition diagram is a pictorial view of a one step transition probability matrix, this is nothing to do with the initial probability distribution it gives only information about whenever the DTMC is a time homogeneous. Suppose the system start from one particular state, what is the probability that; the system will move into the another states with the probability. And it would not give it would not give more than that information, but this much information is useful then you are going to study the properties of the discrete time Markov chain as well as.

When you are want to find out the limiting distribution that is the distribution of Xn as n tends to infinity the diagram will be very useful to conclude, whether the limiting distribution exists or not; if it exists whether it is going to be unique or not and so on. So, those things can be visualized easily by seeing the state transition diagram.