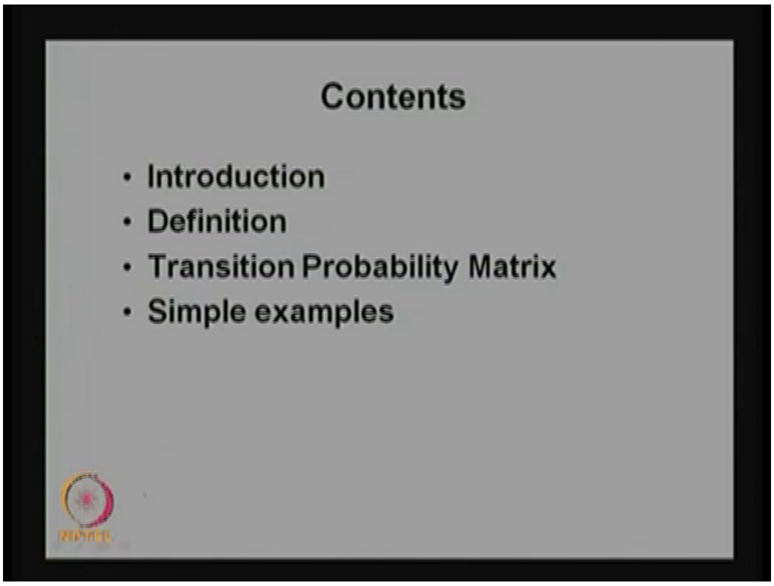
**Introduction to Probability Theory and Stochastic Processes**
**Prof. S. Dharmaraja**
**Department of Mathematics**
**Indian Institute of Technology, Delhi**

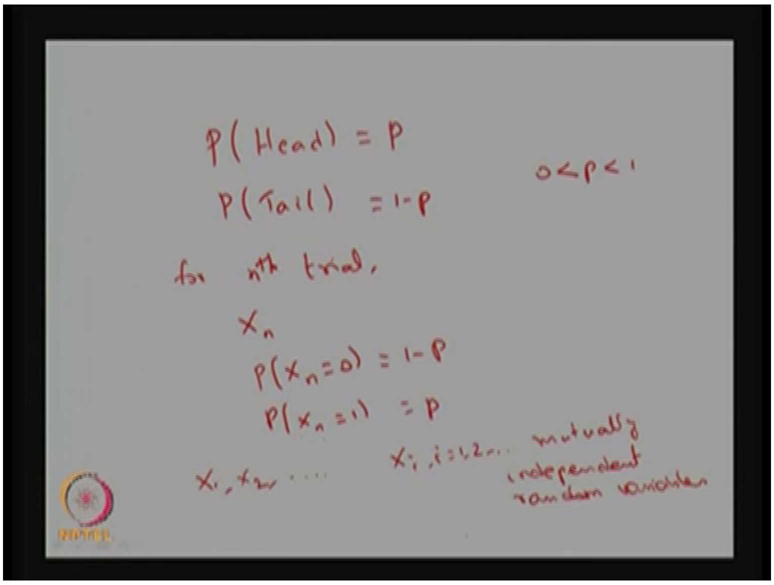
**Module – 10**
**Discrete-time Markov Chains (DTMCs)**
**Lecture – 60**

(Refer Slide Time: 00:19)




This stochastic process, in this we are going to discuss a Discrete time Markov Chain. And this is a lecture 1, in this lecture I am going to discuss the introduction how about the discrete time Markov chain then followed by the definition, and the important one concept called the one step transition probability matrix and few simple examples also. Consider a random experiment of a tossing a coin infinitely many times. Each trial there are 2 possible outcomes namely head or tail.

(Refer Slide Time: 01:03)

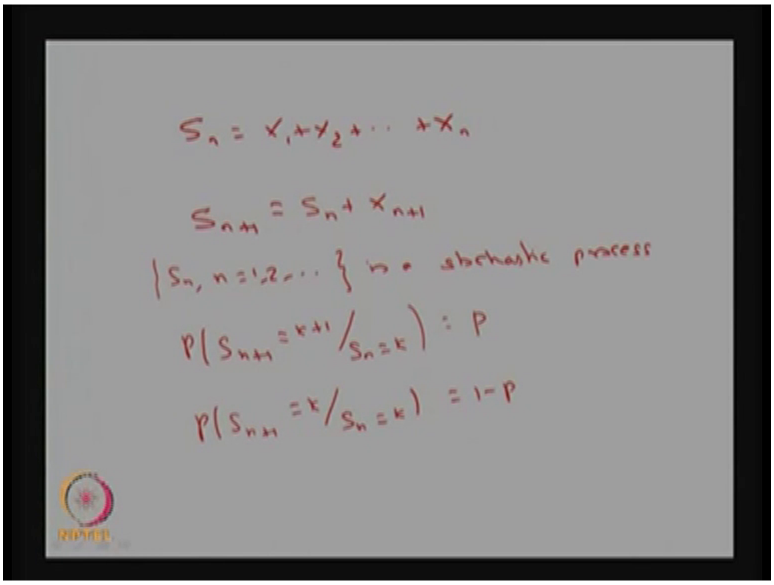



Assume that the probability of head that probability you assume that that is p, and the probability of tail occurring in each trial, that you assume it as 1 minus p. You assume that the P is lies between 0 to 1. Denote for the n th trial because you are tossing a coin infinitely many times, for the n th trial you denote the random variable X n is the random variable whose values are 0 or 1 with the probability, the probability of X n takes the value 0. That is same as in the n th trial you are getting the tail, that probability is 1 minus P.

And the probability of X n takes a value 1 that probability is make it as P for the head of P's. And you already you assume that the probability is lies between 0 to 1. Thus, you have a sequence of random variable x 1 x 2 and so on, and this will form a stochastic process. And assume that all the x i's are all the x i's are mutually independent random variable. So, this is a random experiment in which you are tossing a coin infinitely many times.

And for any n th trial you define the random variable X n with the probability it takes a value 0, with the probability 1 minus p and it takes a value 1 with the probability P and that is equivalent of appearing a head with the probability P and occurring the tail with the probability 1 minus p. Now I am going to define yeah another random variable that is a partial sum of first n random variables n n x i's.

(Refer Slide Time: 03:23)



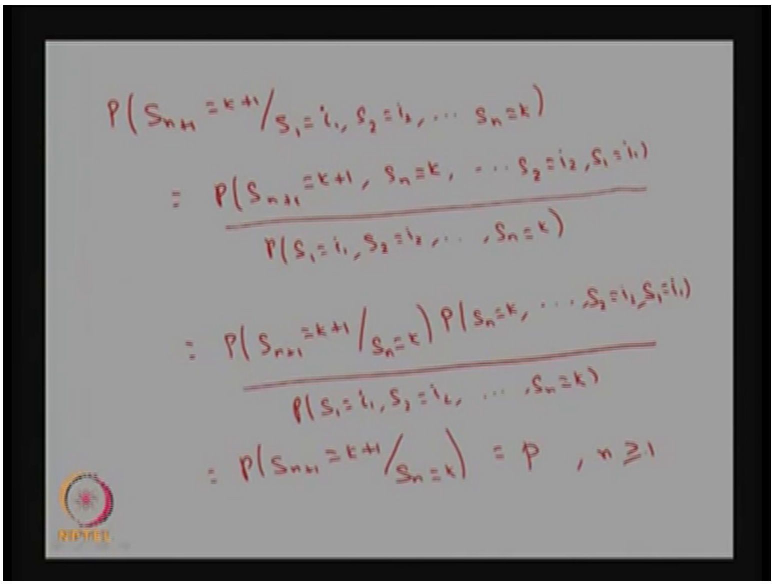

So, the S n will be sum of a first n random variables therefore, the sum S n gives the number of heads appear in the first n trials. It can be observed that S n plus 1 is same as S n plus x n plus 1, since S n is the partial sum of a first n trials outcome. So, the S n plus 1 is nothing but S n plus X n plus 1. You can also observe that since S n is the sum of a first n random variables, and S n plus 1 is S n plus X n plus 1.

And also all the X i's are mutually independent random variables, S n is independent with X n plus 1; that means, here the S n plus 1 th random variable is the combination of a 2 independent random variables whereas, the S n is the till n th trial how many heads you appeared, plus whether it is a head or tail accordingly this values is going to be 0 or 1. Therefore, if you see the sample path of S n plus 1 it will be incremented by 1 if x n plus 1 takes a value 1 or it would have been the same value earlier if this S X n plus 1 takes a value 0.

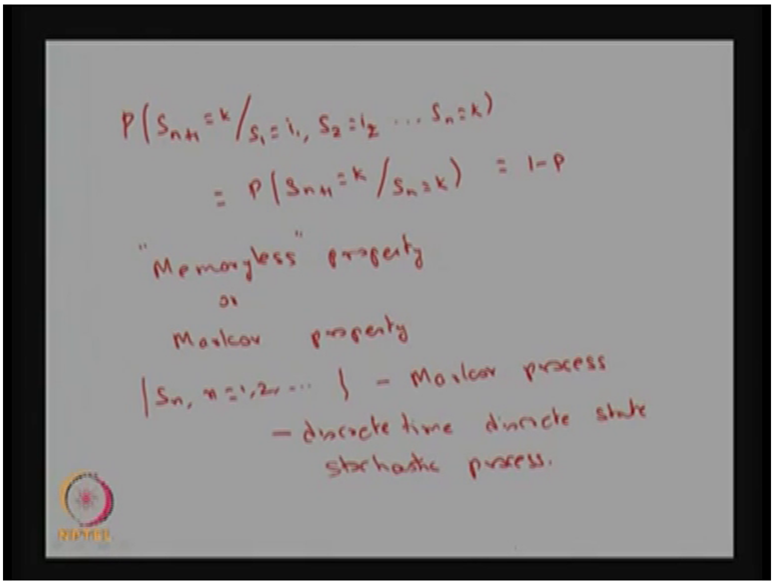
And also you can observe that S n plus 1 is a depends on S n and only on it, it is not a depends on S n minus 1 or S n minus 2 and so on, because it is accumulated the number of trials values over the n, therefore, S n plus 1 is a depends on S n and a only on it. The S n for different values of n this will form a stochastic process. This will from a stochastic process, and now we can come to the conclusion the probability of this is a stochastic process the probability of S n plus 1.

Suppose this values is k plus 1, given that S n was k; that means, the S n plus 1 valued been 1. Therefore, the appearance of the head appears in the n plus 1 th trial, and that probability is going to be P. Similarly, you can make out suppose S n plus 1 value will be k such that S n is also k, then that is possible with n plus 1 th trial you got the tail; therefore, that probability is 1 minus P. This is satisfied for all n so, you can make out this is satisfied for n is great nor equal to, even I can go for n is great nor equal to 1.

(Refer Slide Time: 06:45)




(Refer Slide Time: 06:55)

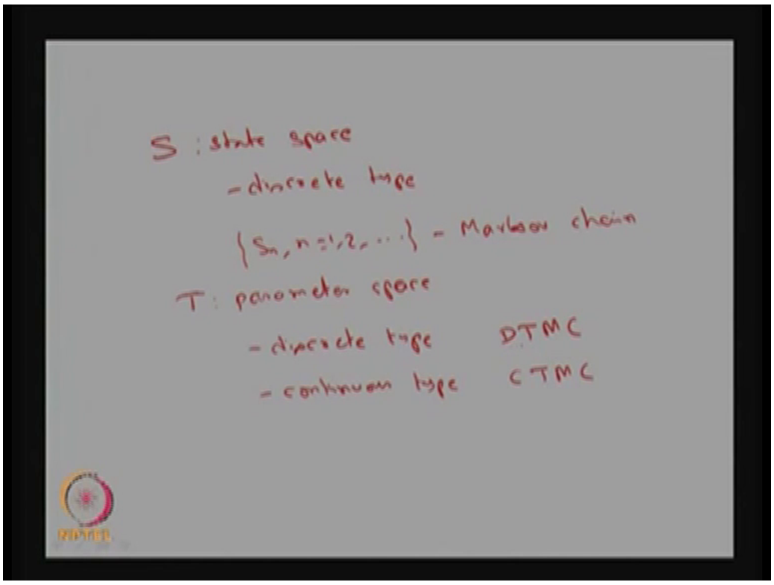


Not only these similarly I, can come to the conclusion, the probability of S n plus 1 is equal to k given that S 1 was i 1, S 2 was I 2 and so on, S n was k that is also can be proved with the probability of S n plus 1 is equal to k given that S n is equal to k. That is same as what is the probability that the value was same k in the subsequent trials. That is possible of appearing a tail in the n plus 1 th trial. Therefore, the appearance of the tail in the n plus 1 th trial the probability is 1 minus P or I can use the notation q.

That means, the probability of n plus 1 th trial that distribution, given that I know the value till the n th trial that is same as the distribution of n plus 1 th trial given with the only the n th distribution, not the earlier distributions. And this property is called a memory less property. The stochastic process the S n satisfies the memory less property or the other word called a Markov property, the distribution of a n plus 1 given that the distribution of a one first random variable second random variable the n th random variable, that is same as the conditional distribution of n plus 1 th random variable given that with the n th random variable only. And this property is called a memory less or Markov property.

The stochastic process the S n satisfying the Markov property or memory less property is called a Markov process. The stochastic process satisfying the memoryless property or Markov property is called Markov process. In this example the stochastic process S n is the discrete time, discrete state stochastic process. Now, I can give based on the state space and the parameter space, I can classify the Markov process or I can give the name of the Markov process in a easy way based on the state space as well as the parameter space.

(Refer Slide Time: 09:57)




So, when the state space S the S is the state space, this is nothing but the collection of a all possible values of the stochastic process, if this is of the discrete type; that means, the collection of a elements in the state space S is going to be a finite or countably infinite, then we say the state spaces of the discrete type. So, whenever the stochastic process satisfying the Markov property, then the stochastic process is called the Markov process. Or you can say whenever the state space is a discrete, then we can say the corresponding stochastic process we can call it as a Markov chain whenever, the state space is a discrete.

Now, based on the parameter space a capital T, parameter space is nothing but the possible values of T whether it is going to be a finite or countably infinite then it is going to be a discrete parameter space or discrete time. Or it is going to be a uncountably many values then it is going to be call it as a continuous type. So, whenever the T is going to be a discrete type, then the Markov chain is going to be call it as a discrete time Markov chain. Whenever, the parameter space is going to be of the continuous type; that means, the possible values of a capital T is going to be uncountably many then we say continuous time Markov chain.

So, in this example the S n, the possible values of a S n is also going to the state space is going to be a discrete type, and the parameter space is also going to be a discrete type therefore, the given example the S n is going to be the discrete time Markov chain. So, in this model we are going to study the discrete time Markov chain the next model 4, model 5 we are going to discuss the continuous time Markov chain. So, in general whenever the stochastic process satisfying the Markov property, it will be called the Markov process. So, based on the state space the Markov process is called as a Markov chain, and they based on the parameter space it is called the discrete time Markov chain or continuous time Markov chain accordingly discrete type or continuous type.