Introduction to Probability Theory and Stochastic Processes

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Lecture - 04

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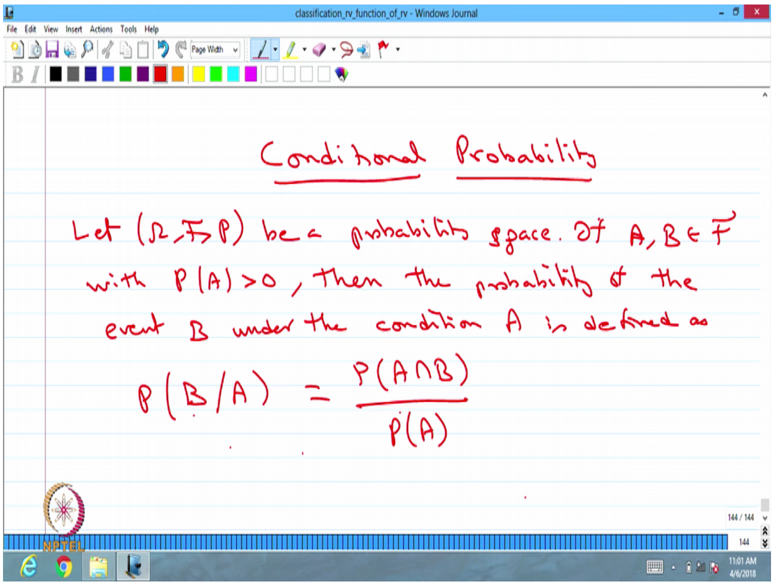

Now, we will move into the next topic that is called conditional probability. Let omega capital F capital P be a probability space, we are not making any assumption over this probability space. If a comma B belonging to F both are events with the P of a is greater than 0; that means, it is not impossible events impossible events means the probability with that event is equal to 0 and, sure event means the probability of a that event is equal to 1, rare event means the probability of a that event is open interval between 0 to 1 so, here it is a non impossible event.

Then the probability of the event B, under the condition the event A, that is defined as B of event B slash event A that is same as P of A intersection B divided by P of A, this is well defined, because the probability of a is strictly greater than 0.

So, this condition probability is going to be probability of B given A is same as probability of A intersection B divided by probability of A, this is called the conditional probability of event B given event A few properties on the conditional probability, but before that let me give the definition of independent events, then I will come to the results on the conditional probability.

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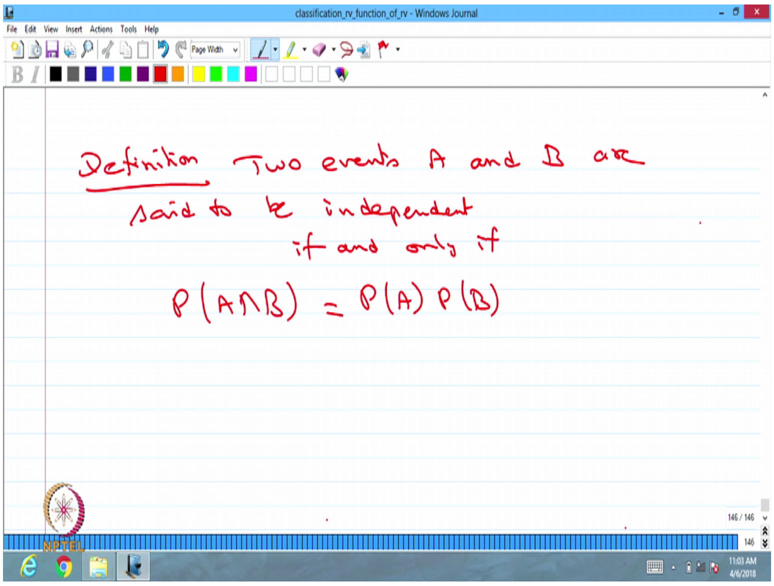

The first definition two events A and B are said to be independent if, and only if the intersection probability is same as the product of the individual probabilities.

We say if and only if condition, whenever this condition is satisfied by any two events, then we call it as both the events are independent. If two events are independent, then this condition will be satisfied, based on this I am going to give a two definition, we always called pair wise independent and other one is called mutual independent.

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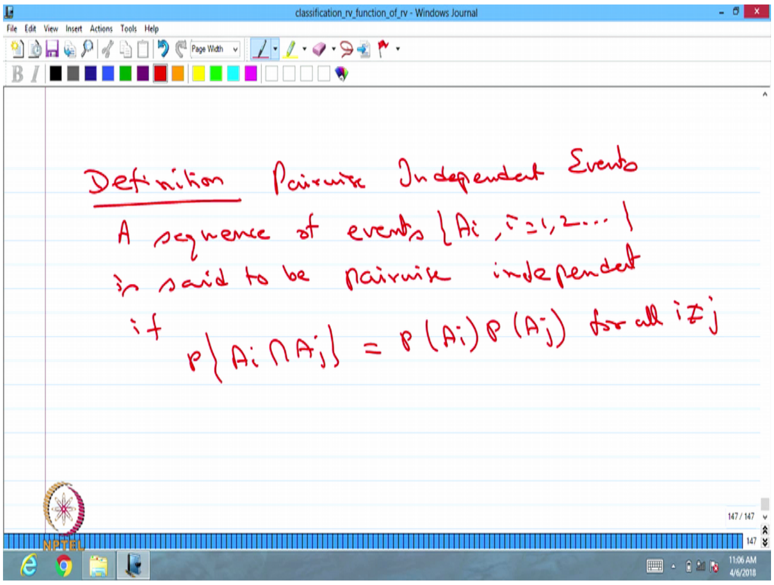

Then its definition that is a pair wise independent events A sequence of A events A I, where i is equal to 1 2 so on is said to be pair wise independent. If you take any two events that satisfies the independent property, that is a P of A i intersection A j that is same as P of A i into P of A j, for all i i is not equal to j.

So, if you take any two events that satisfies the independent condition, then we conclude this sequence of events or this collection of events are called pairwise independent, if it satisfies the independent condition.

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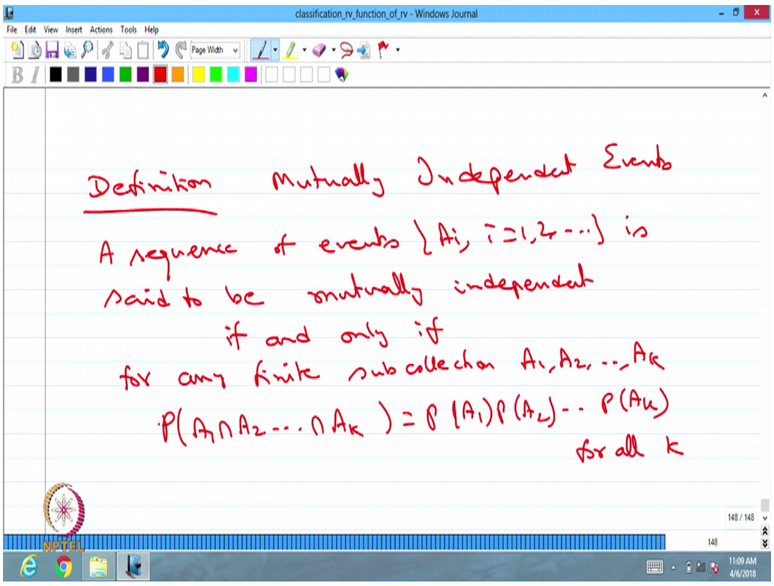

The next definition, that is mutually independent events, A sequence of events, A i's is said to be said to be mutually independent, mutually independent. If and only if for any finite, for any finite sub collection that is a A 1 A 2 so, on A k.

The probability of A 1 intersection A 2 and so, on intersection A k that is same as P of A 1 multiplied by P of A 2 and so on, P of A k for all k, that is very important that means, if you take a k equal to 2, so if you take any 2 events in this collection that satisfies independent property, or independent condition. Or, if you take any 3 events that also satisfies the independent condition, like that if you take all the possible number of events, that is the sub collection from that collection of events, satisfies the independent property. Then we conclude: they are mutually independent events sometimes, we would not use the word mutually, if you say that a few events are independent; that means, by default they are mutually independent events.

You see the previous definition that is the pairwise independent events, but that is satisfied only for any two events not 3 or 4 or 5 independence properties not going to be satisfied; that means, the mutual independent events implies their pairwise independent, but the pairwise independent need not imply the mutual independent events, for the collection of a events.

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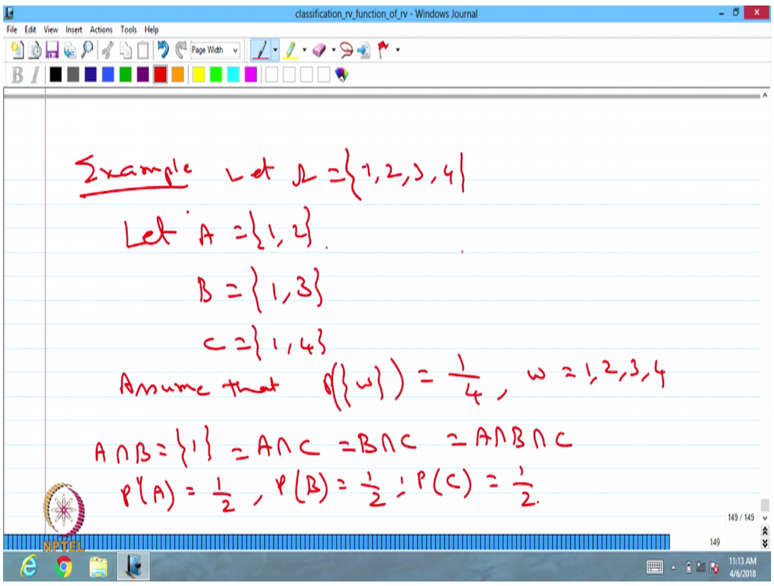

So, let us go for one simple example for pairwise independent and mutual independent, then we will move into the properties on the condition property, so the example as follows. Let omega consisting of 1 2 3 4. Let A be the event consisting of only two elements 1 and 2.

Similarly P is the element consisting of 1 and 3 and C is the element consisting of 1 and 4. And we assume that we assume that the probability of singleton element w that is equal to 1 divided by 4, for w is equal to either 1 or 2 or 3 or 4.

Now, we will verify whether this event satisfies the pairwise independent as well as mutual independent, or only satisfies pairwise independent not the mutual independent. Let us go for finding A intersection B that is nothing, but the sample one. And similarly if you go for A intersection C that is also singleton element 1.

If you go for A intersection B intersection C that is also single element 1, where as you compute the P of A that is 2 by four that is 1 by 2 and P of B that is also 2 by 4, that is 1 by 2 and P of C P of C, that is also again 2 by 4 that is 1 by 2.

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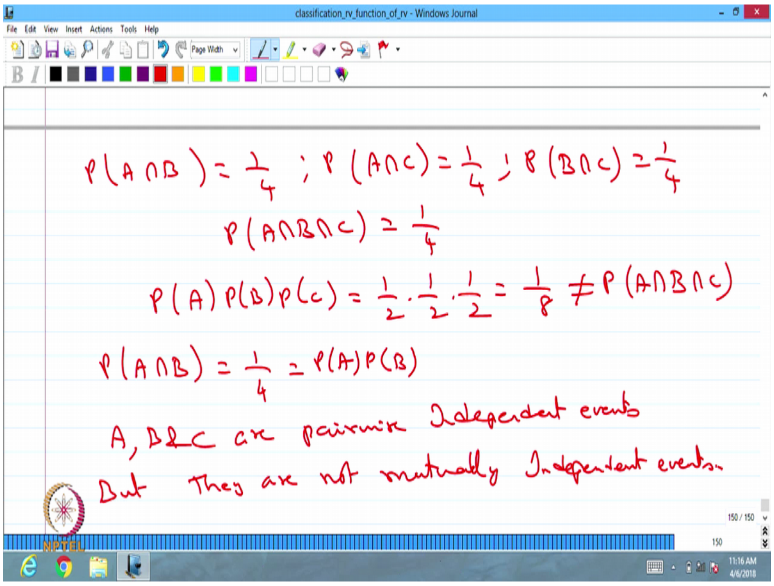

Now, if you compute P of A intersection B that is also 1 by 4. Similarly if you compute B of A intersection C that is also 1 by 4; similarly, P of A intersection C that is also 1 by 4 whereas, P of A intersection B intersection C that is also 1 by 4.

For example if you test P of A into P of B into P of C that is going to be 1 by 2 into 1 by 2 into 1 by 2 that is 1 by 8, which is not same as P of a intersection B intersection C.

Whereas A if you go for P A intersection B that is same as 1 by 4, that is same as a P of a into P of B.

Similarly, probability of a intersection C that is same as P of A into P of C. Similarly P of B intersection C that is same as B into P of C that means, it satisfies the independent condition for any two events not for the all 3 events that means, in this example A B and C, or pair wise independent events.

But they are not mutually independent events, because it does not satisfies independent property for three events. So, this very simple example conclude pairwise does not imply the mutually independent events in general.

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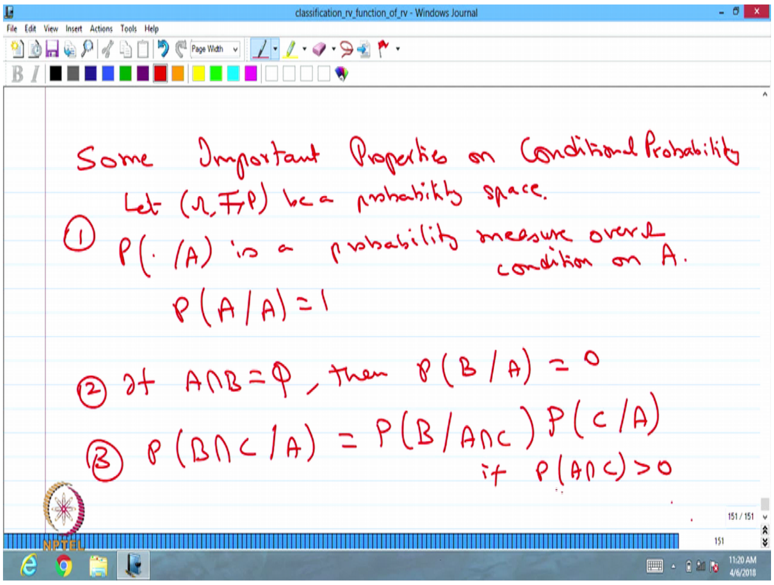

Now, let us go for some properties on condition property, some important properties on conditional probability. The first one so, before that we have probability space, let omega capital F P P F probability space, in that probability space we going to give the properties the first one, P of the conditional probability that is probability measure.

The condition probability that is also a probability measure over omega condition on the event A that means, it satisfies the axiomatic properties of the probability, that is a three properties P of A's is are always greater than or equal to 0 for all A belonging to F. And P of empty set P of whole set is 1 and, mutually disjoint events P of union is same as a submission of P of S.

The same thing here also satisfied here, the P of A given A that is going to be 1, where as in the axiomatic definition probability the P of omega is 1 here, the condition probability that is also probability space here P of A given A that is equal to 1. Second important property that is if A intersection B is empty set, then the condition probability of event B given A that is doing to be because same as probability of B intersection A divided by probability of A.

Since, A intersection B is empty set P of empty set is 0 therefore, probability of B given a that is equal to 0, third result or third property P of B intersection C given A, that is same as the conditional probability of event B, given A intersection C multiplied by P of event C given A. If probability of A intersection C is larger than 0.

Whenever you want to compute a probability of B interaction C given A, that can be computed in the form of probability of B given a intersection C multiplied by P of that is multiplied by probability of event to C given A, whenever probability of A intersection C is greater than 0.

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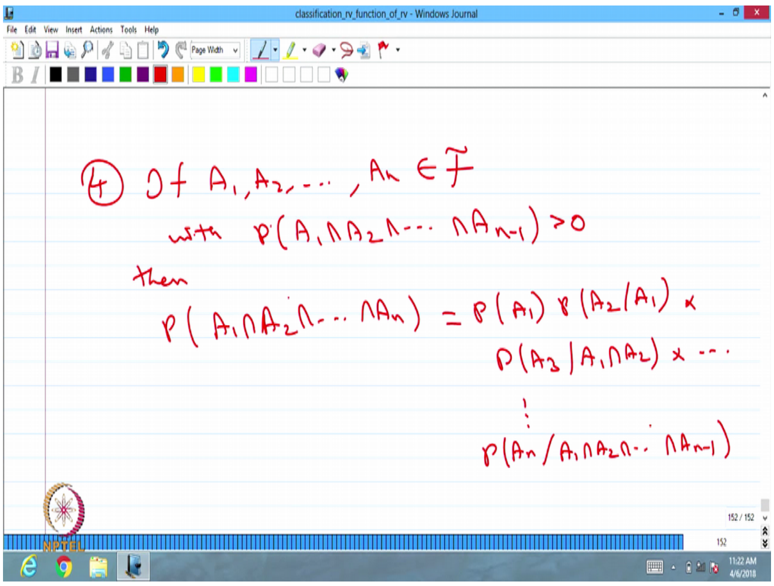

The next result related to the conditional probability. If A 1 A 2 and so on, A n n events belonging to F with probability A 1 intersection A 2 intersection, A n minus 1, if that probability is t larger than 0.

Then one can define probability of intersection of n events the assumption is A intersection of A n minus 1 events probabilities greater than 0, we have n events belong F. Then one can find the probability of intersection of n event is same as probability of A 1 multiplied by probability of A 2, given A 1 multiplied by probability of A 3 given A 1 interaction A 2, multiplied by the same way. The last element is probability of A n given A 1 intersection A 2 intersection so on, intersection A n minus 1.

This is possible whenever you have A n events and, the probability of intersection of n events provided probability of intersection of n minus events is greater than 0, one can always find the product of condition probability with the interaction of the events. This is valid for any countable number of events satisfying this condition, there no assumption over the events to apply this result.

Now, suppose these events are mutually independent, or mutually independent events, then the results are going to be day simplified as follows.

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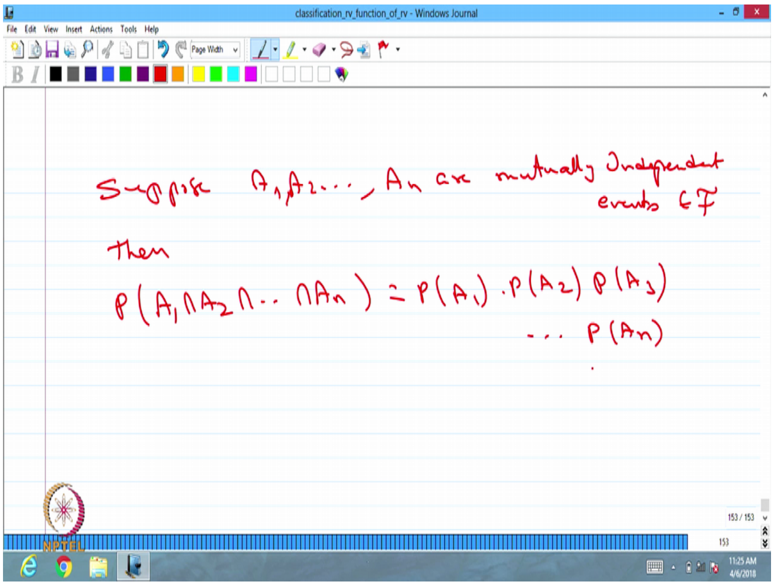

That means, suppose A 1, A 2 A n are mutually independent events, all are belonging to F suppose, this events are mutually independent event belonging to F.

Then the P of A 1 intersection A 2 intersection and so on, A n that is same as, we are retaining the other condition intersection of P of A n minus 1 n minus 1 events that is probability is greater than 0, we are keeping that in P of A 1, intersection A 2, intersection

A n. That is same as you see the previous one, first one is P of A 1, the second 1 P of A 2 given A 1. Since A 1 and A 2 and A n's are mutually independent events the intersection.

The condition probability A 2 given A 1 that is going to be a probability upon A 2 therefore, this is going to be probability of A 2. Similarly, probability of A 2 given A 1 interaction A 2 that is again to be probability A 3 and so on; the last one it is P of A that means, if events are mutually independent, then the intersection probability same as the product of individual probabilities. So, this can be proved also; so I am just using that result.