Introduction to Probability Theory and Stochastic Processes

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Lecture – 58

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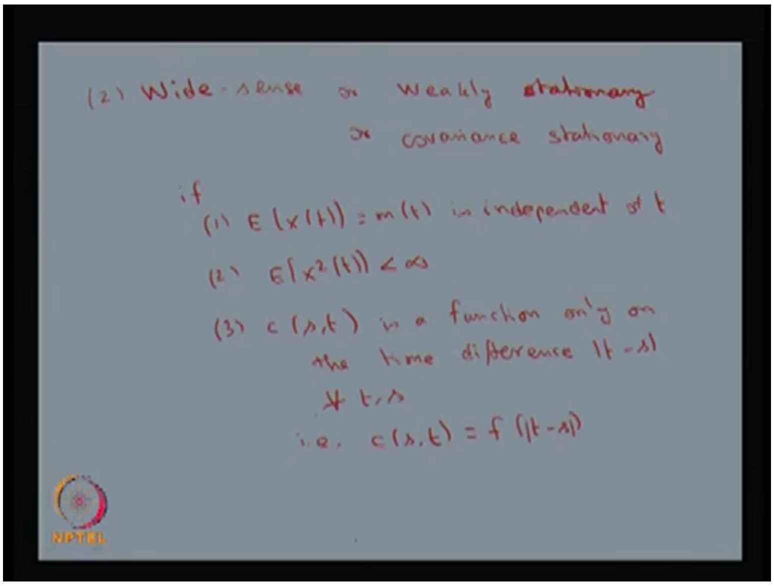

The next definition is a wide sense. The next one is a wide sense or weakly stationary or is another word covariance stationary process. When we say, the given stochastic process is going to be a wide sense or weekly stationary or covariance stationary, if it is going to satisfy the following properties, following conditions mean function that is, m of t is independent of t.

The second condition, the second order moment is going to be finite. Basically the stochastic process is going to be a second order moments are going to be finite. Third condition if you find the covariance function c of s comma t that is, a function only on the time difference, t minus s for all t comma s. If you find the covariance function for any 2 random variables x of s and x of t then that is always going to be a function of the only the difference t minus s not the actual t or actual s; that is, inverts in mathematically you can write c of s comma t that is going to be a function of t minus s in absolute.

If this 3 property is going to be satisfied by any stochastic process, then we say that stochastic process is going to be a wide sense or weakly stationary or covariance stationary. This is entirely different from the strict sense stationary. The strict sense stationary you are finding the joint distribution of n random variables, then find the joint distribution of a n random variable shifted by h and for all h greater than 0, and for all n if that property is satisfied then we say that is a strict sense stationary process whereas, here we check the mean function is going to be independent of t.

And the second order moment is going to be a finite value and the covariance function is going to be a function of only the difference of t minus s therefore, any stochastic process satisfying the strict sense stationary process, strict sense stationary property that does not imply the wide sense stationary property. As well as the wide sense stationary process need not be satisfied all the strict sense property therefore, you cannot imply one stationary process that does not imply the wide sense and the wide sense stationary process that does not imply the stick sense stationary process.

So, in the strict sense process, what we are saying is it is a stochastic process whose joint distribution does not change; when shifting time or space as a result, the parameters such as the mean and variance if they exist. Also do not change over the time or push or the strict sense stationary process. Now I am going to give few examples for the stationary process, maybe it could be a strict sense stationary process or it could be a wide sense stationary process.

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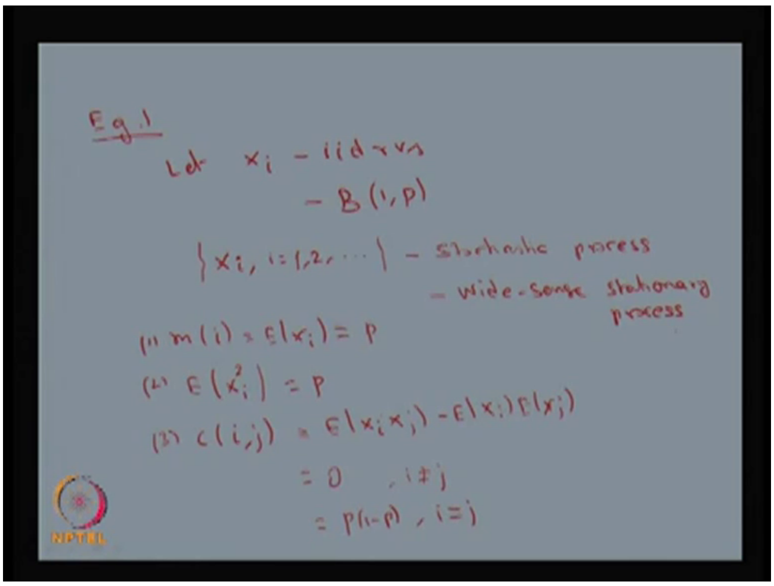

The first example let xi is going to be a iid random variables, independent, identically distributed random variable and assume that each one is going to be Bernoulli distributed random variable with the parameter p, a notation it is a binomial distribution with the parameters. So, 1 and p that is same as each Xi's are Bernoulli distributed random variables with the parameter p.

Now, I am creating a stochastic process with those such iid random variable, in which each random variable is a Bernoulli distributed random variable therefore, this is going to be a stochastic process. Now you can verify whether it is going to be a strict sense stationary process or a wide sense stationary process. The assumption is all the random variables are mutually independent and each random variable is identically distributed, which is a Bernoulli distributed.

So, this is just for examples sake we have taken, and if you find out the mean function for each random variable that is, going to be expectation of xi and that is going to be the mean of Bernoulli distribution is going to be p which is independent of i. The second condition if you find out what is the second order moment of, second order moment of the second order moment is going to be, 1 square into p and 0 into 1 minus p. So, therefore, that is also going to be p.

So, if you find out c of some i comma j instead of s comma t, you have i comma j that is nothing but expectation of x of i x of j minus expectation of x of i expectation of x of j. If you find out this quantity this is nothing but xi and xj and it since they are independent random variable therefore, expectation of xi into xj is nothing but the expectation of xi into expectation of xj that is same as this 1 therefore, this is going to be 0.

For all i is not equal to j, for i is equal to j that is nothing but expectation of xi square minus expectation of x i whole square that is nothing but the variance. And the variance of the random variable Bernoulli distribution that is going to be npq therefore, that is going to be p into 1 minus p for i is equal to j. And this value is independent of this values is going to be a function of i minus j you can make out therefore, since these all 3 properties of the weakly stationary property or wide sense stationary property is satisfied therefore, this is going to be a wide sense stationary process.

In fact, even if the random variables are simply iid's then to we can check that the processes is wide sense stationary, for illustration purpose we have discussed Bernoulli process. More examples on continuous time stochastic processes are discussed in the problem sheet.

Now, we can cross check whether this is going to be a strict sense stationary process also.

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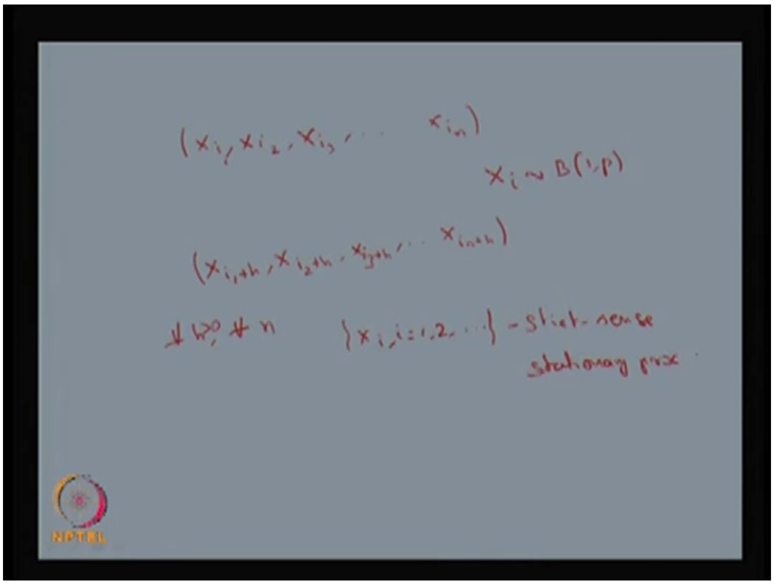

If you find out the joint distribution of a suppose, you take a few random variables x of i 1 x of i 2 x of i 3 x of i n. So, this is the n such random variable and each random variables are Bernoulli distributed with the parameter p and all are independent therefore, the joint distribution is going to be the product of individual distribution.

And if you shift these i 1 with some number h and x of i 2 plus h and x of i 3 plus h and so on x of i n plus h. You shift those random variable with the h if you find out the joint distribution and since each one is a independent random variable therefore, the joint distribution by shifted by h that is also going to be the product of those n random variables product therefore, the distributions are again going to be identical; because they are because each random variable is identical as well as mutually independent therefore, the joint distribution is going to be product. And all are going to be identical therefore it is going to be power m of the distribution.

So, this is going to be satisfy the stick sense property that is a joint distribution of this random variable and joint distribution of this random variable are going to be same for all h as well as for all n also. Since, it is satisfied for all h greater than 0 and for all any integer n therefore, this is going to be a the same collection of a random variables the stochastic process is going to be a strict sense stationary process.

So, this is the cooked up example in which this stochastic process is going to be a strict sense stationary process, as well as the wide sense stationary process, but there are many situation in which stochastic process may be a strict sense; not the wide sense and the some stochastic process may be a wide sense stationary process not the strict sense stationary process. And how this particular stochastic process become a strict sense and a wide sense because of each random variable is a mutual independent as well as identical therefore, it is going to be a strict sense stationary process as well as a the wide sense stationary process.