Introduction to Probability Theory and Stochastic Processes
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Lecture – 52

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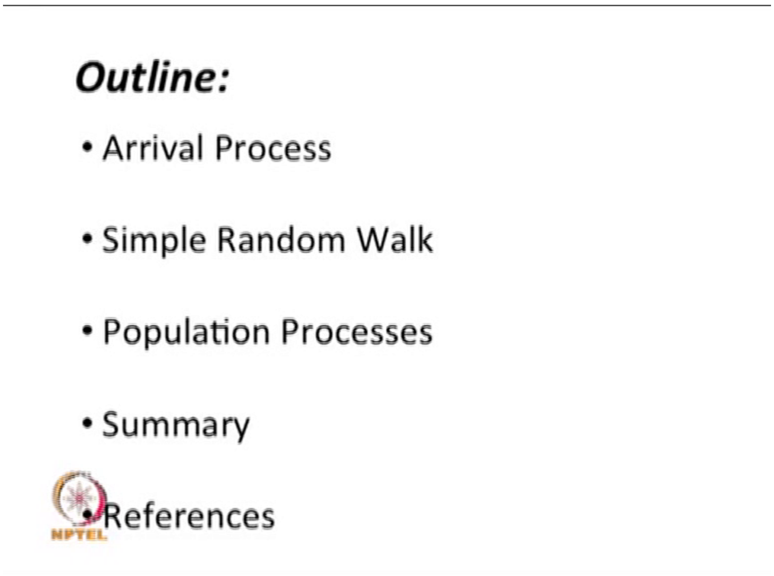

In this lecture we are going to discuss some simple stochastic process, starting with the discrete time arrival process that is a Bernoulli process and, a continuous time arrival process that is a Poisson process, followed by that we are going to discuss the simple random walk. Then we are going to discuss a yeah one simple population process which arises in the branching process, then we are going to discuss the Gaussian process so, with that the lecture 2 will be over.

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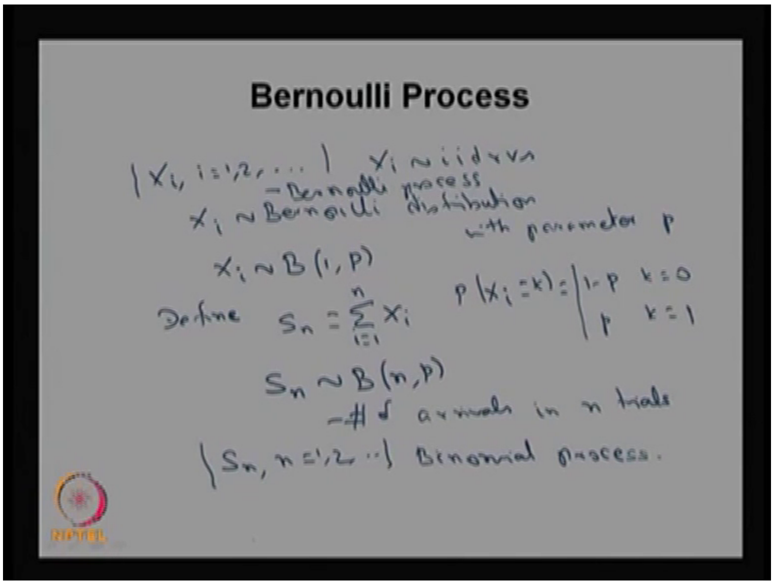

What is Bernoulli process? Bernoulli process can be created by the sequence of random variable. Suppose you think of a random variable X i where i is belong I takes the value 1 2 and so on therefore, this is going to be a collection of random variable.

And each random variable or X i’s and you can think of X i's are going to be a i i d random variables. And each is coming from the Bernoulli trials that means, each random variable is a Bernoulli distributed, each random variable is a Bernoulli distribution and with the parameter p so, the same thing can be written in the notation form X i takes the X i’s are in the notation it is a capital B 1 comma small p that means, it is a binomial distribution with the parameters 1 and p that is same as each X i’s or Bernoulli distributed with the parameter 1 and p.

So, now I can so this is going to be a stochastic process, or we can say it is a stochastic sequence. Now, I can define another random variable for every n S n is nothing but sum of first n random variables, suppose you think X i is going to be the outcome of the ith trial so, the X i can take the value 0 or 1 that means, with the probability the X each X i can take the value k, if k is equal to 0 with the probability 1 minus p and the k taken k can take the value 1 with the probability p.

Therefore each since each X i’s are i i d random variable, you can come to the conclusion S n is nothing but binomial distribution with the parameters n comma p. Suppose you assume that X i is going to be number of whether the arrival occurs in the ith trial, or not if X i takes the value 0; that means, no arrival takes place in the ith trial if X i takes the value 1 that corresponding to the ith trial there is an arrival.

So, the S n represents S n denotes the number of arrivals in n trials. So, now you can create a stochastic process with the S n, where n takes a value 1 2 and so on therefore, this is going to be a binomial process.

So, the X i is takes the value 0 or 1 with the probability 1 minus p and p. So, each 1 is going to be Bernoulli distributed therefore, this is going to be a Bernoulli process this X i's are going to from Bernoulli process. The way you have created S n is equal to some of first n random variable and each S n is going to be a binomial distribution with the parameters n and p therefore, this S n that sequence of S n for n is equal to 1 2 3 binomial process.

Therefore since you have a collected arrivals over the over the possible values of 1 2 and so on therefore, this is going to be a one of the discrete time arrival process. So, similarly we are going to explain, what is the continuous time arrival process whereas, here binomial process, this is going to be a discrete time arrival process.

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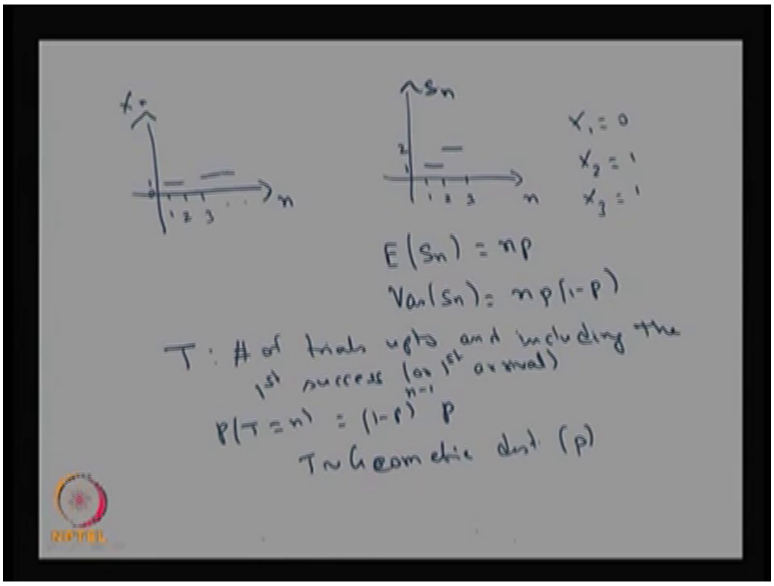

Suppose you would like to see the trace of a S n so, before you go to the trace of S n we can go for what is a trace or sample path of X i, for different values of a n is equal to 1, n is equal to 2, n is equal to 3 and so, on if you see each X i's are takes a value 0 or 1 therefore, it can take the value 0 or X 1 can take the value 1, or X 2 can take the value 0, or this can take the value 1, again it can take the value and 1 and 0. So, the possible values of X i's are going to be 0 and 1 therefore, each X i is can take the value 0 in the horizontal line, or it can take the 1 till you get the next trial.

Similarly, if you make the sample path or the trace of S n, the sample path or trace of S n since S n is going to be a some of first n random variable therefore, the based on the X i takes the value, suppose the X 1 takes the value 0 and the suppose X 2 take the value 1 and the suppose X 3 takes the value 1 and so on.

So, since the X 1 is equal to 0 therefore S 1 is 0, then at S 2 is same as X 1 plus X 2 therefore, he takes a value 1 and S 3 is equal to X 1 plus X 2 plus X 3 therefore, that is going to be again you are adding the values therefore, it is going to be a 2 therefore, this is 1 and this is 2.

So, based on the X 4 it is going to be 0 or 1, either it can take the value 2 itself or it can go to the 3 therefore, if you see the sample path of S n it is going to be incremented either incremented by 1, or it takes the same value, till the next n.

Therefore, not only you can find out the S n, you can not only you can find out the sample path of you can you can get the mean and variance, because each S n is going to be binomial distribution with the parameters n and p therefore, the expectation of S n is going to be n times p and the variance of S n is going to be n times p into 1 minus p.

So, you can able to see the sample path of X i's as well as S n over the different values of n. In discrete time sample paths are sequences, I can also define the new random variable capital T is nothing, but number of number of trials up to and including the first that means, suppose it takes the value n that means, for subsequent n minus 1 trails, I got the failures or no arrival takes place in the subsequent n minus 1 trail and the nth trial I get the first arrival; that means, the T is a random variable to denote how many trials to get the first success or the first arrival, or the first arrival

So, ifs the it is going to take the first arrival in the nth trail, then the probability of T takes the value n, that is same as 1 minus p into n minus 1 into p, because all the trials are independent and, subsequent n minus 1 trial gives no arrival and, the nth trail you get the first arrival. Therefore, this is going to follow a geometrical geometric distribution with the parameter p.

So, since you know the distribution of T, you can find out the mean and variance, because the mean of a geometric distribution is going to be 1 divided by p and the variance of T is going to be 1 minus p divided by the p square.

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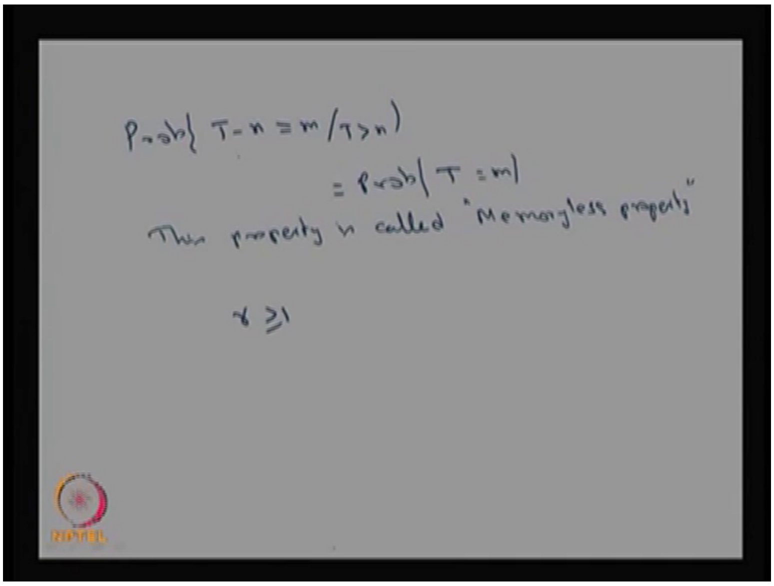

Similarly, I can go for finding out what is the probability that, it till nth trial I did not get the first, or I did not get the first arrival. So, if n plus mth trial if I am getting the first arrival, what is the probability that it is going to take. After m trials it get the first arrival, that probability you can able to get that is same as the probability of the T takes the value m.

So, this property is called, this property is called a memory less property. Since T is a geometrically distributed and the geometric distribution satisfies the memory less property that can be visualized in this example the probability of a T minus n is equal to m given that the T takes a value greater than n that is same as what is the probability that the T takes a value small m. That means, the right hand side result is the independent of the n and, it is the same as the distribution of that means, the residual arrival number of arrivals that is same as the original arrival distribution therefore, this satisfies the memory less property.

So, this is the geometric distribution satisfies the memory less property in the discrete time and, there is another distribution satisfies the memory less property in the continuous time, that is a exponential distribution. So, the way I have related the binomial distribution from the Bernoulli process, then I get the binomial process, also I will I was able to create the geometric distribution, you can create the or you can develop the Pascal distribution, or negative exponential distribution.

The way I have defined the capital T is going to be the number of trials to get the first success, or first arrival instead of that if I make another random variable to go for, how many trials are needed to get the rth success where r take can take the value greater than or equal to 1. If it is the rth first success is going to happen in the nth trail, if r is greater than 1, then I can go for a defining what is the negative binomial distribution, for that particular random variable if r is equal to 1, then that is random to be the same the random variable capital T.