Introduction to Probability Theory and Stochastic Processes
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Lecture – 49

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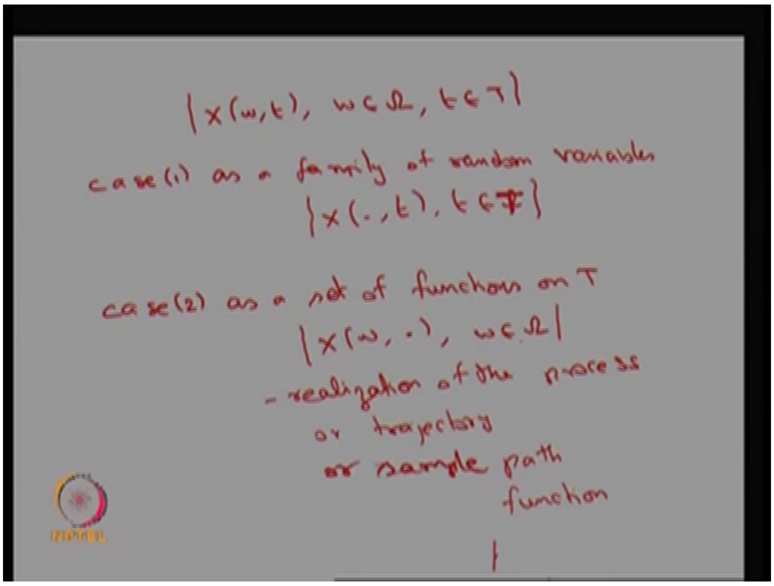

So, the based on the values of the way I have explained the random variable or the stochastic process is going to be X of w comma t where w is belonging to omega and t is belonging to T. There are 2 approaches can define the stochastic process. The first one, that is a we name it as a case 1. I can say it as the collection of random variable, as a family of random variables, as a x dot t, where t is belonging to capital T.

So, this is the way I can create the random variable and this is the easier approach in the sense, once I know the different t for fixed t, it is going to be a one random variable and I have collected a family of random variable for different values of small t. Therefore, this is the way we can create the stochastic process and this is the easier approach also.

The next one that is a case 2 that is nothing but as a set of functions on capital T; that is nothing, but a collection of X w comma for w is belonging to omega; that means, I have made a function on capital T. And once I fix one w, I will have a one function and if I fix another w, where w is nothing but a possible outcomes.

Therefore, if I have a different possible outcome that is going to be create a different stochastic process. There are, therefore, I can create a stochastic process of x of w comma t either fixing a t or fixing the w. Accordingly, I can have a 2 different ways of creating a stochastic process.

And the case 2, the way I have made a collection of random variable by fixing the w, then I made a function of functions on t. Therefore, this is going to be the method of realization of the process or it is going to be call it as a trajectory or it can be call it as a sample path or it can be call it as a sample function also.

So, these are all the different ways the case 2 can be called; that means, once you know the one possible outcomes therefore, you are tracing the stochastic process along the one possible outcomes. Therefore, that is going to be call it as a realization of the process or trajectory or the sample path.

So, the conclusion is a, you can always define a stochastic process as a collection of a random variable for different value of a t or you can go for a collection of a functions on t for a different values of the possible values of possible outcomes; that is, a w belonging to omega. So, these are all the 2 approaches can create the stochastic process. Not only we can go for making a one dimensional random variable or one dimensional stochastic process.

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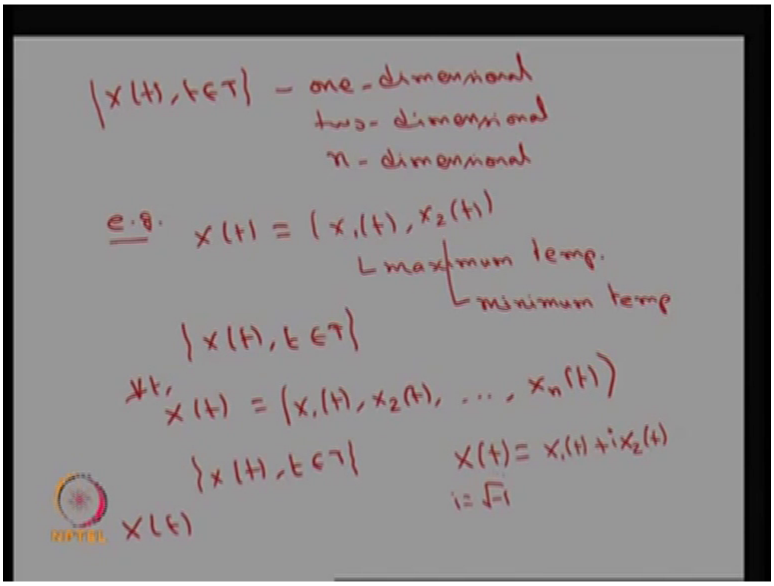

So, you can create a stochastic process. It could be a 1 dimensional or it could be a 2 dimensional or it could be a n dimensional also. So, first we have discuss what is stochastic process and how to create the stochastic process whether it exist and so on. Then, we have given the parameter space and the state space, then we have given what are all the ways we can create the 2 different approaches, you can create the stochastic process. Now, we are discussing what is the dimension of the stochastic process.

So, either the default, it could be a one dimensional or it could be a 2 dimensional or it could be a n dimensional. Let me give a one simple example in which it is going to be a 2 dimensional; that means, I have a random variable x of t. That is going to be a x 1 of t comma x 2 of t in which x 1 of t is nothing, but the maximum temperature and x 2 of t could be minimum temperature, the maximum and minimum temperature possible of a place at any time t.

And this together is going to be a one random variable; that means, this is a random vector which consists of a 2 random variables x 1 of t and the x 2 of t; that means, for x for fixed t, you have a one random vector x of t. And therefore, you have a random vector for over the t and this random vector will form a stochastic process. Therefore, this is going to be a 2 dimensional stochastic process.

Therefore, in general, you can define a n dimensional stochastic process with for fixed for every t, you have a random vector x of t that is going to be a x 1 of t comma x 2 of t and so on. It is going to be the nth element is xn of t, that is going to be a n tuple in which each one is going to be a random variable for fixed t. And this is going to be a random vector for fixed t and this is going to be a n dimensional stochastic process in which each one is going to be a one dimensional random variable for fixed t.

That means, you can go for making a one dimensional random variable. Then you have a collection of random variable form a one dimensional stochastic process or you can have a 2 dimensional. Like that, you can have a n dimensional stochastic process. In the course, what we are going to discuss always it is a one dimensional stochastic process.

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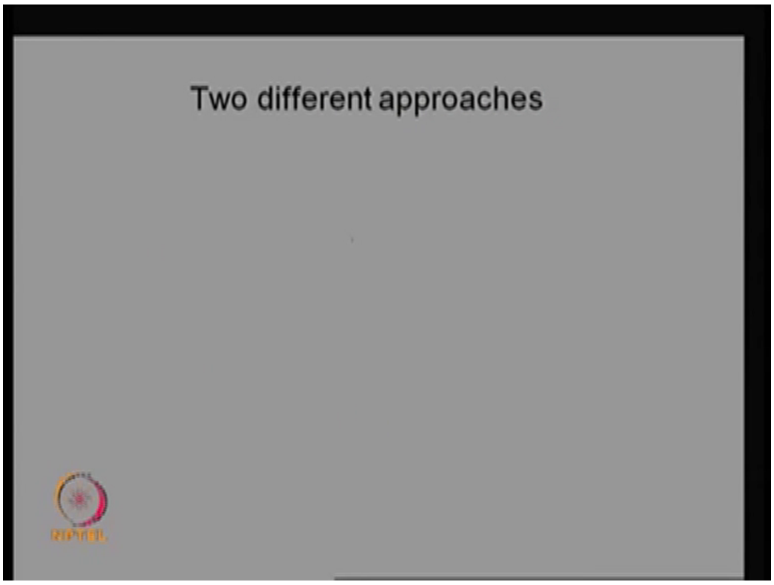

We can always create a complex valued stochastic process also in the form of x of t. Let me define it here. The x of t is going to be x 1 of t plus i times x 2 of t, where i is nothing, but the complex quantity square root of minus 1; that means, the x 1 of t is a real valued random variable for fixed t and x 2 of t is also a real valued random variable for fixed t.

The way I have made it, the x of t this is going to be a complex valued random variable for fixed t. Therefore, the x of t over the t that is going to be form a complex valued stochastic process. Because, for fixed t x of t is going to be a complex valued random variable, the corresponding stochastic process is called complex valued stochastic process in the one dimensional form. Like that, you can go for the multi dimensional complex valued stochastic process also.

But, in this course, we what we are discussing you only the real valued one dimensional random variable. Most of the times sometimes we are discussing real valued 2 dimensional or n dimensional stochastic process that do with the real valued random variable not the complex valued. So, now we are going for classification of a stochastic process.

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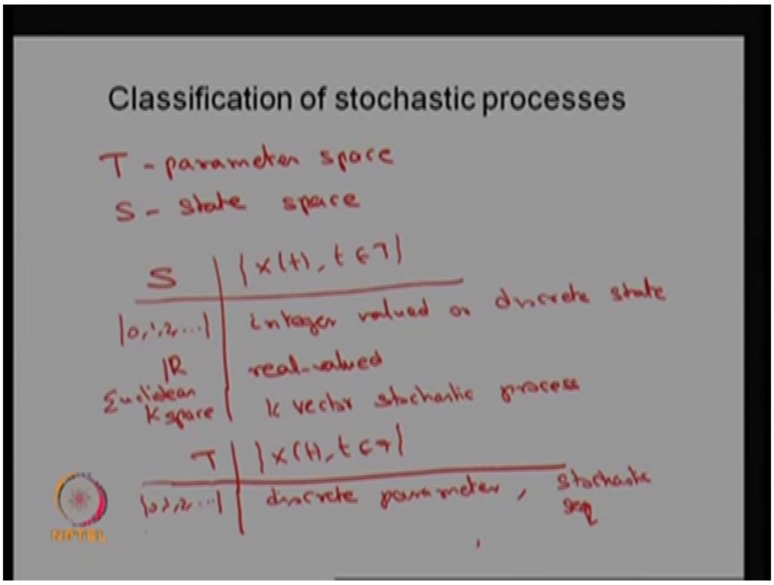

The way I have explained the parameter space capital T, the capital T is a parameter space and capital S is going to be the state space; that is nothing, but the collection of a possible values of x of t and the possible values of a small t belonging to capital T, that form a parameter space, some books they use the notation parameter set also and, capital S is going to be the state space.

Now, based on this, we are going to classify the stochastic process. Suppose, let us start with the capital S. Suppose, the possible values of S and what is the name of the stochastic process if S is going to take the only countably infinite or countably finite values, then it is going to be call it as the corresponding stochastic process is going to be call it as a integer valued stochastic process or we can call it as a discrete state stochastic process.

So, whenever the possible values of S is going to be a countably finite or countably infinite, then we say, it is a integer valued stochastic process or a discrete state stochastic process. Suppose, the possible values of S is going to be the real values, then we call it as a real valued stochastic process.

Suppose, if it takes Euclidean space with the k dimensional Euclidean k dimensional space, then we call it as a k; k vector space, k vector stochastic process; that means, the each random variable going to have a one dimensional random variable. And like that, you have k random variables for fixed t.

Therefore, you have a you have k vector stochastic process. Therefore, it is going to be call it as a k vector stochastic process in which each element is going to be a one dimensional random variable for fixed t.

So, the collection that the k tuple values stochastic process is going to be call it as a k vector stochastic process. Similarly, we can go for based on the capital T, what is the name of the stochastic process for different values of t; that means, if it is going to take the value countably finite or countably infinite or it is going to take only the integers values, then we say it is a discrete parameter stochastic process or we can there is a another name. It is called the stochastic sequence.

Also, whenever the possible values of a T is going to be a countably finite or countably infinite, then we call the corresponding stochastic process as the stochastic sequence or it is a discrete parameter stochastic process. Otherwise, if it takes a uncountably, many values in the T, then it is going to be call it as a continuous parameter and it is or it is going to be call it as a stochastic process itself.

Therefore, based on the discretized, it uses the word sequence or if it is going to be uncountably many values of capital T, then it is going to be call it as a stochastic process. So, based on the classification, I can go for making a one table in which the possible values of s will take a column and the possible values of capital T will take a row.

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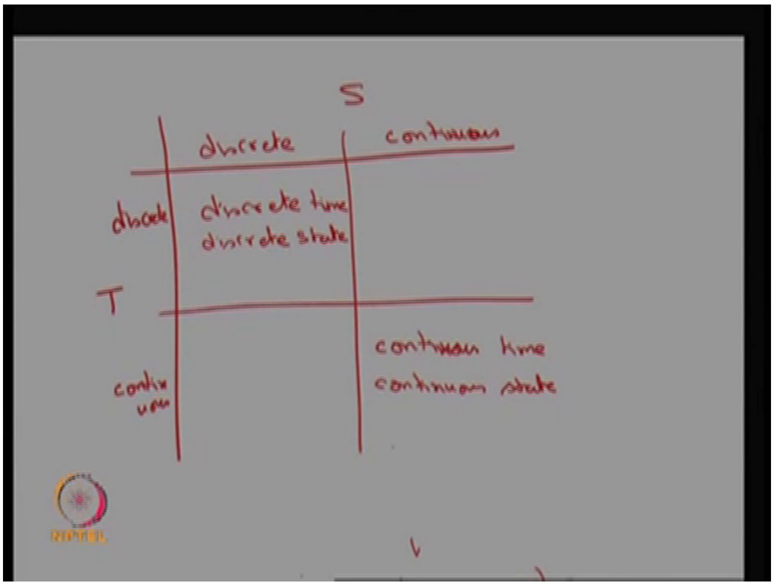

So, either it could be a either it could be a countably finite or countably infinite that I uses the word are discrete. If the possible values of T is going to be uncountably many. Either it is set of all intervals or it will be a whole real line itself or it is going to be a union of many intervals. In that case, it is going to be call it as a continuous a parameter.

Similarly, if the possible values of S is going to be a countably finite or countably infinite, then the state space is going to be call it as a discrete. Similarly, if it is going to be uncountably many values, then it is going to be call it as a continuous. So, accordingly, you can classify the stochastic process into the four type in which, if the T is going to be a discrete, as well as S is going to be a discrete, then it is going to be a discrete time discrete time or discrete parameter. Both are one and the same.

So, discrete time discrete state stochastic process. Similarly, if the t is a discrete and the state space is continuous, then we can call it as a discrete time continuous state stochastic process. Similarly, this is going to be a continuous time discrete state stochastic process and this is going to be a continuous time continuous time continuous state stochastic process; that means, the based on the possible values of a T and the possible values of S.

Any stochastic process can be classified into the four types in which it is going to be a discrete discrete or continuous continuous or discrete continuous continuous continuous based on their time and the state space.