Introduction to Probability Theory and Stochastic Processes

Prof. S. Dharmaraja
Department of Mathematics
Indian Institute of Technology, Delhi

Module - 09
Introduction to Stochastic processes (SPs)
Lecture – 48

In this model what we are going to discuss is stochastic process, then we are going to discuss the classification of a stochastic process followed by a few simple examples which arises in the real world problem.

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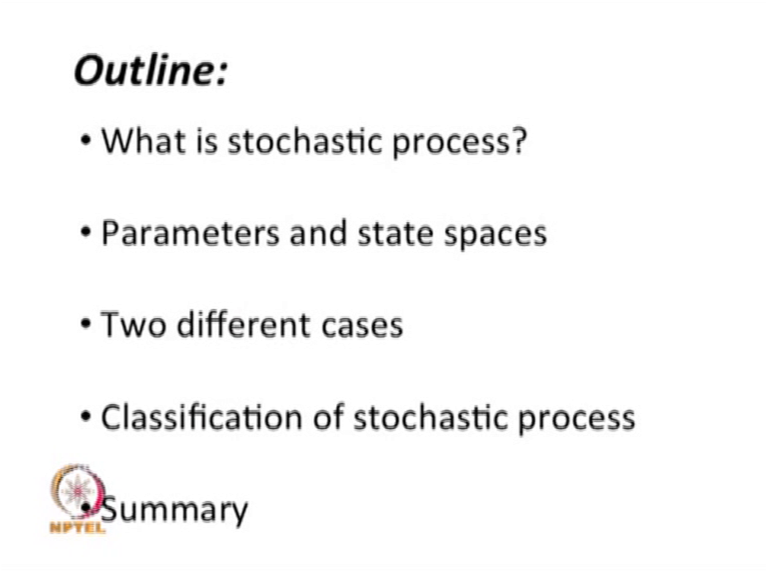

So, the content of this lecture is going to be as I said let me first give the definition of a stochastic process, then I will explain how to create or how to develop the stochastic process, and how to what is a meaning of a parameter and the state space.

Then I am going to give what are all the approaches in which the stochastic process can be described and the classification of a stochastic process based on the parameter and the state space. Then at the end of this lecture we are going to discuss the some of the few simple stochastic processes, and the summary of the lecture one, and there are few reference books also listed for this course to be it preparation. What is stochastic process?

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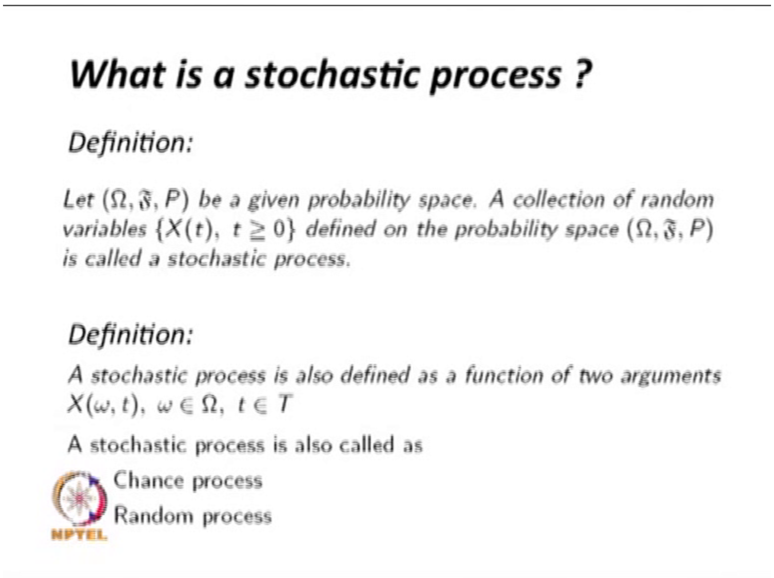

Let me give the definition let omega f P be a given probability space; that means, you know what is a random experiment, from the random experiment you know what is the omega. And from the collection of possible outcomes you got the sigma algebra that is capital F, and you have a probability measure also. Therefore, this triplet is going to be the probability space and you have a given probability space. From the given probability space, you have the collection of random variables. That is X of t, where t is belonging to capital T defined on the probability space.

That is omega f capital P, that is called a stochastic process; that means, you have a probability space, from the probability space you have collected a random variables with the t belonging to capital T, and this collection is going to be call it as a stochastic process.

Now, the question is whether we can create a only one stochastic process, or how to create a stochastic process from the sigma algebra; that means, suppose you have a omega from the omega you can always a create here sigma algebra that is a capital F. That is a collection of a subsets of omega satisfying the condition. If you make a union of a few elements, then they if you make the elements if you take a few elements then the union of elements is also belonging to one of the element. And if you take any one of the elements in the f then the complement is also belonging to f.

So, if these conditions are going to be satisfied, then that collection of a subsets of omega is going to be call it as sigma algebra. So, from the omega we have created a random variable that is X of t 1 that is nothing but a random variable of that is nothing but a real valued function, which is defined from omega to r such that it satisfies the condition X of t 1 of inverse of minus infinity to the closed interval X.

That is belonging to f for all X belonging to r; that means, whatever with a X belonging to r if the a inverse images from minus infinity to some point X, if that is belonging to capital F then that real valued function is going to be call it as a random variable. Like that if you make a different random variable for different t where all the t i's are belonging to. So, I can go for t i are i is so, all the t is are belonging to capital T.

So,; that means, if I have a collection of a random variables for the different values of t then that collection is going to be call it as a stochastic process. Now the question is whether we can create a only one stochastic process from a given probability space or more than one stochastic process can be created from the same probability space.

The answer is yes, you can always a create more than one random variable from the same probability space; that means, a for a different a collection of a capital T you can have a different stochastic process. More than one stochastic process can be created from one probability space.

Now, the next question if I change the sigma algebra what happens. If I change the sigma algebra capital F, then I may land up collecting some other stochastic process in which those real valued function is going to be a random variable for that that particular omega and f and P.

And the for a given probability space the stochastic process is going to be changed for a different collection of a t belonging to capital T; that means, once you know the f then you will have a some collection of first random variable that will form a stochastic process. If you change the another f, then you may get the different stochastic process. And also for a given probability space you can have a more than one stochastic process by the way you define a collection of random variable the way you have a capital T accordingly you will have a different stochastic process.

Now, the way I have given the collection of random variable, I can say it in a different way that is a stochastic process, is also defined as a function of 2 arguments; that is, X of w comma t where w is belonging to omega and t is belonging to capital T; that means, the same way I can define the collection of random variable as a collection of a w comma t, where w is belonging to omega and t belonging to capital T and this is also going to be form it as a stochastic process; that means, all always the w is belonging to omega; that means, the w is belonging to the possible outcomes and the t is belonging to capital T, and this is going to set the given probability this is going to set the stochastic process.

The other names for the stochastic process are going to be chance process. There are some authors, use the word chance process. There are some authors they use a notation that is called a random process. So, either the stochastic process can be called it as a chance process or the random process also. Now what we are going to see once you have a collection of random variable; So, based on the X offered the values of X of t, and the values of the different values of t, we are going to define what is a parameter space, and what is a state space. What is a meaning of a parameter space?

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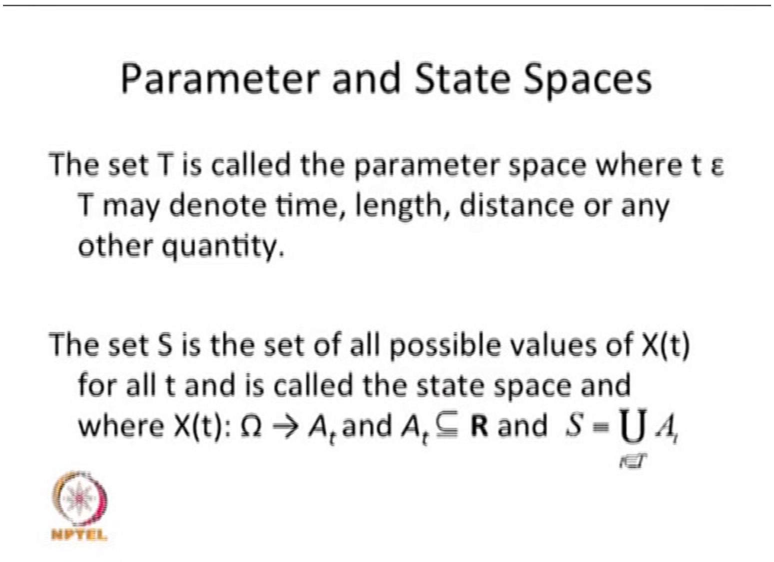

The set we use the notation capital T that is called the parameter space, the set capital T is called the parameter space, and it usually represented as the time most of the time or it can be represented as the length, or it can be represented as the distance and so on.

So, usually we go for a t as the time so, the set t is called the parameter space. Similarly, I can define the state space as the set capital S. That is nothing but all possible values of X of t for all t. So, this set is called the state space. X t is a random variable from omega into a suffix t, where A suffix t is a subset of capital R.

Then the At is are going to be the elements of it is going to be a contained in the real line, then the S is nothing but union of t belonging to capital T all the At is that is going to deform a state space; that means, a for a fixed t, you will have a collection of a possible values that is going to be the at and for variable t, you collect all the union and that possible values of X t is going to form a set and that set is called the state space.

Similarly, the all possible values of a small t belonging to capital T and that set is going to be call it as a parameter space. So, based on the parameter space and the state space, we can go for classification, now I can explain: what are all the possible values of S can take.

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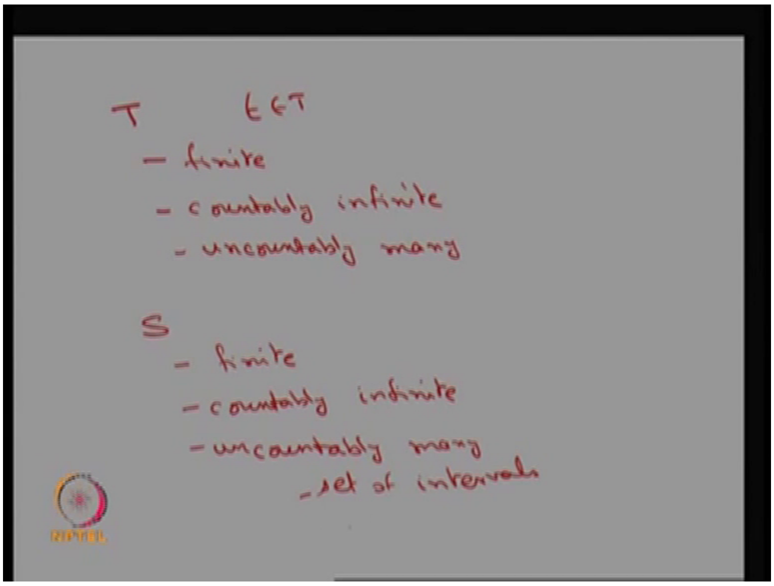

So, this T is going to be the collection of capital T, therefore, this can be a finite; that means, a countably finite, or it could be a countably infinite also. Or it could be uncountably many elements of a small t. So, that set can be a finite set or it could be countably infinite or it could be uncountably many elements also. T can also be multi-dimensional set.

Similarly, the state space a capital S, that can be a same way it could be a finite, or it could be a countably infinite, or it could be uncountably many elements. So, since the state space are going to be the collection of all possible values of X t, and X t is a real valued function and then it is going to be a random variable.

Therefore, these way elements are going to be always real numbers. So, either it could be a finite elements or it could be a countably infinite elements. And it is going to be uncountably many elements; that means, it could be a set of intervals on a real line or it could be a the whole real line itself.