Introduction to Probability Theory and Stochastic Processes
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Lecture – 05

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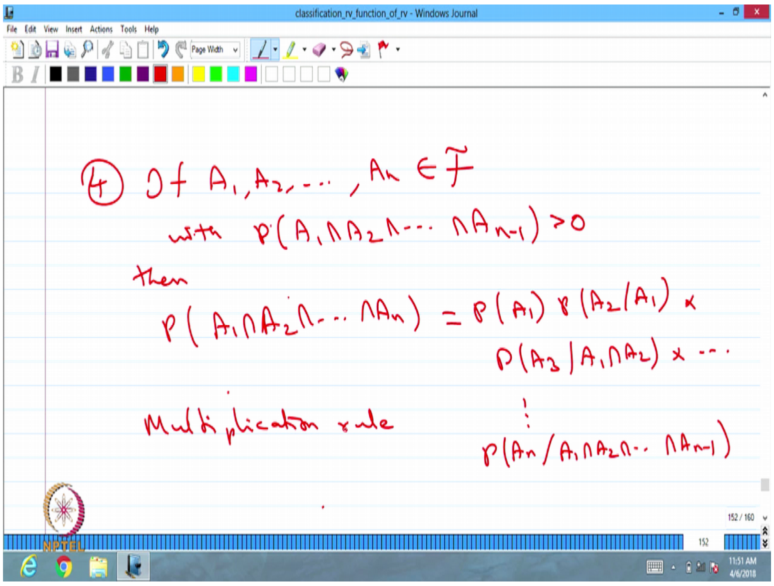

This result on this condition this probability of intersection of events is same as the product of this probability. This rule is called multiplication rule. Later I am going to introduce the total probability rule, whereas, this one is called multiplication rule that is for the probability of intersection of events is same as the probability of individual events.

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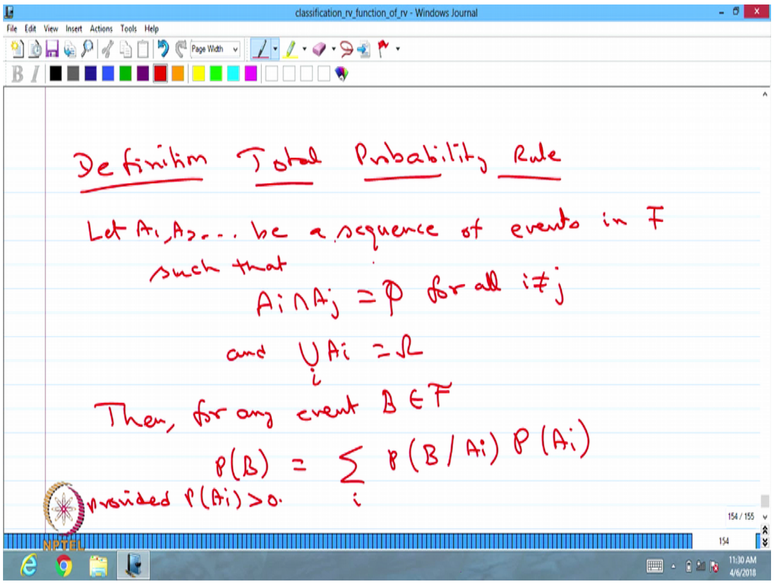

Now we are moving into the next important definition before going to the base theorem; that is called total probability rule or we can (Refer Time: 00:50) as a theorem also. So, this called total probability rule, what it says, let A 1 A 2 so on.

Be a sequence of events in F; such that A i intersection A j, that is going to be empty; that means, their mutually disjoint for all i is not equal to j not, only that if a make a union of A i is that is going to be the whole set; that means, indirectly this is partition of omega. This sequence of events could be finite or countably infinite.

Therefore, I making union of i's union over i it could be finite or countably infinite such that they are mutually disjoint union is going to be the whole set, that is basically partition events. Indirectly what you are saying is their mutually exhaustive also this events are mutually exhaustive satisfying this condition, then for any event capital B belonging to F probability of B can be computed by the summation of conditional probability on A i's multiplied by P of A i's. This summation over i provided P of A i's has to be strictly greater than 0.

So, the total probability rule says if you have partition events on omega, the probability can be computed for any event with the condition probability on partition events multiplied by the probability of those partition events for all i's. This is going to be call it as total probability rule. You can give the proof of this, even though I made it as a definition, we can prove it.

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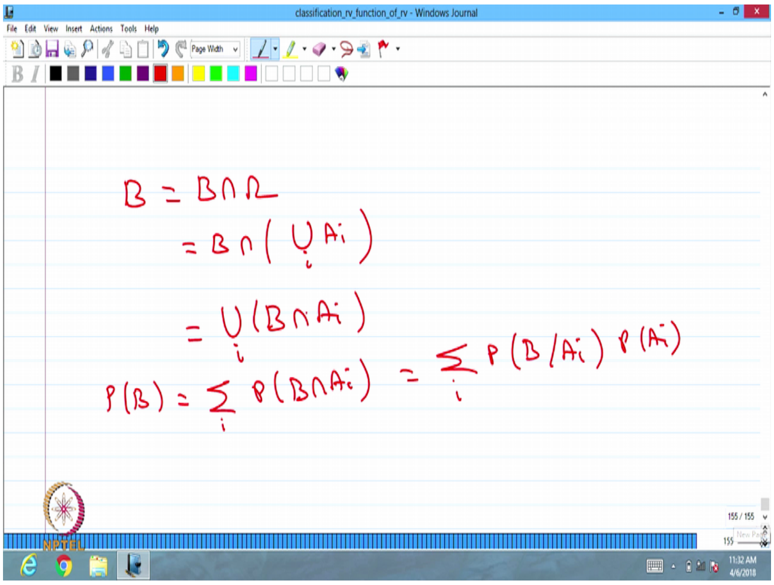

The any event B can be written has B intersection, whole set that is same as we can write rewrite event B intersection, the omega can be rewritten as a union of i's over is A i's. Then this is same as union of i's of B intersection A i's.

Therefore, the probability of B is nothing but, since B intersection A is are mutually disjoints. Therefore, this is same as summation of P of B intersection A i's, that is same as you can apply the condition probability, therefore, it is going to be probability of B given A i's multiplied by probability of A i's. This is possible when P of A is are going to be greater than 0.

So, whenever the P of A is are going to be greater than 0, then probability of B can be computed summation over i P of B given A is multiplied by the probability of A i's. Using this total probability rule, we are going to define next rule that is called Bayes rule, as a next definition, that is called Bayes rule.

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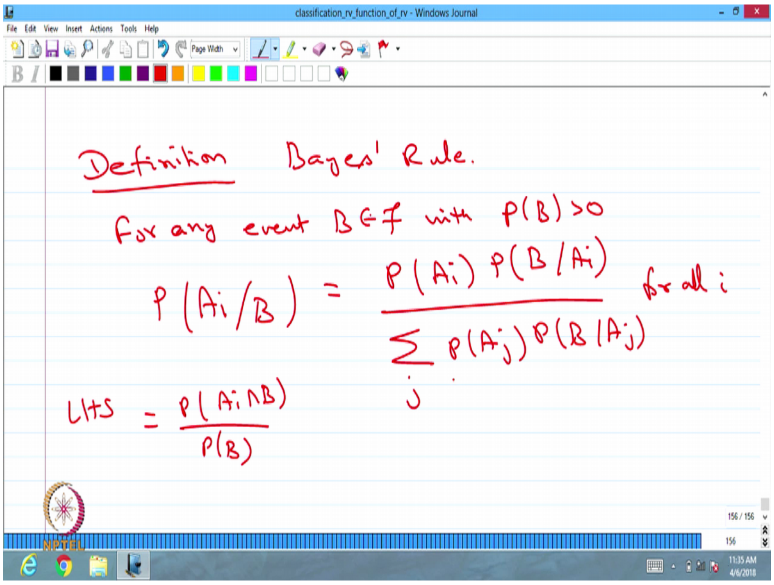

What this rule says, the continuation of the total probability rule that is for any event capital B belonging to F with; the probability of B is also going to be greater than 0, already we made the assumption P of A is are greater than 0, now we are making P of B also going to be greater than 0. In that case, one can find the P of the partition event A i given B that is same as P of A i's multiplied by P of B given A i divided by summation over j P of A j multiplied by probability of B given A j. This is valid for all i. Because we have already define A i's are the partition events of omega.

So, you can go for finding the condition probability of A i's given B provided probability of B is greater than 0. That is same as the numerator is probability of A i's multiplied by probability of B given A i, where as the denominator is all possible values of the same thing in the numerator. Therefore, this ratio is going to be the probability of A i given P you can prove it easily, the left hand side is same as probability of A i intersection B divided by probability of B.

So, in the numerator you can apply the condition probability and the denominated you can use total probability rule therefore, you can get the answer. So, these are very important result on whenever you know the partition events, you can find the probability of any event using total probability rule as long as you know the condition probability on partition events, and the individual probability of A i's partition events. Then using the same result you can find out the condition probability of partition event given any event B provided probability of B is greater than 0.

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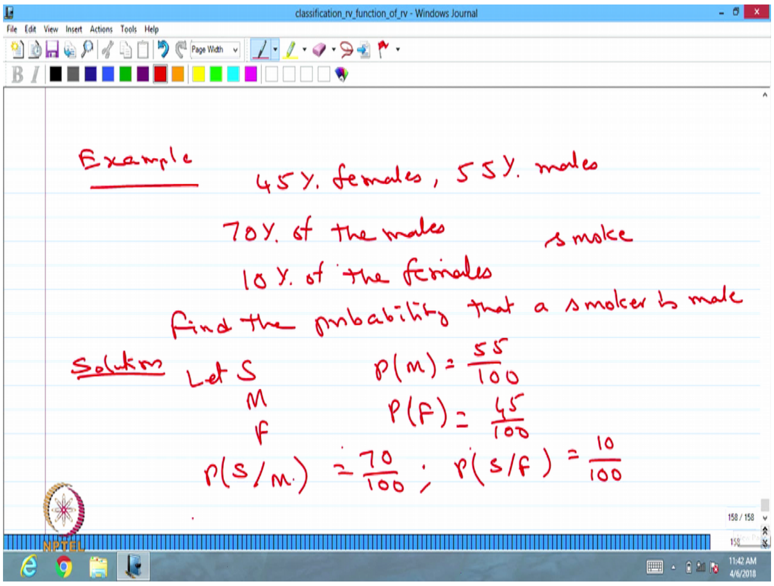

We will go for the simple example, it is known that the population of certain city consisting of 45 percentage females, and 55 percentage males.

Suppose that 70 percentage of male of the males, and the 10 percentage of the females smoke, suppose that 70 percentage of the males and 10 percentage of the female's smoke, the question is find the probability that find the probability that a smoker is male. Let me read the question again. It is known that the population of the certain city consisting of 45 percentage of 45 percentage females and 55 percentage males. Suppose that 70 percentage of the males and 10 percentage of the female's smoke, find the probability that smoker is male. Let us solve this problem by treating, let S be the event S be the event that the person is smoker.

Let M be the event that the person is male, F be the event that person is female. Let S be the event that person is smoker, M be the event that person is male, F be the event the person is female. Therefore, you can find the probability of M, that is 55 percentage therefore, 55 by 100, probability of f that is 45 percentage. So, 45 by 100, and you can find the probability of S given male that is 70 divided by 100. Similarly, you can find probability of S given f that is also given, that is 10 percentage therefore, 10 by 100 these are all the given information.

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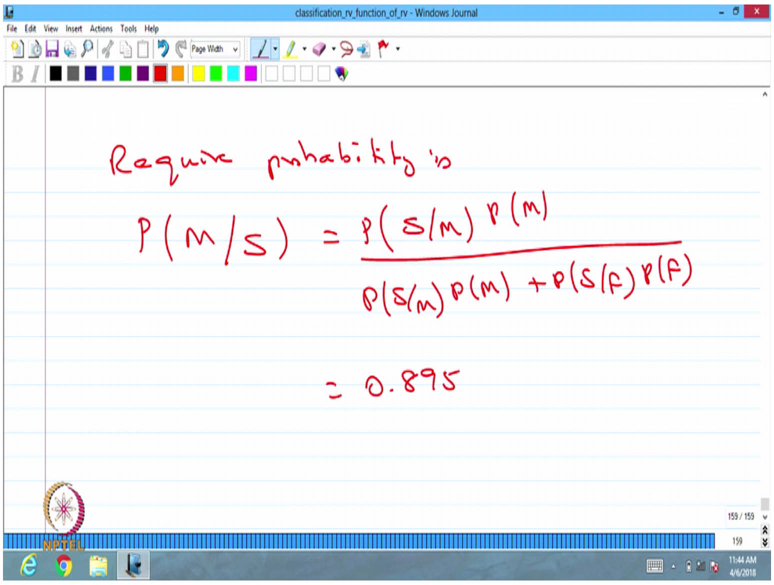

Now, the required probability is probability of M given S, that is same as probability of S given M multiplied by probability of M, divided by probability of S given M multiplied by probability of M, plus probability of S given f probability of F. So, you can substitute all the values then you can get the answer that is 0.895 by substituting all the probability values you can get the condition probability of M given S. So, by using the total probability rule as well as the Bayes rule we are getting the result of condition probability of M given S.

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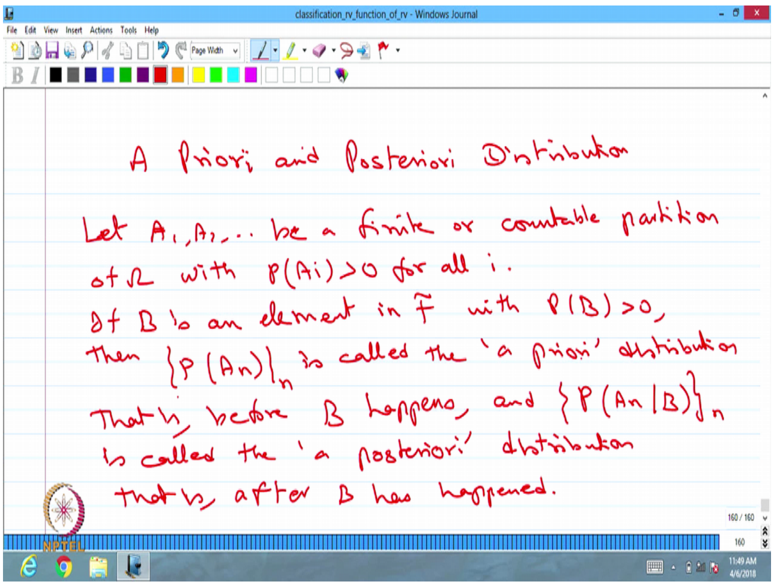

So, there is very important result on using the Bayes rule, that is called a priori and posteriori, posteriori distribution, there is very important concept on priori and posteriori distribution. What it says? Let A 1 A 2 and so on be a finite or countable partition of omega; that means, intersection is empty and union of A is same as omega with P of A i's greater than 0 for all i , it is basically partition events of omega.

If B is an event if B is an element or event in f with the probability of B is strictly greater than 0, then the distribution or the P of A n's for all n, that is called the priori distribution. Because this information is known to you earlier, therefore, this is called priori distribution knowing the probability of partition events for all n.

That is before B happens and the P of a n, given B this collection for all n, that is called the posteriori distribution. Because this distribution is known after B happens, where as P of a ns is a before B happens, therefore, it is called priori distribution and probability of a n given B that is after B happens therefore, it is called posteriori distribution, that is after B has happened.

So, this is very important result on probability, the priori and posteriori distribution base on Bayes rule, the one is call the probability of partition events these are called the priori distribution, and after the event B happens, the probability of a n given B for all n, that is going to be called as posteriori distribution; This as the wide application in statistics.

So, with these we are completing basic, basics of probability starting from random experiments sample space probability space probability measure. Then some results on the probability space, then independent events, then mutually independent and the pair wise independent events, then after that we have introduce condition probability, then we have introduce two important result one is total probability rule, and the multiplication rule. Then finally, we have discussed the Bayes rule.