Introduction to Probability Theory and Stochastic Processes
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Lecture – 59

I will have another example in which it is going to be a only the it would not be a strict sense itself.

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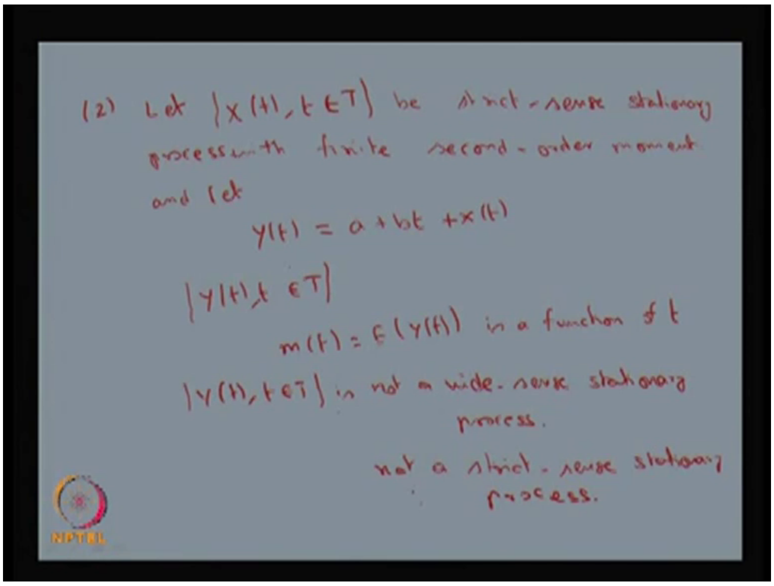

Let me start with the example in which this stochastic process is a strict sense stationary process. The given X t is a strict sense stationary process with a finite second order moment. So, you do not want the finite a second order moment for the strict sense stationary process, but I have taken as a example. The given X t is going to be a strict sense stationary process along with the finite second order moment.

Now, I am going to define the another stochastic process with the random variable Y of t; that is a a plus bt plus X of t so, this is going to be a stochastic process. This is a stochastic process Y t. Now we want to check whether the Y t is going to be a stationary strict sense stationary process or not, as well as whether this is going to be a wide sense stationary or not. The X t is the strict sense stationary process, suppose you find out the mean for this random variable the mean for this random variable, if you find out the mean for the Y of t where a and b are constant therefore, this is going to be a function of t.

Since a and b are constant the mean of Y of t is a function of t therefore, this is a function of t, since it is not satisfying the first property of the first condition to become a wide sense stationary process. Therefore, the Y of t is not a wide sense stationary process. We started with the strict sense stationary process, and we created a new stochastic process Y of t, that is a plus bt of X t plus X t where a and b are constant. Now if you find out the mean of y of t mean function that is going to be a function of t. That is nothing, but that depends on t therefore, Y t is not going to be a wide sense stationary process whereas, a X t is a strict sense.

Now, similarly you can cross check whether the joint distribution of a Y of t, and shifted b Y h t shifted by h you can conclude, this is also not going to be a since it is a function of a t, since it is a the mean is going to be a function of t and the Y of t also involves the function of t as well as a X of t. Even though X of t is a strict sense stationary process, the way you made a a plus bt plus X t, you will land up the joint distributions are going to be different by the t with the shifted t plus h it would not be satisfied. Therefore, you can conclude Y of t is not a strict sense stationary process also.

That means from this example we can conclude whenever you have a strict sense stationary process , if you make a a plus bt plus X t definitely the y of t is not going to be a wide sense stationary process and as well as a strict sense stationary process. We go for the third example.

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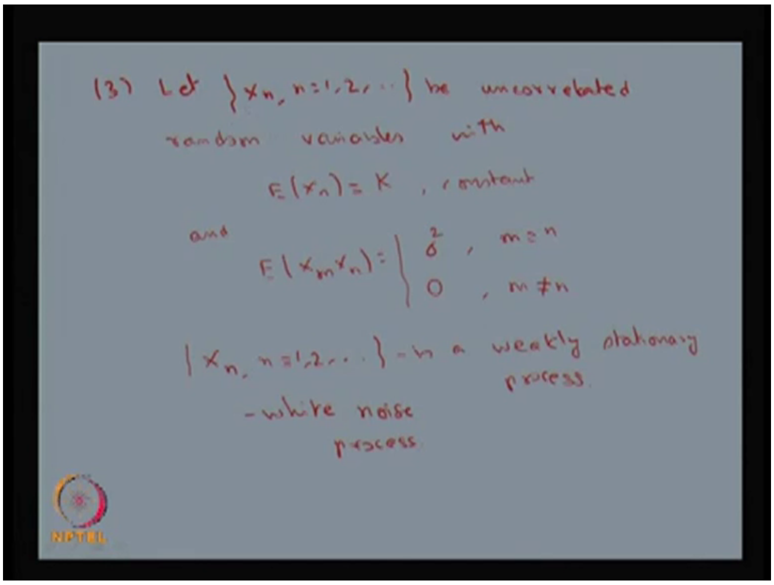

In this third example, let me start with the stochastic process be here this each random variables are uncorrelated random variables with the mean of each random variable is going to be some constant K, which may be assume it to be 0 in some situation. So, in general you keep the mean of each random variable is going to be some constant K.

And you make X of m X of n, that is going to be it is variance for m is equal to n, and all other quantity you make it 0. Not only this each random variables are uncorrelated random variable; that means, if you find out the correlation coefficient that is going to be 0, and the mean is going to be constant and the expectation of the product of any 2 random variables if they are different it is 0 and obviously if they are same.

Since you make the assumption therefore, this is going to be a variance sigma square. If you cross check all the properties of a all the conditions of the wide sense stationary property starting with the main function and second order moment exists that is finite and covariance function of any 2 random variables is going to be a function of only the difference. There all those 3 conditions are going to be satisfied therefore, you can come to the conclusion , I am not working out here this is going to be a weakly stationary process or wide sense stationary process. Or it is going to be call it as a covariance stationary process also.

And this stochastic process is also called white noise process. This is very important in the signal processing you keep the uncorrelated random variable with this assumption, the mean is going to be a constant which may be 0. And the product of expectation is going to be the this values, and this is going to be a weakly stationary process in the sense it satisfies a all 3 conditions of that weak sense or wide sense stationary process, and this stochastic process is called a white noise process.

Note that this stochastic process, we did not make the distribution of a each random variable X n what is the distribution of X n is not defined here, without that we we give the all the assumptions of the mean and variance. Therefore, this is going to be very useful in the time series analysis as well as the signal processing. And this particular stochastic process is called the white noise. And sometimes we make the assumption the X n's are going to be normally distributed random variable also.

But in general we would not to define we would not to give what is the assumption; what is the distribution of X n. Without that with this stochastic process is going to be call it as a white noise process.

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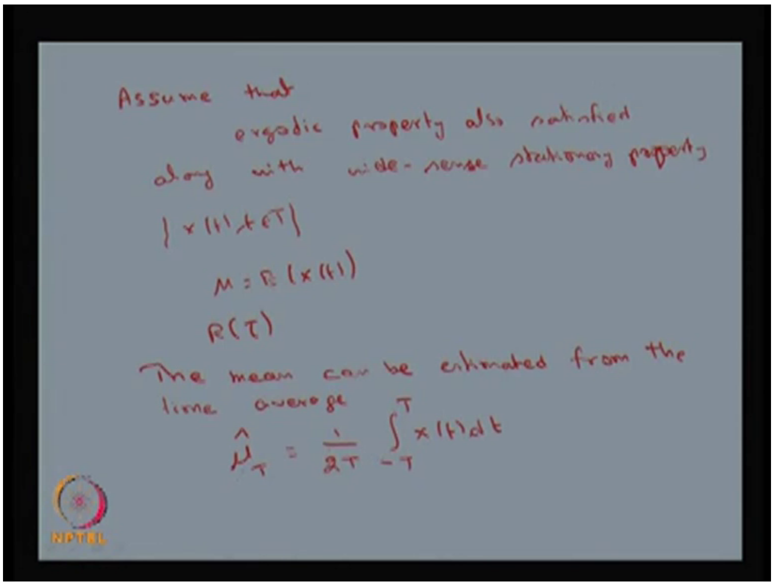

Addition to the wide sense stationary process one can assume, one can assume that ergodic property also satisfied along with wide sense stationary property. For illustration purpose we have discussed a Bernoulli process; that means, the given stochastic process is a wide sense stationary process. As well as it is the ergodic property is also satisfied.

In that case, the mean function is going to be a sum independent of t, that you can make it as the mu. And a auto covariance function is going to be a r of tau only because it is a wide sense stationary process. Therefore, the mean is independent of t and the autocorrelation function is going to be a function with the only tau. And we have a ergodic property therefore, you can find the mean can be estimated from the time average so, this is possible only if the ergodic property satisfied.

So, the mean can be estimated with the up arrow; that means, the estimator estimation of a mean, that is same as one divided by 2 times t, and a minus T to T of X of t dt. So, this is possible as long as the stochastic process is so, in general I define t belonging to capital T, that T is different from this t. So, here you have the time interval of length to t within that to t if you find out the time average, and that time average quantity is going to be the estimation for the mean; that means, if mu T converges in the squared mean to mu as a t tends to infinity then the process is going to be a mean ergodic. That stochastic process is going to be call it as a mean ergodic process.

Similarly, one can estimate other higher order moments also provided the process is ergodic with respect to those moments. So, here I have made the ergodic with respect to the mean therefore, you are estimating the mean with the ergodic property. Similarly, if this given stochastic process is satisfying the ergodic property with the higher order moment, then those measures also can be estimated in the same way. So, here the mu T converges in means in squared mean to mu as a T tends to infinity. So, that is the conclusion we are getting from the ergodic property along with the wide sense stationary property.

With this let me stop the todays lecture.

Thanks.