Introduction to Probability Theory and Stochastic Processes
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Lecture - 39

Now, we discuss the correlation coefficient.

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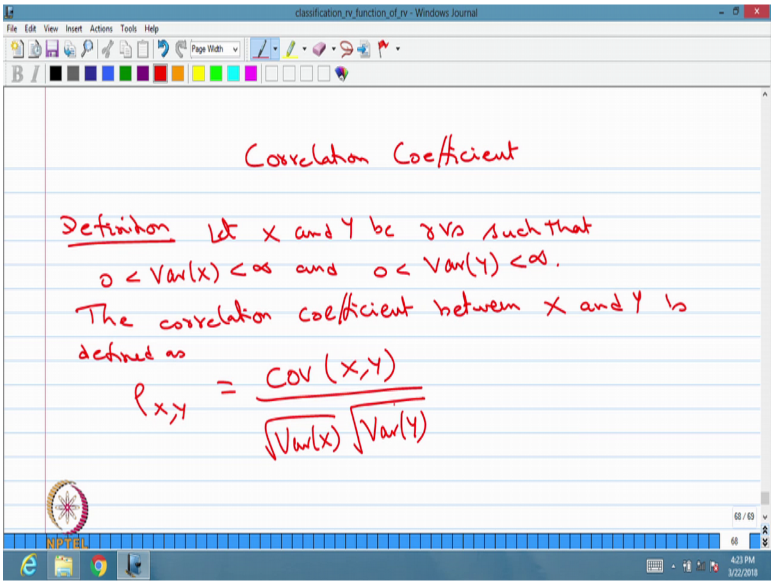

Let me start with the definition correlation coefficient definition, let X and Y be random variables such that 0 less than variance of X is less than finite and 0 less than variance of Y less than finite quantity. That means, the variance exist which is not equal to 0 for both the random variables X and Y.

Then the correlation, then the correlation coefficient between the random variables X and Y, is defined as with the notation rho, you can use the notation suffix, or we do not need if the suffix is there. That means, these two random variable if the suffix is not there the underlined random variables are two random variables, the correlation coefficient that is same as covariance between the same two random variables, divided by the positive square root of variance of X multiplied by positive square root of variance of Y. Since, this two con comes in the denominator, we are making the restriction the variance has to be strictly greater than 0 for both X and Y.

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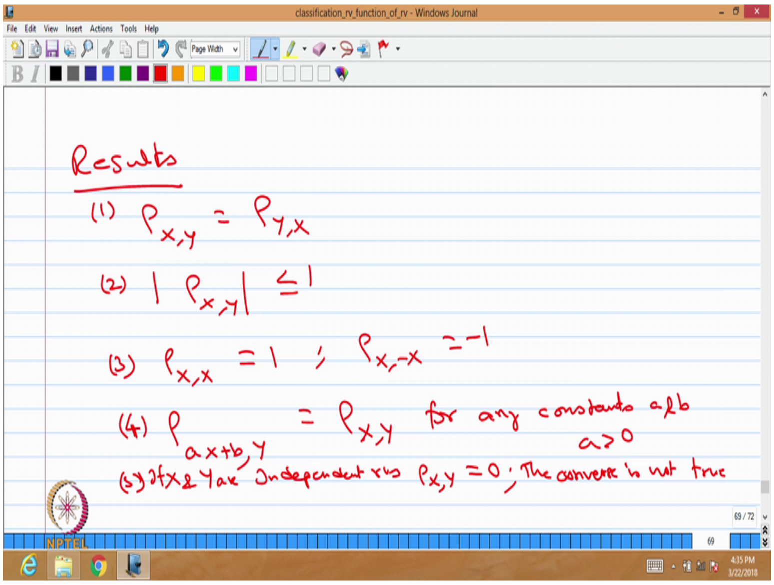

This value is going to be call it as a correlation coefficient between the random variables X and Y, I am going to give few results over the correlation coefficient. The first result the correlation between X comma Y, that is same as correlation between Y comma X by interchanging the role of X and Y it is going to be the same value. Since, the value is divided by covariance divided by square root of variance of X square root of variance of Y, the absolute value of correlation coefficient is always less than or equal to 1; that means, the value is lies between minus 1 to 1.

Third result if you compute the correlation coefficient between the same random variables between same random variables. That means, covariance of X comma X that is nothing, but the variance of X and the denominator it becomes a variance of X. So, you will get the answer 1; that means a correlation coefficient between the same random variable that becomes 1. Suppose you go for correlation between X with minus X, then that quantity is going to be minus 1.

The fourth result that is the correlation coefficient of a X plus b, where a and b are constant with the random variable Y that is going to be if we use the results of covariance between any two random variable, when 1 random variable is a X plus b the other one is Y use that result, you will get it is correlation coefficient of X comma Y for any constants a comma b with a is greater than 0 this can be proved. But we are not giving the proof of this result, but this results will be used again and again.

As a fifth result suppose the random variable X and Y are independent, if random variable X and Y are independent, then we can conclude the correlation coefficient between these two random variables is going to be 0. Because, the correlation coefficient is same as covariance divided by positive square root of variants positive square root of variance of Y and, we know that if two random variables are independent the covariance of X comma Y is going to be 0. Therefore, the correlation coefficient is also going to be 0, when two random variables are independent.

Whereas, the converse is not true the converse is not true that means, if the correlation coefficient is 0, for two random variables that does not imply those two random variables are independent.

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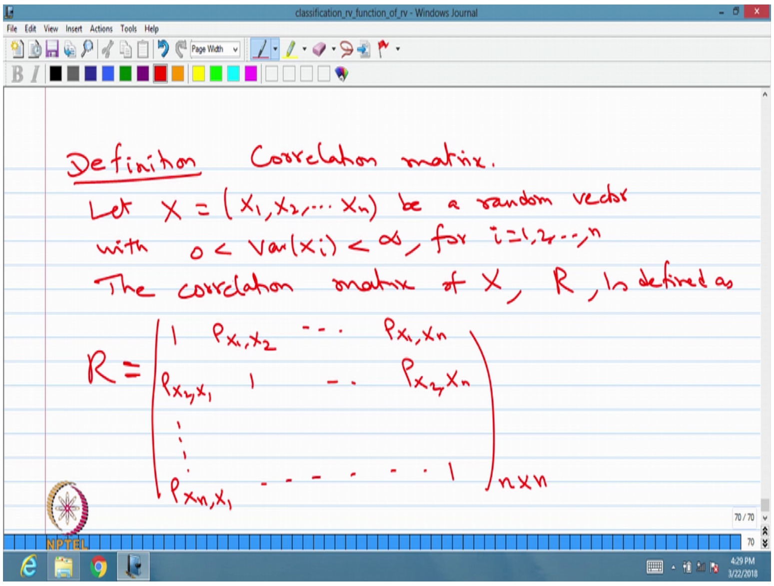

Now, we are going to define the matrix related to the correlation coefficient that is, now we are going to define a matrix which is related to the correlation coefficient that is correlation matrix. Let capital X is a random vector, whose elements are X 1, X 2 and so on, X n be a random vector with individual variance of the random variable has to be not equal to 0 and it is a finite quantity, for i is equal to 1 2 and so on, all n random variables.

Then the correlation matrix of the random vector capital X, which is denoted by the matrix called capital R that is defined as capital R earlier. We have used the notation sum that is for the covariance variance matrix and here, we are using capital R for correlation matrix, whose elements are the diagonal element is 1 and, the second element is first row second element is correlation coefficient between X 1 with X 2 and so on. The first row last element that is n-th element is correlation coefficient between X 1 with X n. Similarly the second row that is a correlation coefficient between X 2 and X 1 and second row second element that is 1 and so on. The second row n-th column that is correlation coefficient between X 2 with X n, like that you can fill up n rows.

So, the n-th row first column that is correlation coefficient between X n with X 1 like that the last element will be 1, again this is a n cross n order matrix, whose diagonal elements are 1 and other elements are for the i-th row j-th column that is a correlation coefficient between X i and X j.

We will go for the example, but we will take the example what we have discussed for the covariance variance matrix. We will take the same example and find out the R matrix.

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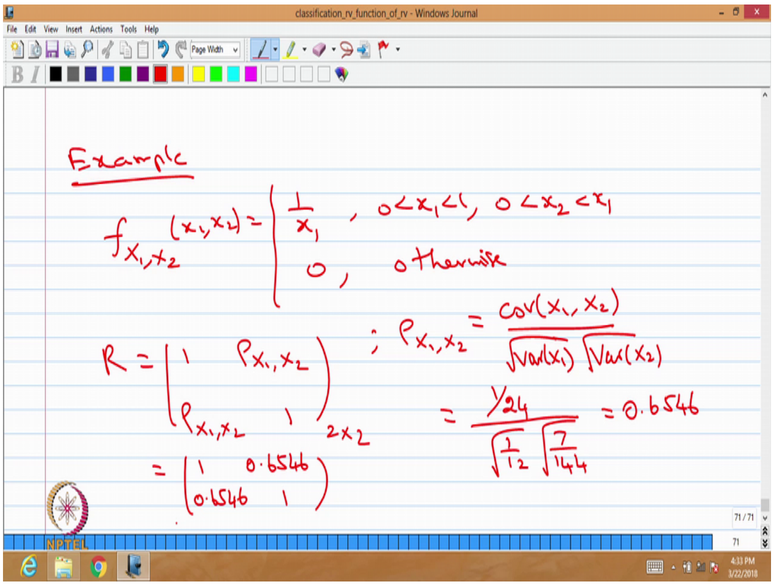

That is example is same as two dimensional random variable with the joint probability density function is 1 divided by x 1, when x 1 takes a value 0 to 1 and x 2 takes a value 0 to x 1 0. Otherwise we have already computed we have already computed the expectation of X 1 and expectation of X 2 expectation of X 1 square expectation of X 2 square, then expectation of X 1, X 2 using that we got the covariance between X 1, X 2 also and variance of X 1 as well as variance of X 2.

Therefore in the same example in the R matrix is going to be 2 cross 2 matrix 1 and the second element is correlation coefficient between X 1 with X 2 and this is correlation coefficient between again X 2 with X 1, that is same as X 1 with X 2, because of symmetric and last element is 1 that is 2 cross 2 matrix. So, we have to compute what is the correlation coefficient between X 1, X 2.

So, correlation between X 1 comma X 2, that is covariance between X 1 with X 2 divided by square root of variance of X 1 multiplied by square root of positive square root of variance of X 2. So, we already got the covariance of X 1 with X 2 that is 1 divided by 24. Therefore, it is 1 divided by 24 divided by variance of X 1 is 1 divided by 12.

Therefore, it is 1 divided by 12 variance of X 2 is 7 divided by 144. Therefore, it is 7 divided by 144, you do the simplification you will get the answer that is 0.6546. Therefore, this R matrix is going to be 1 0.6546 06546 1 that is going to be the correlation matrix for the same example which we have discussed earlier.