Introduction to Probability Theory and Stochastic Processes
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Lecture – 31

Now, we are going to give a few important results some important results on distributions of functions of several variables.

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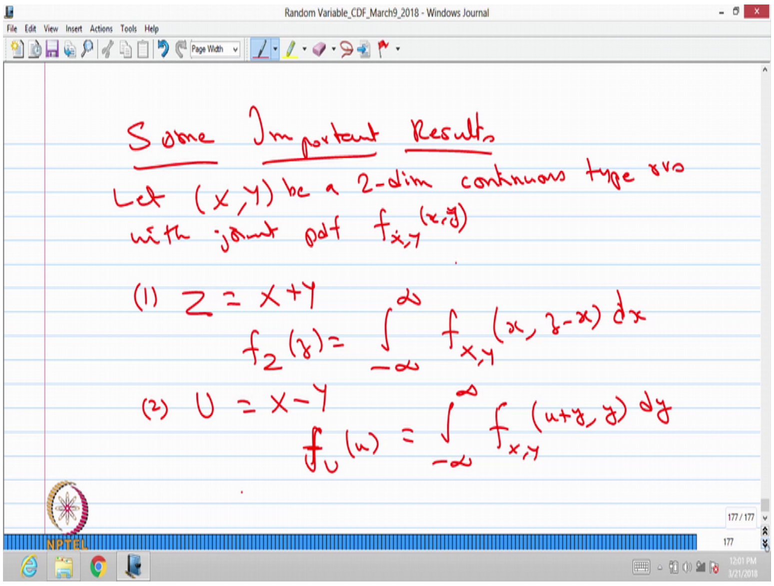

The first one let me start with a continuous type random variable let x comma y be a two dimension continuous type random variables with joint probability density function is given or is known the first result. Suppose I make a random variables x as x plus y we can find the distribution of Z directly that is the probability density function of a Z is going to be integration from minus infinity to infinity.

The joint probability density function of x comma y by replacing x by x whereas, y by Z minus x, so this is going to be the probability density function of Z when Z is x plus y and x and y are two dimensional continuous type random variable with a joint probability density function small f.

The second result suppose I have a another random variable U that is nothing, but the difference of two random variables x comma y. Then the probability density function of U as a function of u that is going to be integration from minus infinity to infinity. The joint probability density function of x comma y by replacing x by u plus y and keep the other variable y as that it is integration with respect to y. This is for the difference of two random variables by knowing the joint probability density function of x and y you can directly get the probability density function of u.

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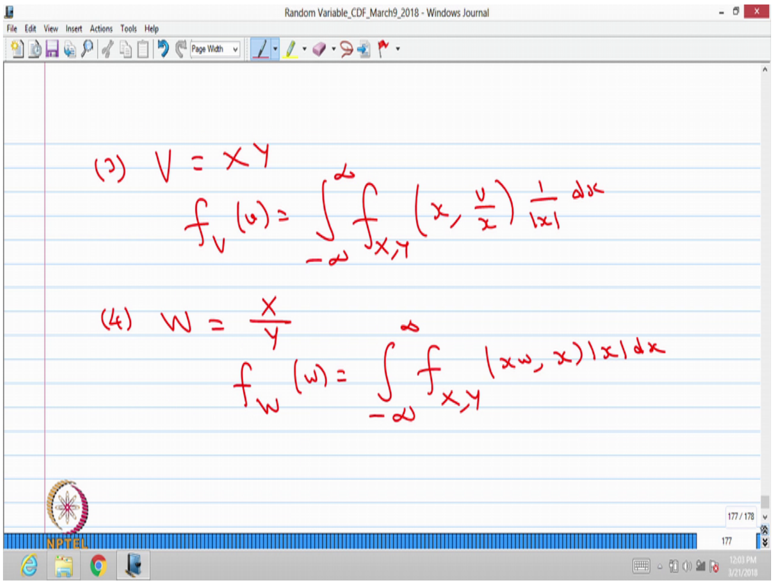

Similarly the third result that is the V is a new random variable that is product of two random variables. You can get the probability density function of a v by integrating the joint probability density function by substituting x as x as it is. The u by v divided by x multiplied by 1 divided by absolute of x d x, this will give the probability density function of v.

Similarly, in the last result the random variable W is defined as x divided by y then you can get the probability density function of w by the similar manner, x absolute of x d x. You see in all this 4 examples we started with the two dimensional continuous type random variable and we define one random variable z or u or v or w.

You can get the probability density function of z or u or v or w by using the previous theorem which I have explained you. The only difference is to apply the theorem you need n dimensional random variable of transformation into another dimension random variable. I have explained the theorem with only two dimensional random variable, the same concept can be extended to n dimension random variables.

So, here we started with the two dimensional random variable we are finding the probability density function of a random variable Z. You can apply the previous theorem by introducing one dummy random variable dummy means you can create a any random variable along with Z is x plus y such a way that a both the assumptions are satisfied and the Jacobian is a non zero. Then find the joint probability density function of Z with the dummy random variable

After that from the joint probability density function you can always able to get the probability density function of any one random variable using that concept you can get the probability density function of Z. That means, we are applying the theorem by suitable inclusion of a one dummy random variable getting the joint distribution then getting the original distribution from the joint distribution. So, that is what a this result says the probability density function of Z is a integration minus infinity to infinity. The joint probability density function of x comma y by replacing a x by x and replacing y by Z minus x.

One more remark over a this results I have not made the assumptions of this random variables are independent or not. If this random variables are independent random variables then you can replace the right hand side with joint probability density function by product of probability density functions.

Even though I have given results for in general if the random variables are mutually independent random variables you can replace the joint probability density function by product of probability density functions. So, these four results you can remember and you can use it whenever you want to go for finding the sum, or difference, or multiplication, or division of two random variables.