Introduction to Probability Theory and Stochastic Processes
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Lecture – 55

Now, I am going to explain how we can create the sample path of the Poisson process using the MATLAB code.

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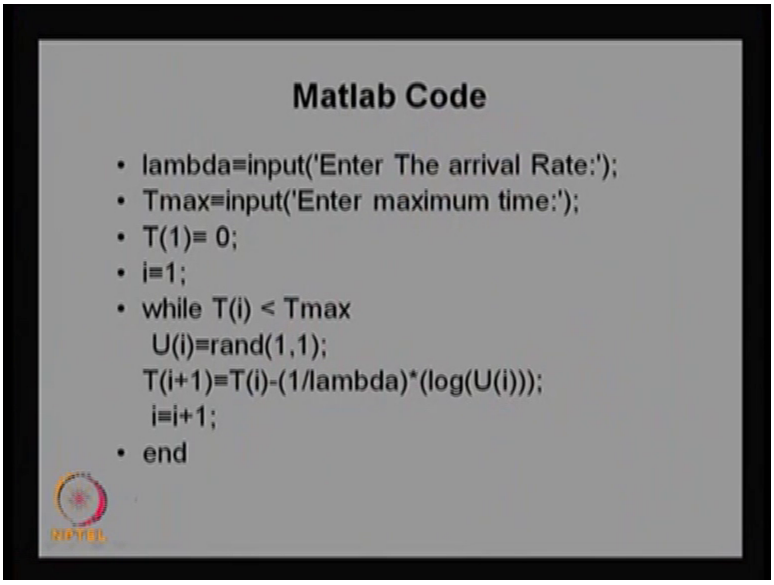

So, since I said the Poisson processes is related with the inter arrival times are exponential distribution. So, I can start with the time 0 there is no customer in the system and I can go for what is the maximum time I need the sample path then, I can keep on create the random variables. From the random variable, I can generate the exponentially distributed the time event.

Then, I can shift the time event by T of i plus 1 by adding the next exponentially distributed time event. Then I can go for plotting the sample path.

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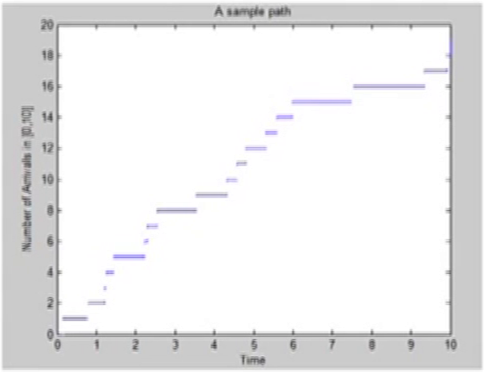
Sample Path



So, this is the one sample path in which over the time from 0 to 10, the number of arrivals occurs in the interval 0 to time 0 to 10 in the form of; that means, there is a one arrival occurs at this time. Therefore, the n of T values is incremented by 1 and it is taking the same value. And, when at the second arrival occurs, then the increment is taken by 2 and so on.

So, and if you see carefully the sample path, you can find out the increment is always by 1 over that time. And there is no 2 arrival or more than 1 arrival in a very small interval of time. And you can come, you can able to see the inter arrival time that is going to be a exponentially distributed with the parameter lambda whatever the lambda I have chosen in this sample path. So, this is the way the sample path of the Poisson process look like. Now, we are going to discuss the third type of stochastic process that is a simple random walk.

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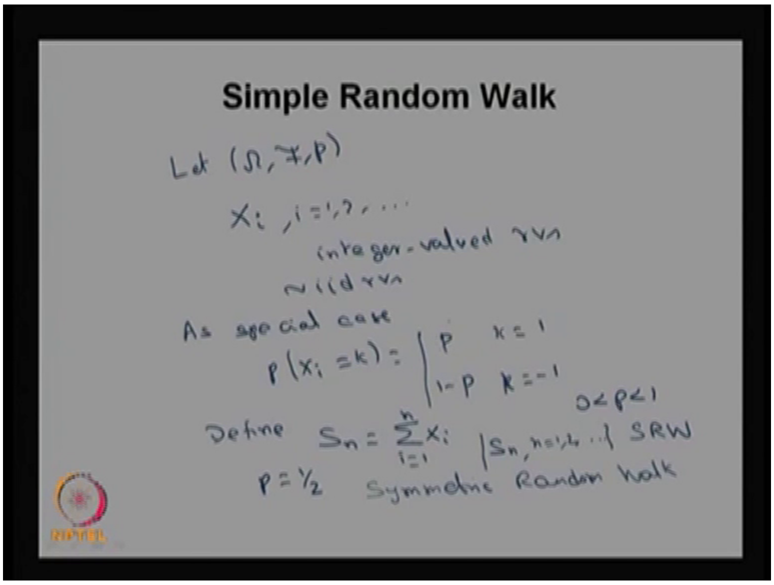

So, how we can create the simple random walk let me explain. You have a probability space, you have a probability space. From the given probability space, you define a sequence of random variable X i's and those random variables are integer valued random variables.

Each X i's are integer valued random variable. Not only that all the X i's are iid random variables, also all the X is are iid random variables. And each one is a integer valued discrete type random variable.

As a special case, I can go for the random variable X i takes a value 1 or minus 1 with the probability P and 1 minus P. This is a special type of random walk. In general, I am going to define the in general random walk also.

As a special case, I will go for the random variable X is takes the value 1 with the probability P and X i takes the value minus 1 with the probability 1 minus P where, P can take the value 0 to 1. Now, I am going to define the random variable S n. That is nothing, but sum of X i's, sum of first n X i's that is going to form a the random variable S n.

And the stochastic process S n or the stochastic sequence S n for different values of n this will form a simple random walk the S n is going to form a simple random walk. Why it is simple? Because, it is going to take integer valued random variable and each values are going to take each random variable is going to take the value 1 or minus 1.

Therefore, this is going to be call it as a simple random walk. In general, in general, the k can take the any integers. Accordingly, you land up having a S n are going to be a random walk and I am going to give the another special case when P is equal to off; that means, each X i random variable takes a value 1 with the probability of or minus 1 with the probability off.

Then, that random walk is going to be called it as a symmetric random walk. Why it is symmetric? Because, with the probability off it takes a forward one step or with the probability off it takes of backward one step. Therefore, that type of a random walk is called a symmetric random walk.

In general,, if it takes a value 1 or minus 1, then it is called simple random walk. If a k can take any integers, then it is going to be call it as a generalized random walk. So, this random walk can be created in a simple example of 2 persons.

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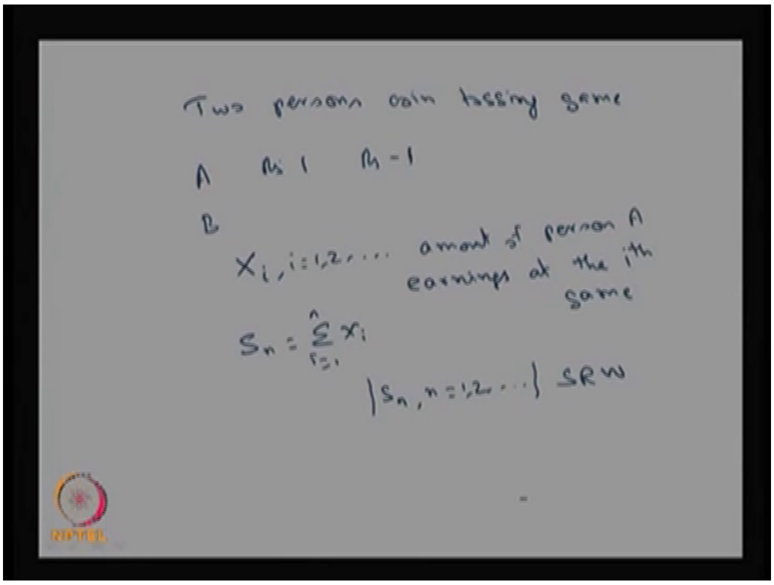

Coin tossing game also this simple random walk can be explained by the example 2 person’s coin tossing example in which you have a person A and B. If at the end of the coin tossing, if he is going to head, then he is going to win rupees 1 or if he is go at the end of the nth coin tossing, if it is going to get that tail, then he is going to be loose.

In this game, if A wins then, B gives rupees 1 to A and if A loses, then a gives rupees 1 to B. So, accordingly, I can go for creating a random variable X n or X i for i is equal to

1 2 and so on. Therefore, X i denotes what is the amount of the person A earning at the ith game.

Similarly, we can construct a stochastic process for player B and calculate the measures of interest. I can go for creating a random variable S n is nothing, but summation of X i where i is equal to 1 to n. Therefore, the S n denotes what is the amount earned by the person A at the end of nth the game.

That is a what is total amount. So, the X i denote how much he is going to earn at the end of each game, whereas, the S n is going to be the total amount earned by the person A at the end of first n games. Therefore, this S n is going to form a simple random walk where, X is are going to take a integer valued with the value 1 and minus 1 with the probability P, it is going to take the it is going to take the value 1 or it is going to take the value minus 1 with the probability 1 minus P.

So, I am just relating the sample random walk with the simple scenario with the 2 person's coin tossing game. If you see the sample path of the S n, first I can go for what is a sample path off.

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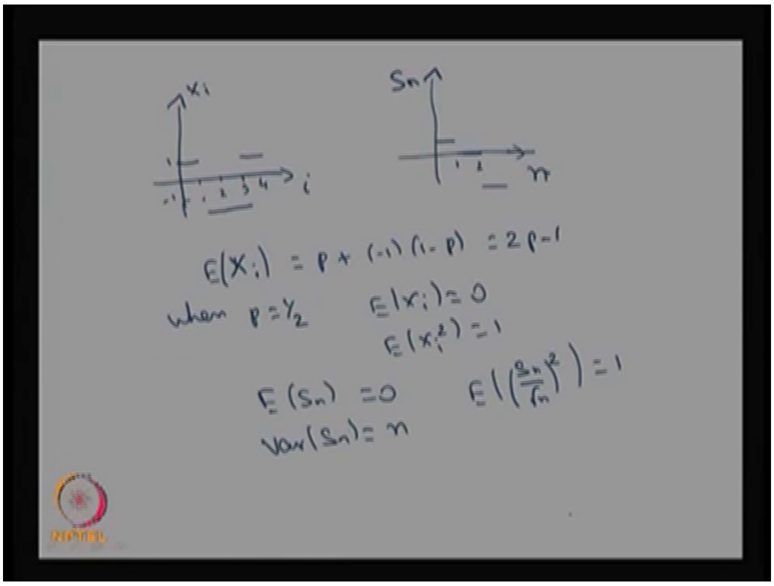

Each X i's each X is can take the value 1 or minus 1. Therefore, it is going to take the value 1 or minus 1. Therefore, if X 1 takes the value 1. It is 1 if X 2 takes the value minus 1 it is like this if a x 2 takes the value minus X 3 takes the value minus 1, then it is here. If X 4 takes the value 1 then it is like this. So, this is a sample path of X i over i, the way I have given the X is.

Now, I can go for sorry now, I can go for writing what is the possible values of n and what is the possible values of S n. So, since X 1 is equal to 1. Therefore, X 1 is going to be 1 and X 2 is going to be minus 1. Therefore, he takes a value 1 plus minus 1.

Therefore, it is going to be 0 and X 2 is going to be minus 1. Therefore, X 2 is X 3 X 3 is going to be minus 1 and the X 4 is going to be 1. Therefore, it is going to be again 0.

So, this is the way the sample path goes over the n. So, this is a one sample path for the possible values of X i takes a value 1 and minus 1. Accordingly, I have drawn the sample path of S n over the n. Since X is are going to take the value 1 and minus 1.

And with the probability P and with the probability 1 minus P takes a value minus 1, I can go for finding out what is the expectation of X i. That is nothing, but X i is equal to P plus minus one times 1 minus P. Therefore, this is nothing, but 2 P minus 1.

So, when I go for discussing the symmetric random walk, when the P is equal to off, then the expectation of each X i is going to be 0. And also, I can able to find out what is a E of X i squares that is going to be 1. Not only that, when P is equal to off, I can able to find out what is the expectation of S n that is going to be 0 and the variance of S n is going to be n.

And I can go for writing what is the expectation of S n by root n power n power 2 that is going to be 1. So, the way I have a got the result for expectation of S expectation of X is and the expectation of S n, I can go for what is the limiting distribution of S n.

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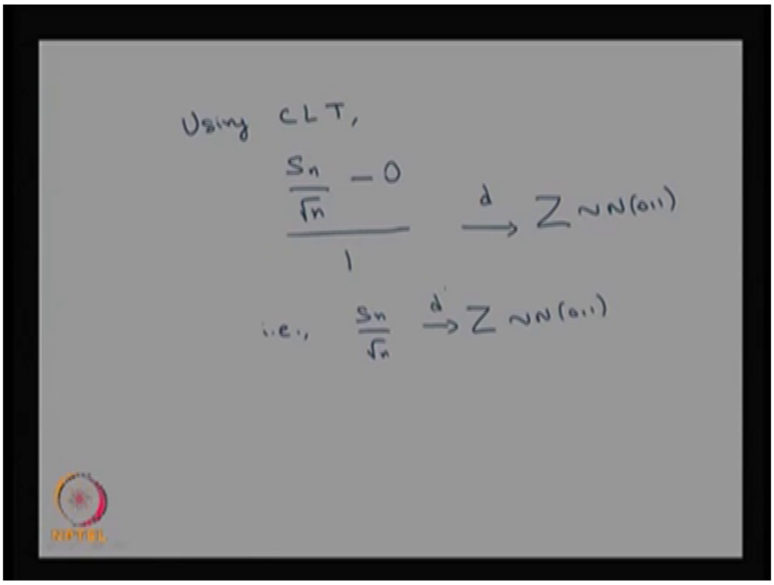

So, using central limit theorem, I know what is the mean for each S n and I know what is the variance of each S n. Also, therefore, using a CLT, I can be able to conclude S n divided by square root of n minus the mean of this random variable is 0 divided by the standard deviation is going to be 1. And this as n tends to infinity.

This will be a standard normal distribution. Whereas, Z is going to be a standard normal distribution as a n tends to infinity and this convergence is via distribution. That means, I can able to conclude the distribution of S n by square root of n has n tends to infinity in distribution.

This sequence of random variable will converges to the standard normal n distribution. I can go for creating what is the sample path of the simple random walk by using the MATLAB code.

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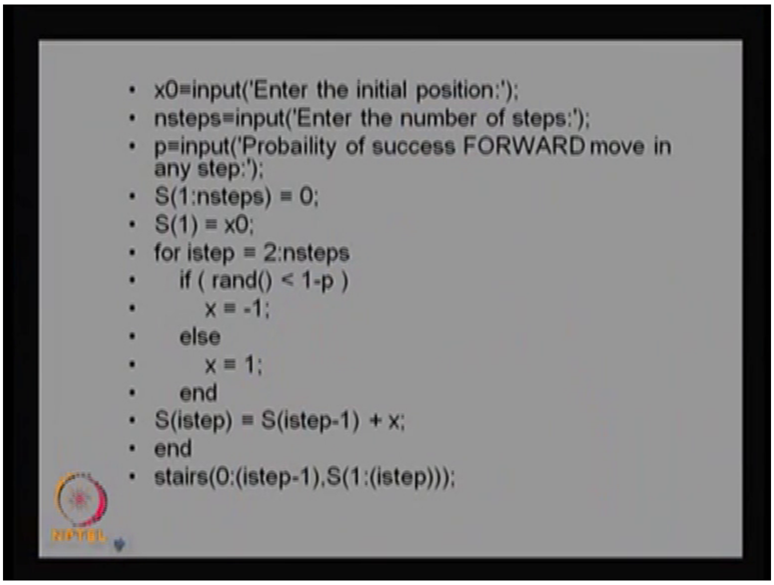

So, for that, I have to fix what is the initial position and what is the maximum number of the steps I would like to go for finding the sample path and what is the probability of success in each a for what is a forward move probability. Accordingly, it is going to take the value 1 with the probability P and it is going to take the value minus 1 with the probability 1 minus P.

So, I am giving the value of the P only and then, I am just going for the possible values of S n by adding the 1 or minus 1. Accordingly, I am just writing the sample path of S is.

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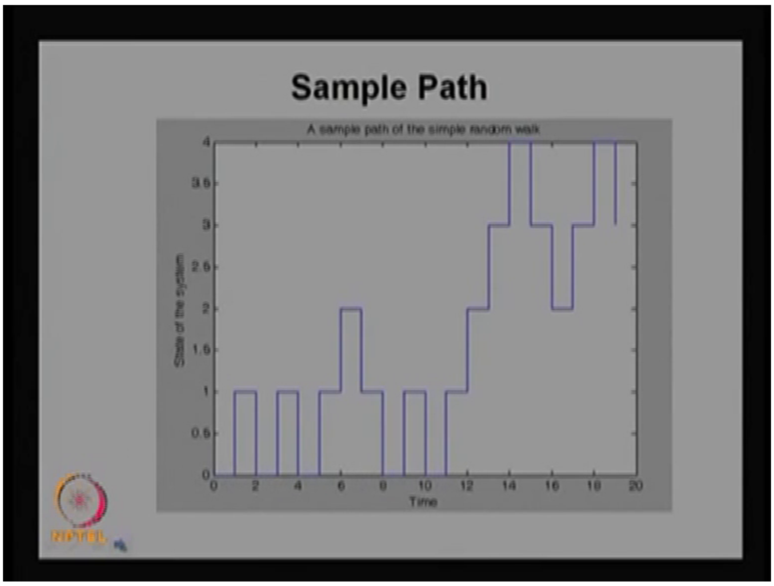

So, if you see the sample path over the time 0 to 20 and each X is are going to take the value 1 or minus 1, accordingly, the S n is going to take the same value or incremented by 1 or decremented by minus 1. According to the values of X is, therefore, this is going to be the one sample path which is depicted using the MATLAB code.

. So, this is the earlier I have shown the same graph.

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This is the S n as n tends to infinity. Here, you can see the different a sample path for as n tends to infinity, you can find out what is the distribution of a S n divided by square root of n as n tends to infinity.

Also, and this figures, it has a 3 different sample path and one can observe what is the amount of a person a have n tends to infinity that depends on whether it is he is going to take the positive value or he is going to have the negative value depends on the first few games that it can be observed from this diagram.

The first few the first few results whether he is going to gain by 1 rupee or he is going to lose by 1 rupee. Accordingly, the possible values of S n will go as n tends to infinity.

Now, we are going to discuss the 4th simple stochastic process that comes in the population model.

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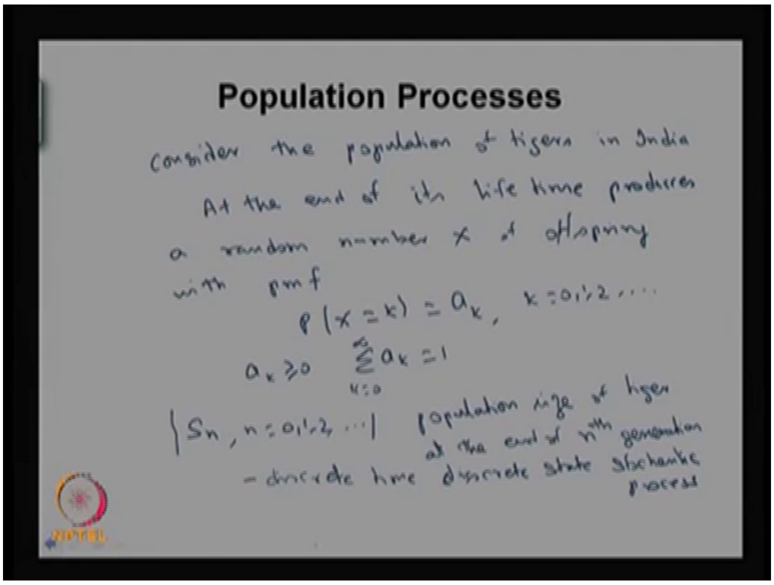

Now, we will see the 4th simple stochastic process arises in the population model. You consider a population of tigers in India. So, that is going to be a for over a time. This is going to be a form a stochastic process. So, I am going to make the assumption at the end of it is life time.

It produces a random amount random number X of offspring with the probability mass function. That is, the probability of X takes the value k that is ak where, it satisfies ak's are going to be greater than or equal to 0 and the summation is going to be 1.

And also, I am making the assumption all the off springs act independently of each other and the end of their lifetime individually can have a pregnancy accordance with the probability mass function. The same probability of X is it takes the value k with this S n with this.

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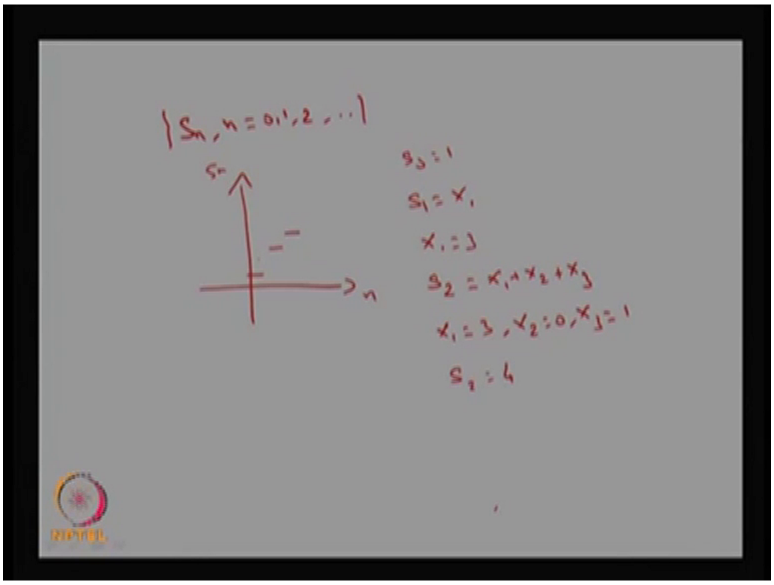

Sn will form a discrete time and discrete state stochastic process, where S n is the population size of tiger at the end of nth generation and if you see the sample path of S n over the different generation.

Suppose, you make it a S 1 is equal to X 1 and suppose the X 1 takes the value 3 and then the second generation S 2 is going to be X 1 plus X 2 plus X 3. And suppose, you make it X 1 takes a value 3 and X 2 takes the value 0 and x 3 takes value 1, then we have S 2 is going to take the value 4.

So, if you see the sample path of S n over the n, it is going to take the value 1then it is going to take the value 3, then it is going to take the value 4 and so on. And this is the sample path of population size of a tiger over the nth generation. And this is going to form a discrete time, discrete state stochastic process.

Thank you.