Introduction to Probability Theory and Stochastic Processes
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Lecture – 50

So, let us see some simple example based on the possible values of capital T and the capital S.

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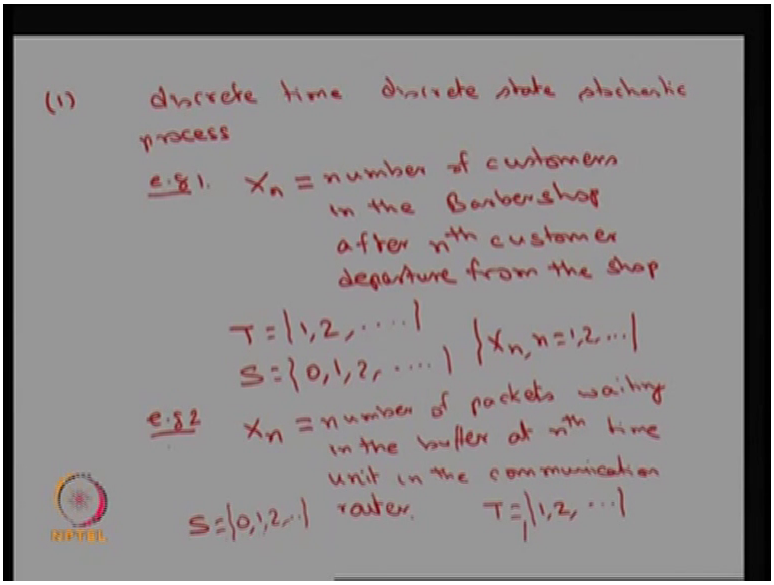

So, the first one is going to be a discrete time or you can use a discrete parameter also, discrete time discrete state stochastic process; that means, the possible values of capital S as well as the possible values of capital T has to be a either it has to be of countably finite or countably infinite elements in it.

Let see the one simple example, let us create a random variable X of x n has nothing but, that is nothing but the number of customers in the barber shop after nth customer departure from the shop.

So, here suffix n that will form a parameter space therefore, the T can be a the possible values of n; that means, whenever the one customers leave the system, how many are in the system after he leaves? So, the possible values of T will be, the first customers when he leaves how many are there he want to find out and so on.

Therefore, the possible values of capital T is going to be 1 2 or 3 therefore, this is a number of making the number of customers in the system. Whereas, the possible values of Xn for possible values of n that is going to be there is a possibility no customers in the system when someone leaves. So, there is a possibility 0 when someone leaves only one customer in the system then it is going to be 1 or 2 and so on. Therefore, there is a possibility it could be a finite also. So, the capital S can be a countably finite or in this case I have made the assumption it is a countably infinite.

Therefore, the capital T as well as T is going to be form of the discrete. Therefore, the corresponding stochastic process x n for possible values of n is going to be 1 2 and so on, and this is going to be a discrete time discrete state stochastic process.

You please note that here the parameter space the capital T is not the time. The parameter space the forming the 1, 2, 3. These are all the customer's the nth customers therefore, n can be 1 2 and so on. Therefore, in usually the capital T is a time whereas, sometimes it could be a distance or length or the number or whatever the other quantity. So, here is the typical situation in which the parameters space is not considering the time.

Therefore, this is going to be a random variable, because you never know how many customers are going to be in the system after the nth customers leaves, therefore, this is going to be a random variable; obviously, it is a real valued function satisfying all the property of the definition and you can see the probability space for this and from the probability space you have created a random variable. And therefore, this random variable is going to be this collection of random variable over the n, that is going to be the discrete time and discrete state. Therefore, this random variable here you can created with the help of a case one by making for fixed n what is a random variable, then you make a collection of random variable.

So, we can create this stochastic process by using the case one or the approach one which is easier one. I can go for creating one more stochastic process for this discrete time and the discrete state stochastic process.

That comes under telecommunication problems. Xn is going to be number of packets waiting in the buffer at nth time unit in the communication router; that means, there is a communication router in which the packets are coming for transmission.

So, after the transmission is over in the buffer the packets are leave the leave the router. So, at any time you do not know? How many packets are waiting in the buffer for the transmission?

So, there is a possibility no packets may be there at some time point, and there is a possibility there are many more packets may be waiting for the transmission in the buffer. So, the possible values of capital T the possible values of capital S that is going to be. There is a possibility no packets in the buffer or one, and so on and similarly the possible values of a capital T, that is also we are making the nth time unit therefore, the time unit could be a first time unit or second time unit and so on.

Therefore, here the S is going to be the discrete as well as the T is going to be a discrete. Therefore, this will this collection of a random variable x suffix n for possible values of n. That is also going to form a discrete time discrete state stochastic process. Because the possible both the values are going to be of a discrete type.

Discussing the simple stochastic process based on the parameter space and the state space, and we have seen the discrete time discrete state stochastic process the first one. Now we are seeing the second one.

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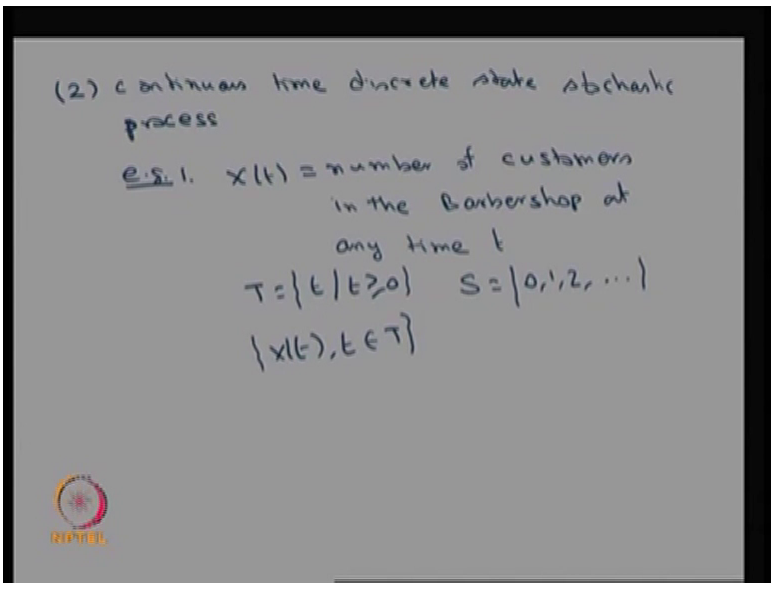

That is continuous time discrete state stochastic process; that means, the possible values of parameter space is going to be a uncountably many values therefore, we get the

continuous time. And the possible values of the state space that is going to be a countably finite or countably infinite therefore, you get the discrete state.

So, we will see the few simple example of this type. The first example that is a x of T, that is going to be the number of customers in the barber shop at any time T that is a difference, in the earlier example we have seen the number of customers in the barber shop for the nth customers a departure.

Now we are seeing the number of customers in the barber shop at any time T. Therefore, we are looking at how many customers at any time T in the barber shop. Therefore, the possible values of capital T that is going to be a collection of T such that the T is greater than or equal to 0. And the possible values of s that is going to be still it is the number of customers therefore, the possible values are 0 or 1 or 2 and can be and there is a possibility it can be a countably finite also.

So, whether the state space is going to be a countably finite or countably infinite we will classify as a discrete state. Therefore, this is a typical example of continuous time and discrete state stochastic process, and the collection of random variable is going to be x of T for all possible values of capital T. So, this is going to form a real valued stochastic process in which a for each T it is going to be a random variable.

So, this is going to be a real valued stochastic process of one dimensional type. And the T is belonging the small T that is going to be the time that is a default one and it is going to be a uncountably many. And therefore, it is going to be a continuous parameter. So, it is going to be called as a continuous parameter discrete state stochastic process also.