**Introduction to Probability Theory and Stochastic Processes**
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**Module - 09**
**Introduction to Stochastic Processes (SPs)**
**Lecture – 47**

Our lecture is Stochastic Processes; so, before we move into the stochastic process, I am going to give what is a motivation behind the stochastic process.

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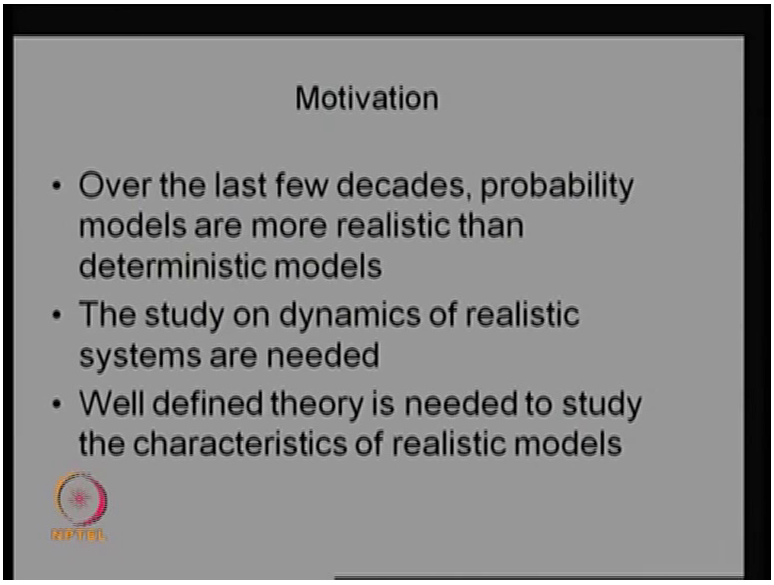


When we see the last few decades problems, more of the probability models are not the deterministic; that means, a you need more probability theory to understand the stochastic, to understand the system, than then only you can study the dynamics of the model. If you see the if you want to study the dynamics of the system, then you need more probability theory. So, the simple probability theory may not be enough to study the more study on the realistic system.

The way this realistic system behaves in a very dynamical way, it is not easy to capture everything through the probabilistic or usual probability models; that means, you need more than the probability model probability theory to understand the system or to study the system in a well behaved way.

For that the one of the important thing is stochastic process. It deals about the collection of a random variable so, that you can study the dynamic of the system in a better way. Even though I am giving very light way of saying the collection of random variable. First we should know how the random variable can be defined so, that you can study the collection of random variable in a better way.

So, for that we are going to spent few examples through that how the more realistic models needs more probability theory other than the usual probability theory so, that the stochastic processed definition and those things I am going to cover it later part.

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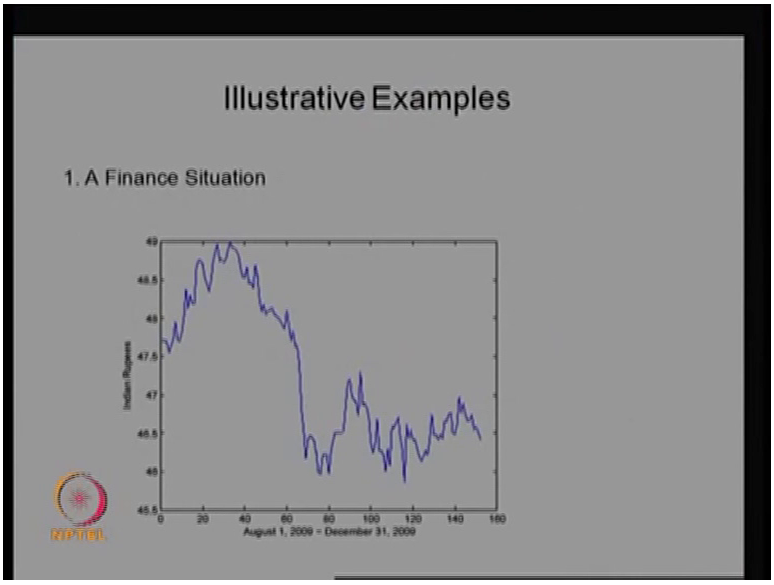


First let us see the first example that comes in the finance situation. This is the actual data which captured over the period of time from August 1 2009 to December 31st 2009; of what is a current price of the 1 U.S. dollars in Indian rupees. So, if you see the graph, you can make out August 1st 2009, the price of a 1 U.S. dollar was 47 rupees 57 or 58 paise.

And if you see the dynamics over the years, over the days from August 1st 2009 to till December 31st to 2009, it keep on changing and it takes some values higher and after that it goes down and it fluctuates and so on. So, this is the actual data which captured from the, which we have captured.

And from that our interest will be what could be the U.S. dollar price after some time. If I know till today what is the price, my interest will be what could be the price after 1 or 2 days or after 1 month or after 6 months. That means, I should know how the dynamics keep moving over the days, and what is the hidden probabilistic distribution is capturing over the time so, that I can identify what is a distribution behind that.

Therefore, I can study the feature production, I can study the dynamics of the this particular model in a much better way; that means, I need what is the background or what is the hidden distribution playing or hidden distribution which causes the dynamics of the system. After identifying what is the distribution my interest could be what could be the some other moment over the time. That means, what could be the average value or what could be the second order moment if it exist and so on; that can be obtain if I know the actual distribution the under the in the underline model.

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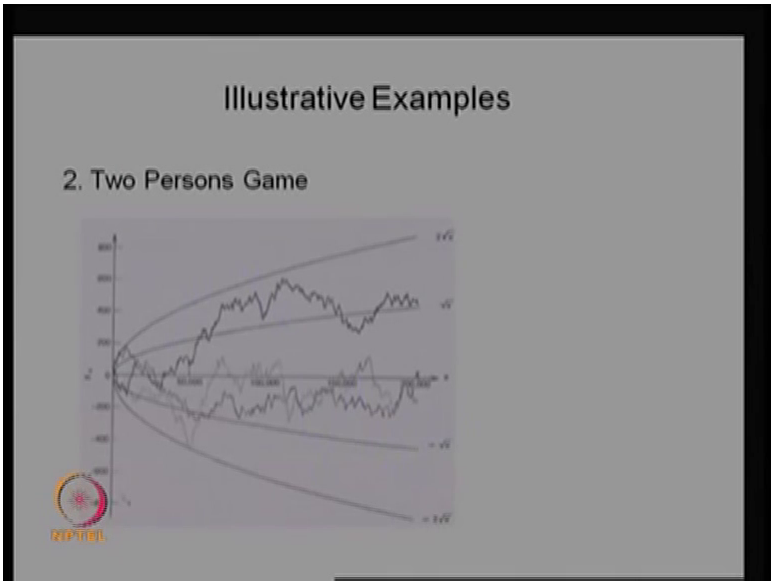


If I see the second example and just changing into the another model in which there are 2 people playing again the person A and person B. Whenever the person A wins, he gets the rupees 1.

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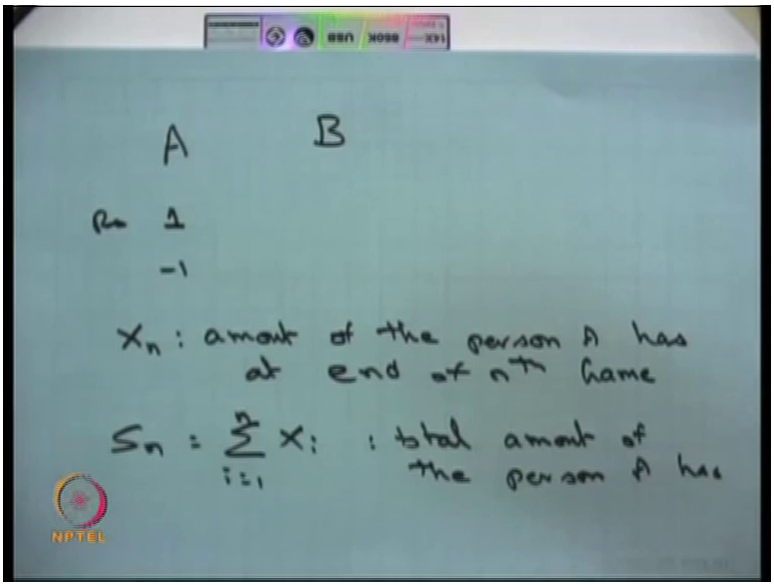


Suppose, the person B wins then he will get the 1 rupee, and at the same time the person A loses 1 rupee the same way. And the play is keep going suppose, you make the random variable as X n is the amount of the person A has at the end of n th game. If you make out the random variable X n for the person A has the amount at the end of the nth game, then at the n the way the game going on the value of the X n will be keep changing. And if you make out another random variable X n is the sum of X i's where i is running from 1 to n, this gives what is the total amount, the total amount of the person A has.

The diagram in which the S n gives what is the way the dynamics goes and over the n. And if you see the diagram, you can make out the whole dynamics goes; the how the game is going on in the first few games, accordingly it changes the positive side or it goes to the negative side. And if the n is goes large then the dynamics of the S n over the n it will be keep changing over the time, and you will get the realization of the S n over the time. And here I have given three different realization and this diagram is a taken out from the book by U. N. Bhatt, the title of the book is elements of applied a stochastic process.

So, this is the one of the motivations behind the stochastic process. And from this our interest will be after the, what is the distribution of S n at any n. And also as n tends to infinity or could be the distribution of S n; that means, you need the distribution of the random variable, and also you need a what could be the distribution has n tends to

infinity or the limiting distribution of S n. If you know the distribution then you can get all other movements as far different n as well as the asymptotic behavior of a the random variable S n.

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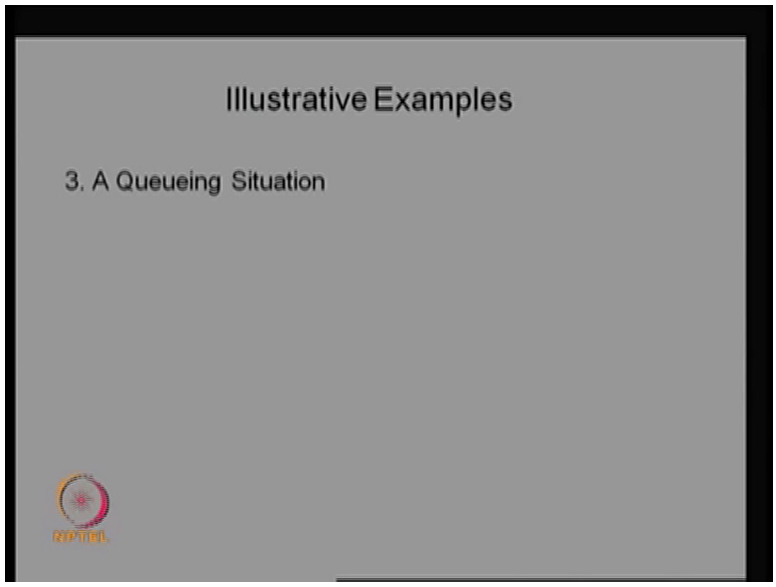


Next I will move into the, another example in which it is the queueing situation. The queueing situation here I have taken it as taken a simple example that is a barber shop.

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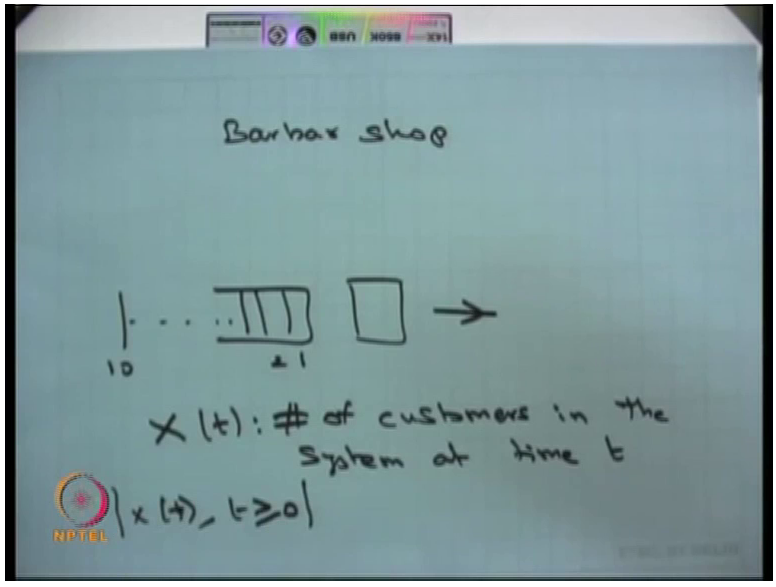


Example in which there is a only one barber shop person, and who does the service for the people who ever entering into the barber shop. And there are only a limiting capacity

in which there is a maximum 10 people can stay in their barber shop and one person will be under service.

Once the service is over and the system will be the customer can leave the system. At any time maximum 10 people can be in the barber shop. And only one person is doing the service for the customers who ever enter into the barber shop. Suppose, you take the random variable as X t is the number of customers in the barber shop or in the system. A time t, the way the dynamics goes the possible values of X t will be starting from 0 to n.

To study the system, you need what is the way the people or the customers are entering into the system. And what is the way the service is going on for the customers. And what is the discipline in which the customers are getting served also. Our interest will be suppose we have the capacity of 10, what could be the waiting time whenever the customers are entering into the system. My interest will be one is how to reduce the waiting time on average in the customers, this is the customers point of view.

As the barbershop point of view how much I can get the more revenue; that means, how I can increase the capacity of the system so, that I can make a more profit over the time. That means, if I know the dynamics of the X t over the t for t is varying from 0 to infinity, I can understand the system over that time as well as I can whatever the probabilistic measures or whatever, the other measures average number of customers or average waiting time and so on, I can find out using this type of random variable. So, later we are going to say this is going to be a one of the stochastic process for this example. Next I am going to consider the 4th example has the telecommunication system.

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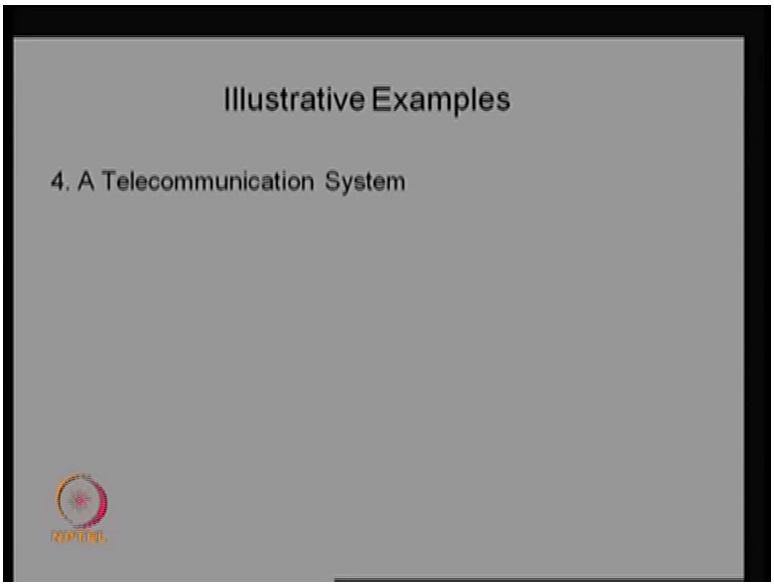


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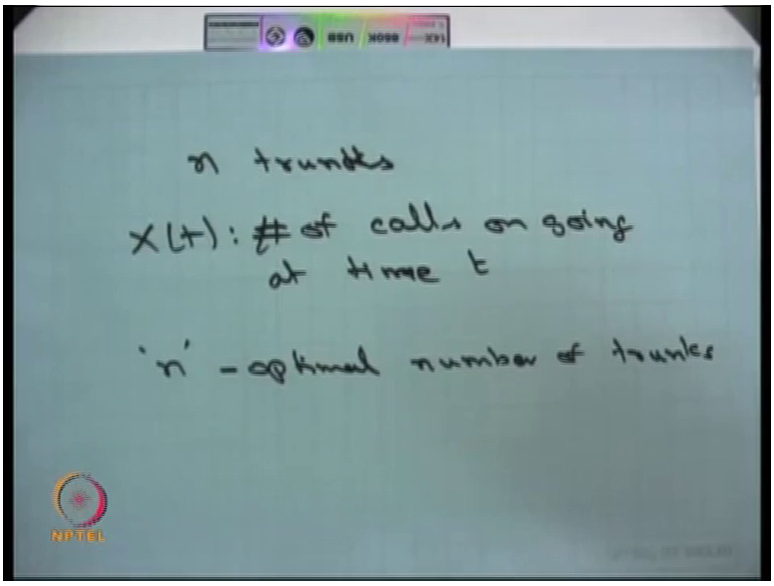


Suppose you think of a system in which you have a n trunks are there. Trunks are nothing but it is a maximum number of a calls will be allowed at any time. Whenever, a call entering into the system and you have given one trunk to the call, and at the end of the call is over the trunk will be back. So, you have a telecommunication system in which n trunks are available at a any; not at any time n trunks are available. Suppose I make a random variable X t as the number of calls on going at time t, here also the dynamics of X t is going to be keep changing from 0 to small n over that time.

And my interest will be how I can do the service such a way that the more calls will be entertained. As well as how I can find out the optimal n such a way that what is the optimal number of trunks; such that I can minimize the waiting time or I can maximize the revenue. So, this is also one of the problems which we come across in the use a daily life and so on. So, my interest is to introduce the stochastic process so, that I can study this type of system in a better.