Introduction to Probability Theory and Stochastic Processes

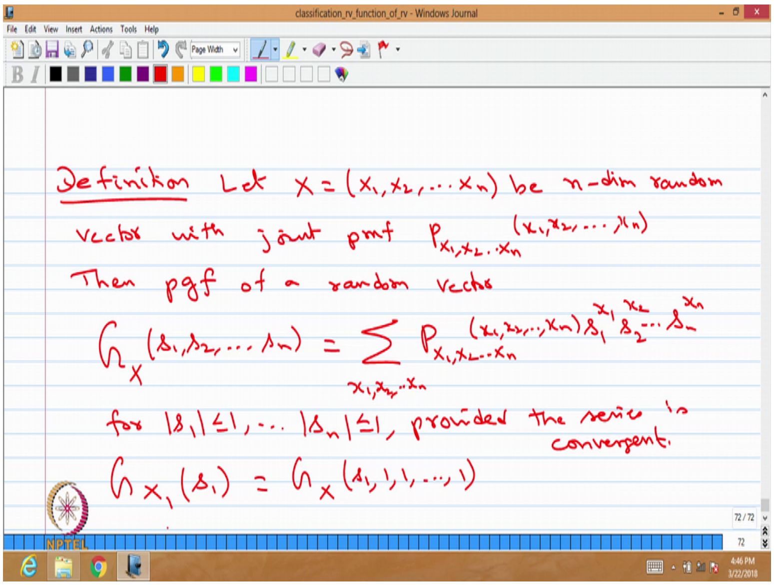
Prof. S. Dharmaraja
Department of Mathematics
Indian Institute of Technology, Delhi

Lecture – 38

In Cross Moments, we will discuss the generating function for the random vector. Earlier we have discussed generating functions for the random variable that is a probability generating function for a random variable, then moment generating function for the random variable, then we have discussed the characteristic function for the random variable.

Now, we are going to discuss these generating functions for the random vector that is the first definition.

(Refer Slide Time: 00:40)



Let capital X be a random vector whose elements are X 1 X 2 and so, on X n be a n dimensional random vector with joint probability mass function that is P of X 1, X 2, X n as a function of x 1, x 2, x n. Then one can define the probability generating function of a random vector is defined as we use a notation G suffix X here the X is a vector whose elements are s 1, s 2, sn.

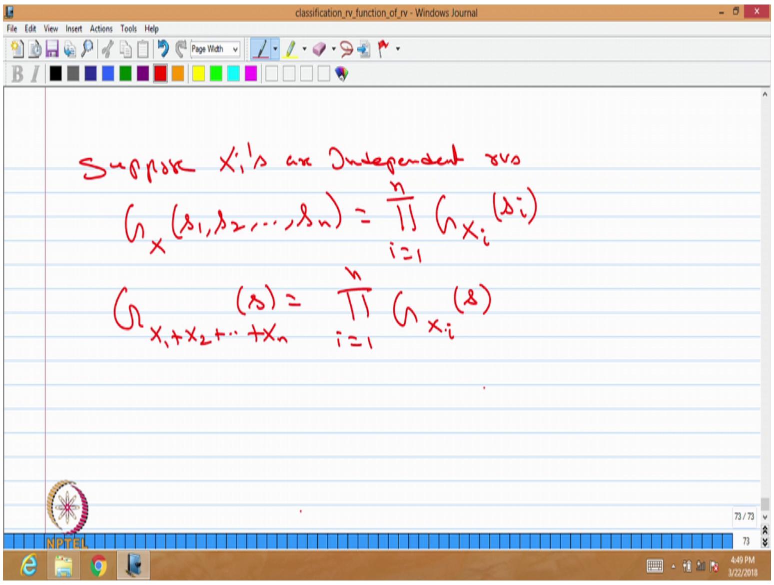
We have defined the earlier the probability generating function of a random variable in that case we have G suffix X as a function of s, now we have a random vector therefore, it is a function of the n variants s 1, s 2, sn that is same as summation of joint probability mass function of X 1, X 2, X n as a function of x 1, x 2, x n multiplied by s 1 power x 1 s 2 power x 2 and so, on sn power xn and here the summation is over all the X i's without restriction.

So, this is going to be a function of s 1, s 2, s n; this is called the probability generating function of a random vector. And this is for absolute of s 1 is less than or equal to 1 and so, on till absolute of sn is less than or equal to 1 provided; provided the series the right hand side series is convergent . So, as long as within the interval of absolute of s 1 is less than or equal to 1 and so, on, absolute of sn is less than or equal to 1. The right hand size series converges then that is going to be call it as a probability generating function of a random vector.

From the probability generating function of a random vector, one can get the probability generating function of any one random variable also. That is suppose I want to find out the probability generating function of a random variable X 1; that is same as by substituting the probability generating function of a random vector in s 1 you keep it s 1; whereas, s 2 onwards you substitute the value 1 till 1, you will get the probability generating function of a random variable x 1.

Similarly, you can get the probability generating function of other random variables; in this way you can get the probability generating function of fewer random vector also by substituting all other variables as 1.

(Refer Slide Time: 05:14)

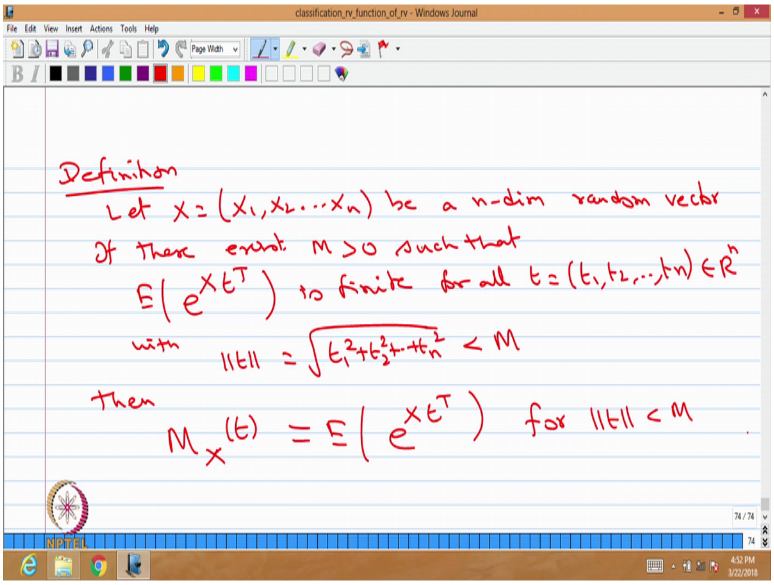


The next result suppose X i's are independent random vectors; suppose the random variables are independent, then you can replace the joint probability density function by replace joint probability mass function by probability mass functions of X i's; since they are independent random variable. Therefore, this summation will simplified into the probability generating function of a random vector as a function of s 1, s 2 and so, on. That is going to be product of probability generating function of each random variable from i is equal to 1 to n.

Because the joint probability mass function can be replaced by the product of probability mass functions of individual random variable. So, the right hand side G suffix X i that is a probability generating function of the random variable X i. In this way, sometimes we may be interested to find out the probability generating function of sum of random variables as a function of s; that is same as if you apply the same concept that is same as

Since they are independent random variable that is going to be the product of i is equal to 1 to n, the probability generating function of individual random variable with the variable s. When X i's are independent random variable the first one is a probability generating function of a random vector, the second expression is probability generating function of sum of random variables that is a one random variable with the variable s that is same as the product of probability generating functions of individual random variable.

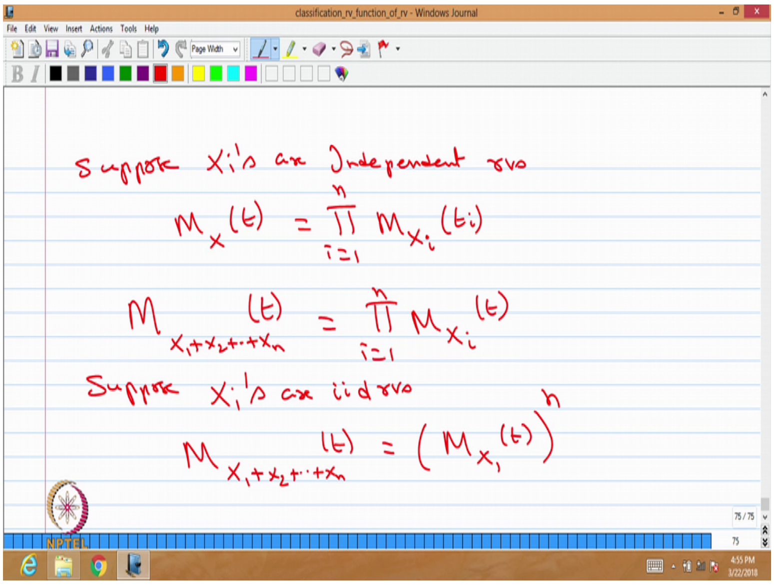
(Refer Slide Time: 07:51)



We will go further second definition that is moment generating function of a random vector. Let X be a vector whose elements are X 1, X 2, X n be a n dimensional random vector; if there exist there exist M which is greater than 0 such that the expectation of exponential of random vector X multiplied by t, transpose that is finite for all t, where t is a vector whose elements are t 1, t 2 and so, on; that is belonging to R n with the num of t is defined as it is a gradient norm that is t 1 square plus t 2 square and so, on plus tn square and that quantity is going to be less than capital M.

If this condition is satisfied, then one can define the joint moment generating function of a random vector with the notation M suffix X; X is a vector small t that is also vector whose elements are t 1, t 2; t 2 that is nothing, but expectation of E power exponential of the vector X, vector t transpose and this result is valid only for num of t is less than capital M. So, this is going to be called as a joint moment generating function of a random vector.

(Refer Slide Time: 10:51)

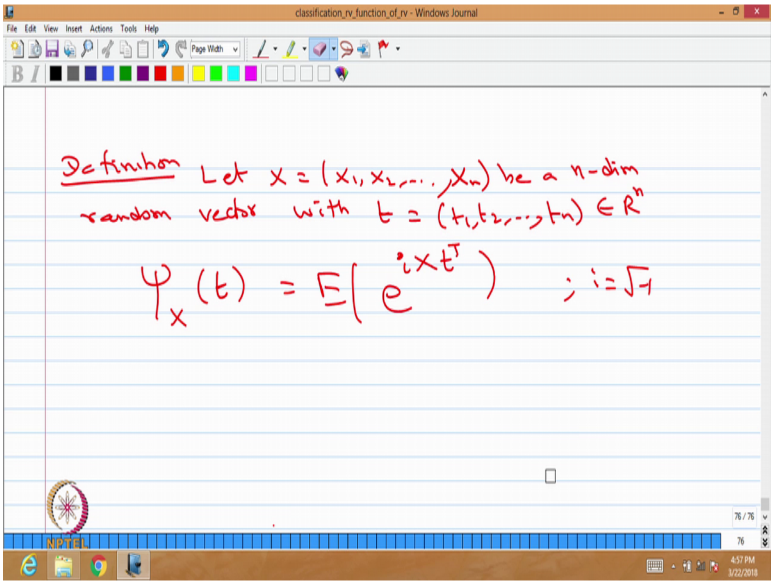


Here also we can go for suppose these random variables are independent; suppose these X i's are independent random variables. Then it is going to be same as the moment generating function of a vector as a function of vector, that is going to be product of i is equal to 1 to n, the moment generating function of individual random variable with the variable t i; t is not a vector t is a element when this random variables are mutually independent.

Similarly, one can go for sum of random variables; similarly one can go for suppose I want to find out the MGF of sum of random variables has a function of t; here the t is a variable not the vector, but I am making the assumption X i's are independent. Again this is going to be if you substitute in the definition and do the simplification, you will get product of MGF's of individual random variables with the same variability. Suppose I will make a additional condition; suppose X i's or iid random variables; that means, independent and identically distributed random variables. In that case, the moment generating function of sum of random variables for the identical distributed random variable the MGF is also going to be identical 0.

Therefore, you find out the MGF of any one random variable as a function of t; power n because all the random variables are identical.

(Refer Slide Time: 13:31)



Now we will move to the third definition that is the joint characteristic function of a random vector. Again the same way let X be a random vector whose elements are X 1, X 2, X n be a n dimensional random vector with the t is a vector whose elements are t 1 comma t 2 and so, on tn that is belonging to R n. You can define the joint characteristic function of a random vector capital X denoted by psi suffix, psi X is a vector has a function of t; t is also vector that is same as expectation of exponential i times vector X, vector t transpose where i is the square root of minus 1.

Here you do not need to any provided condition similar to the characteristic function of a single dimension random variable. So, this is a characteristic function of a; this is a joint characteristic function of a random vector. So, whatever we discussed the results on independent iid random variables and so, on everything will be satisfied for here also. Therefore, no need to do the simple results what we have done in further moments in getting function because it is just replacing t by i times t; therefore, we leave it as it is with the definition.

In the later, when we are solving the problems we are going to use the joint probability generating function of random vector. Similarly we will be using joint moment generating function of a random vector and also joint characteristic function of a random vector; therefore, here I stop the stop it with the definitions and the simple results.